

Monday		
13:30-14:00		Welcome - Room TD124
14:00-14:20	Tuomo Lehtilä	Partition strategies for the Maker-Breaker domination game
14:20-14:40	Nacim Oijid	Parameterized complexity of Maker-Breaker Domination game
14:40-15:00	Paul Hametner	The Minimum Degree Question for the Maker-Breaker domination game
15:00-15:20	Jiayue Qi	Which creatures have the MBD number equal to the domination number?
15:20-15:50		Coffee Break
15:50-16:10	Yannick Mogge	Speed and size of dominating sets in domination games
16:10-16:30	Arthur Dumas	Solving the P_5 -game on forests
16:30-17:30		Open problem session

Tuesday		
09:15-09:30		Welcome - Room TD130
09:30-09:50	Miloš Stojaković	Generalised Turán Game
09:50-10:10	Florian Galliot	Structure and tractability results for the Avoider-Enforcer game
10:10-10:30	Valentin Gledel	On the complexity of Waiter-Client and Client-Waiter
10:30-10:50	Éric Duchêne	Poset positional games
10:50-11:20		Coffee Break
11:20-11:40	Aleksa Džuklevski	On Erdős-Szekeres Maker-Breaker Games
11:40-12:00	Antoine Dailly	The Closed Geodetic Game: algorithms and strategies
12:00-12:20	Hiroataka Ono	Colored Node Kayles: Algorithms and Computational Complexity
12:30-14:00		Lunch Break
14:00-18:30		Work session - Rooms TD124, TD126, TD130
19:00		Games and dinner at "Le jardin d'Aline"

Wednesday		
09:15-09:30		Welcome - Room TD130
09:30-09:50	Svenja Huntemann	Degrees are Useless in SNORT When Measuring Temperature
09:50-10:10	Danijela Popović	A game that mimics any misère game
10:10-10:30	Ethan Saunders	Hex Realizability of Game Values Over Posets
10:30-10:50	Alfie Davies	The Joy of Herding Cats
10:50-11:20		Coffee Break
11:20-11:40	Harmender Gahlawat	Romeo and Juliet is EXPTIME-complete
11:40-12:00	Owen Crabtree	Complexity of Jelly-No and Hanano games with various constraints
12:00-12:20	Mirjana Mikalački	The burning game
12:30-14:00		Lunch Break
14:00-18:30		Work session - Rooms TD124, TD126, TD130

Thursday		
09:15-09:30		Welcome - Room TD130
09:30-09:50	Bojan Bašić	A meeting of prisoners wearing hats and foundations of mathematics
09:50-10:10	Strahinja Gvozdić	Hat guessing game on graphs
10:10-12:20		Work session
12:30-14:00		Lunch Break

Tuomo Lehtilä – Partition strategies for the Maker-Breaker domination game

The Maker-Breaker domination game is a positional game played on a graph by two players called Dominator and Staller. The players alternately select a vertex of the graph that has not yet been chosen. Dominator wins if at some point the vertices she has chosen form a dominating set of the graph. Staller wins if Dominator cannot form a dominating set. We give results on the Maker-Breaker domination game, when Staller begins, on some graph classes by applying different graph partitions.

The talk is based on joint work “Partition strategies for the Maker-Breaker domination game” with Guillaume Bagan, Eric Duchêne, Valentin Gledel, Tuomo Lehtilä, and Aline Parreau. <https://arxiv.org/abs/2406.15165>

Nacim Oijid – Parameterized complexity of Maker-Breaker Domination game

The Maker-Breaker domination game is one of the most studied positional games on vertices. In its first study, it was shown that computing the outcome of the game is PSPACE-complete on bipartite and split graphs, but polynomial on trees and cographs. We initiate here the study of its complexity. We start with the number of moves, the most common parameter in combinatorial game theory. We prove that in a general Maker-Breaker positional game, determining whether Breaker can claim a transversal in k moves is $W[2]$ -complete, in contrast to the problem of determining whether Maker can claim a hyperedge in k moves, which is known to be $W[1]$ -complete since 2017. These two hardness results are then applied to the Maker-Breaker domination game, proving its $W[2]$ -completeness when Dominator must dominate the graph in k moves and its $W[1]$ -completeness when Staller must isolate a vertex in k moves. Next, in the search for the bounds between graph classes in which the problem is hard, and the graph classes in which it is tractable, we study several graph parameters which are related to the distance between a graph and graph classes in which the problem is known to be solvable in polynomial time, namely trees or cographs. We provide FPT algorithms parameterized by the neighborhood diversity, the modular width, the P_4 -freeness, the distance to cluster and the feedback edge number.

Paul Hametner – The Minimum Degree Question for the Maker-Breaker domination game

The Maker-Breaker domination game is a two player positional game played on a graph G . In each of their turns the players Staller and Dominator claim one not yet claimed vertex until all the vertices of G are claimed. If at the end Dominator completely occupies a dominating set she wins, otherwise Staller wins. For a positive integer d define $\beta(d)$ to be the smallest integer n for which there exists a graph G on n vertices with minimum degree d on which Staller wins going second. Bounding $\beta(d)$ from below and in particular determining $\beta(3)$ is an ongoing effort by Oliver Roche-Newton and myself. In this talk I present some of our developed methods and a selection of intermediate results.

Joint work with Oliver Roche-Newton.

Jiayue Qi – Which creatures have the MBD number equal to the domination number?

We describe the graph characterization for those having $\Gamma_{MB} = \Gamma$; try to proceed with at least one direction of the proof if time permits.

Yannick Mogge – Speed and size of dominating sets in domination games

We consider Maker-Breaker domination games, a variety of positional games, in which two players (Dominator and Staller) alternately claim vertices of a given graph. Dominator’s goal is to fully claim all vertices of a dominating set, while Staller tries to prevent Dominator from doing so, or at least tries to delay Dominator’s win for as long as possible.

We prove a variety of results about domination games, including the number of turns Dominator needs to win and the size of a smallest dominating set that Dominator can occupy, when considering e.g. random graphs, powers of paths, and trees. We could also show that speed and size can be far apart, and we prove further non-intuitive statements about the general behaviour of such games.

We also consider the Waiter-Client version of such games.

Joint work with Ali Deniz Bagdas, Dennis Clemens and Fabian Hamann.

Arthur Dumas – Solving the P_5 -game on forests

Positional games are two-player games on hypergraphs introduced by Erdős and Selfridge in 1973. Within this family, the H -game, in its Maker-Breaker version, is played from a graph G and a target graph H . Starting with Maker, the players take turns in claiming an unclaimed edge of G . Maker wins if she succeeds in claiming a set of edges that contains a copy of H and Breaker wins if Maker fails. In Maker-Breaker positional games, there is always a winning strategy for exactly one of the two players. The objective is then to solve the problem which takes as input two graphs G and H and gives the winner of the H -game on G , and potentially to set out a winning strategy.

Recently, Galliot presented a general result on Maker-Breaker positional games, a consequence of which is the resolution in polynomial time of the H -game when H is a graph with three edges or less. When H is a graph with four edges, it was also recently shown that the $K_{1,4}$ -game played on forests is polynomial. Our work follows this result, by studying the case where H is a chain with four edges (i.e. a P_5) and G is a forest.

Theorem 1. *There is an algorithm that computes the outcome of the P_5 -game on a forest F , in linear time in the size of F .*

We also give an explicit winning strategy for the winner.

Joint work with Eric Duchêne, Mathieu Hilaire, and Aline Parreau.

Miloš Stojaković – Generalised Turán Game

We study a game motivated by the Generalised Turán Problem. Let H and F be graphs, and n an integer. Two players, Constructor and Blocker, alternately claim unclaimed edges of the complete graph on n vertices. Constructor is not allowed to claim a copy of F , and his goal is to maximise the number of claimed copies of H . Blocker plays without restriction, and his goal is to minimise the number of copies of H Constructor claimed. When both players play optimally the score of the game, denoted by $gc(n, H, F)$, is the number of copies of H Constructor claimed.

We look for the game score for several natural choices of H and F .

This is joint work with Máté Vizer and Balázs Patkós.

Florian Galliot – Structure and tractability results for the Avoider-Enforcer game

Avoider-Enforcer is the “weak” convention of avoidance positional games. Avoider and Enforcer take turns claiming vertices of the hypergraph: Avoider loses if she claims all the vertices of some edge, otherwise she wins. No results were known on the algorithmic complexity of Avoider-Enforcer until 2023, when Gledel and Oijid established PSPACE-completeness for hypergraphs of rank 6. On the opposite end of the spectrum, we establish some tractability results for smaller rank. For hypergraphs of rank 2, we obtain a structural characterization of the outcome, which

can be computed in polynomial time as a consequence. For hypergraphs of rank 3 that are linear (meaning any two distinct edges intersect on 0 or 1 vertex), we manage the same when Avoider is the last player, but we only get partial results when Enforcer is the last player. A useful tool in these studies is our characterization of the outcome of a disjoint union.

Joint work with Valentin Gledel and Aline Parreau.

Valentin Gledel – On the complexity of Waiter-Client and Client-Waiter

Client-Waiter and Waiter-Client are asymmetrical positional games played on hypergraphs. At each turn, one of the players, Waiter, proposes two vertices to the other player, Client. Client chooses one of them and the other is discarded. The difference between Client-Waiter and Waiter-Client lies in whether Client wins or loses if he fills a hyperedge with his chosen vertices.

In this talk, we will first quickly describe the proof techniques used to show that Client-Waiter is PSPACE-complete for hypergraphs of rank 6 and more, and in P^{NP} for hypergraphs of rank 3. Then, we will explain how sunflowers can be used to prove that Waiter-Client is FPT parametrized by the rank of the hypergraph.

Éric Duchêne – Poset positional games

We propose a generalization of positional games with a restriction on the order in which the elements of the board are allowed to be claimed. They include in particular games like Connect Four.

Such games are called poset positional games, which are positional games with an additional structure – a poset on the elements of the board. Throughout the game play, based on this poset and the set of the board elements that are claimed up to that point, we reduce the set of available moves for the player whose turn it is – an element of the board can only be claimed if all the smaller elements in the poset are already claimed.

In this talk we will provide of a first analysis of these games, with a prime focus on the most studied convention, the Maker-Breaker games. Some general results about poset positional games will be given, as well as complexity results for particular posets, according to their width, height, or the size of the winning sets.

Joint work with Guillaume Bagan, Eric Duchêne, Florian Galliot, Valentin Gledel, Mirjana Mikalački, Nacim Ojjid, Aline Parreau and Miloš Stojaković.

Aleksa Džuklevski – On Erdős-Szekeres Maker-Breaker Games

The Erdős-Szekeres Maker-Breaker game is a two-player game where both players alternately place points in the plane such that no three points are colinear. The first player (Maker) starts the game by placing her point and wants to obtain an empty convex polygon of a given size k such that the vertices of the polygon are chosen from these points and the second player (Breaker) wants to prevent it. Kumar Das and Valla showed that for $k \leq 8$ Maker has the winning strategy. We improve this result by showing that Maker has a winning strategy for each $k \in \mathbb{N}$. We also consider a biased game in which we allow the Breaker to have the advantage of placing s points in each turn and show that the Maker can still win for each k and $s \in \mathbb{R}$. One version of the game that has been considered in the literature is the bichromatic version, in which the points placed by the players are of two different colors and Maker wants to obtain a described polygon with the additional requirement that all its vertices are of her color, i.e., that they are points placed by her. In the bichromatic version we show that Maker still wins for all k as long as the bias of Breaker is less than 2 (which was only known for $k \leq 5$ and bias 1), and show that for $k \geq 8$ and a bias of at least 12, the Breaker wins.

This is a joint work with Alexey Pokrovskiy, Csaba D. Tóth, Tomáš Valla and Lander Verlinde.

Antoine Dailly – The Closed Geodetic Game: algorithms and strategies

The *geodetic closure* of a set S of vertices of a graph is the set of all vertices in shortest paths between pairs of vertices of S . A set S of vertices in a graph is *geodetic* if its geodetic closure contains all the vertices of the graph. Buckley introduced in 1984 the idea of a game where two players construct together a geodetic set by alternately selecting vertices, the game ending when all vertices are in the geodetic closure. The GEODETIC GAME was then studied in 1985 by Buckley and Harary, and allowed players to select vertices already in the geodetic closure of the current set. We study the more natural variant, also introduced in 1985 by Buckley and Harary and called the CLOSED GEODETIC GAME, where the players alternate adding to a set S vertices that are not in the geodetic closure of S , until no move is available. This variant was only studied ever since for trees by Araujo *et al.* in 2024. We provide a full characterization of the Sprague-Grundy values of graph classes such as paths and cycles, of the outcomes of several products of graphs in function of the outcomes of the two graphs, and give polynomial-time algorithms to determine the Sprague-Grundy values of cactus and block graphs.

Joint work with Harmender Gahlawat and Zin Mar Myint.

Hirota Ono – Colored Node Kayles: Algorithms and Computational Complexity

Colored Node Kayles is a combinatorial game played on an undirected vertex-colored graph $G = (V, E)$, where each vertex $v \in V$ is assigned a color from *black, gray, white*. Colored Node Kayles is the game that involves two players: Black and White, who alternate turns. On their turn, Black selects a gray or black vertex, while White selects a gray or white vertex. The selected vertex and all its neighbors are subsequently removed from the graph G . The game continues until no valid moves remain, with the last player able to move declared the winner. The game Node Kayles, a special case of Colored Node Kayles played on graphs consisting solely of gray vertices, has been extensively studied. Due to its simplicity and generality, results for Node Kayles have been applied to a broad range of other combinatorial games, though the partisan variant Colored Node Kayles has received far less attention. A restricted version of Colored Node Kayles, referred to as Bigraph Node Kayles, was implicitly introduced by Schaefer in his seminal 1978 work, where one side of a bipartite graph is colored black and the other side white. In this work, we formally define Colored Node Kayles and study the computational complexity of determining the winner on a given graph. We prove that deciding the outcome of Colored Node Kayles is PSPACE-complete, even when restricted to planar graphs with maximum degree 3. Furthermore, we show the W[1]-hardness of Colored Node Kayles concerning the number of turns, and we present several other hardness results, which include results on computing game values in combinatorial game theory. On the positive side, we demonstrate that Colored Node Kayles is fixed-parameter tractable (FPT) for certain structural parameters of the graph. This is a joint work with Tesshu Hanaka and Kanae Yoshiwatari.

Svenja Huntemann – Degrees are Useless in Snort When Measuring Temperature

SNORT is a colouring game played on any simple graph where Left colours vertices blue and Right red such that opposite colours are not adjacent. The temperature of SNORT, intuitively the urgency of going first, is known to be unbounded in general. We show that further the difference between the temperature and the degree of the graph is also unbounded. This is joint work with Tomasz Maciosowski.

Danijela Popović – A game that mimics any misère game

In this talk we will define Hackenforb, an impartial game in which players alternately remove an edge from a given graph, and whenever a connected component appears that belongs to the set of forbidden graphs given in advance, that whole component is erased in the same move. The

game ends when no more edges are left. We will show that, for any impartial game where each player has the possibility to end the game in each move (in effect, to resign whenever he wants), there exists an instance of Hackenforb that emulates it.

Joint work with Bojan Bašić.

Ethan Saunders – Hex Realizability of Game Values Over Posets

We analyze the game of Hex using methods from combinatorial game theory. Each position in Hex can be assigned a game value over some poset. A game value V is called Hex realizable if there is some Hex position whose value is V . Similarly, a value V over some poset is called Shannon realizable if there is some Shannon game whose value is V . I will present some results on the Hex realizability and Shannon realizability of game values over posets.

Joint work with Peter Selinger.

Alfie Davies – The Joy of Herding Cats

The game of NOWHERE TO GO is a combinatorial game on a graph where each player takes turns moving their token and deleting an edge from the graph, trying to isolate their opponent. We show a decomposition of this game into a conjunctive sum of CAT HERDING, introduced by Rylo Ashmore. Even though the latter game is loopy, some of the game theoretic values are curious; in particular, we find a number of graphs equal to numbers.

Harmender Gahlawat – Romeo and Juliet is EXPTIME-complete

Romeo and Juliet is a two player Rendezvous game played on graphs where one player controls two agents, *Romeo* and *Juliet*, who aim to meet at a vertex against k adversaries, called *dividers*, controlled by the other player. The optimization in this game lies in deciding the minimum number of dividers sufficient to restrict Romeo and Juliet from meeting in a graph, called the *dynamic separation number*. We establish that this game is EXPTIME-complete, settling a conjecture of Fomin, Golovach, and Thilikos [Inf. and Comp., 2023] positively.

Owen Crabtree – Complexity of Jelly-No and Hanano games with various constraints

This work shows new results on the complexity of puzzle games Jelly-No and Hanano with various constraints on the size of the board and number of colours.

Hanano and Jelly-No are one-player, 2D side-view puzzle games with a dynamic board consisting of coloured, movable blocks disposed on platforms. These blocks can be moved by the player and are subject to gravity. The goal of both games is to move the coloured blocks in order to reach a specific configuration and make them interact with each other or with other elements.

Jelly-No was proven by Chao Yang to be NP-Complete under the constraint that all movable blocks are the same colour, whereas Hanano was proven by Michael C. Chavrimootoo to be PSPACE-Complete under this constraint. However, the question of the complexity of Jelly-No with more than one colour was left open.

In this work, we settle this question, proving that Jelly-No is PSPACE-Complete with an unbounded number of colours. We further show that, if we allow black jellies (that is, jellies that do not need to be merged), the game is PSPACE-complete even for one colour. We further show that one-colour Jelly-No and Hanano remain NP-Hard even if the width or the height of the board are small constants.

Keywords: Combinatorial games ; Complexity ; Hanano Puzzle ; Jelly-No Puzzle ; Motion planning ; NP-Hard ; PSPACE-Complete.

Joint work with Valia Mitsou.

Mirjana Mikalački – The burning game

Motivated by the processes of graph burning and graph cooling, we introduce the new graph game, *the burning game*, played by two players, *Burner* and *Staller*. Given a graph, vertices can be burned or unburned, but once burned, they stay in this state until the end of the game. The players alternate in choosing the previously unburned vertices of a given graph, with different aims: Burner wants to burn the graph as quickly as possible, and Staller wants to prolong this process as much as possible.

At time step $t = 0$, all vertices are unburned. In each time step $t \geq 1$, first all neighbours of burned vertices become burned, and then one of the players burns one unburned vertex in this time step as well. The game ends in the first time step t in which all vertices of G are burned. We analyse the burning number, i.e. the minimum number of steps required for the graph to burn, give some basic properties and characterizations.

This is joint work with Nina Chiarelli, Vesna Iršič, Marko Jakovac and Bill Kinnersley.

Bojan Bašić – A meeting of prisoners wearing hats and foundations of mathematics

The topic of this talk evolved from an innocent-looking game featuring prisoners and a warden, where each prisoner wears a hat with an integer number written on it. The question whether the prisoners can devise a strategy to win against the warden has an easy (affirmative) solution, confirming that this game is indeed as innocent as it looks like. We then analyze what happens if numbers on the hat can be any real numbers instead of integers. With this modification, the game still looks equally as innocent, but it turns out that answering the question whether the prisoners still can devise a strategy uncovers a firm link between this topic and roots of mathematics itself.

Strahinja Gvozdić – Hat guessing game on graphs

Given a graph G on n vertices, we consider the following game: each vertex is occupied by one person wearing a hat in one of q colours. Each person can see only the hat colours of the people in their neighbourhood, and cannot see their own hat colour. When the sign is given, all the players try to guess the colour of their hat and if at least one of them guessed correctly, we say that the players won the game. The players can agree on the strategy before the hats are placed on their heads, however they are not allowed to communicate after this. For a given graph G , by $\text{HG}(G)$ we denote the maximum number q of hat colours for which there is a winning strategy. We will present some known results and open questions about the HG number, as well as introduce different versions of the game.