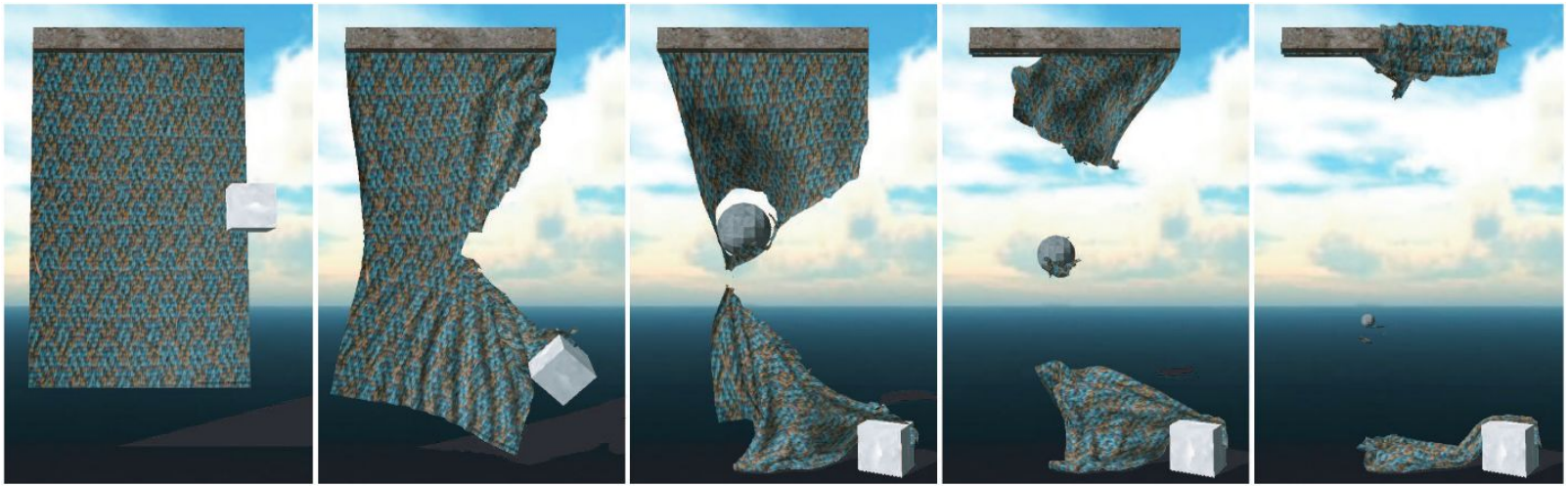
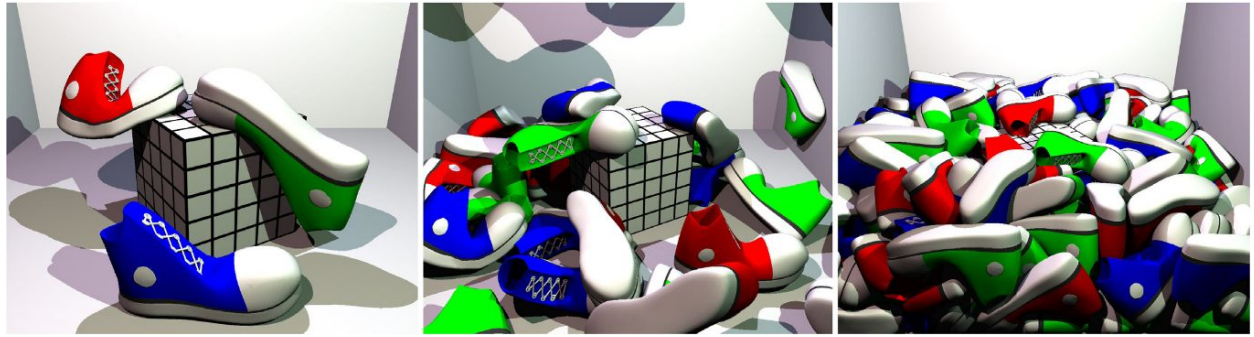
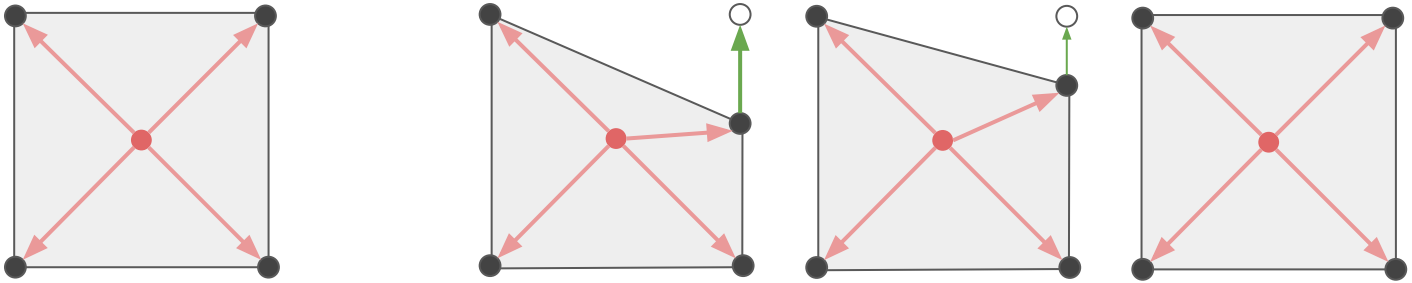
Position Based Dynamics (2006)

Matthias Müller, Bruno Heidelberger, Marcus Hennix, John Ratcliff



Meshless Deformations Based on Shape Matching (2005)



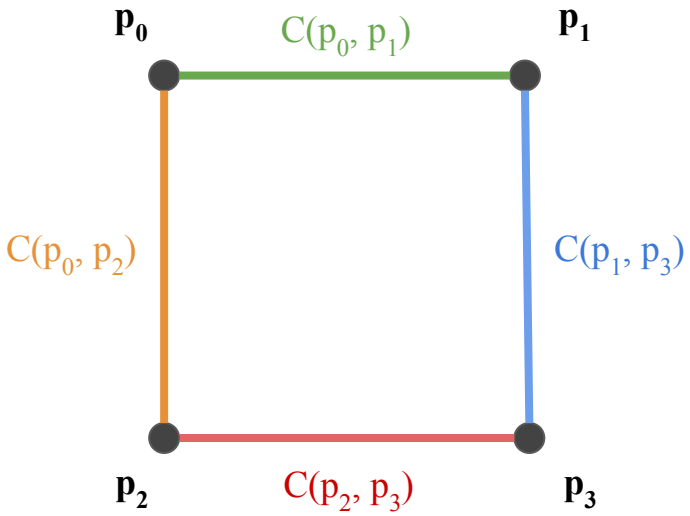


stiffness : **α** [0,1] 0 → no stiff 1 → rigid body

Position Based Dynamics (2006)

Particle/Vertex i $\in$ [1, . . . , N]
position : xi
velocity : vi
mass : mi

Contraints j $\in$ [1, . . . , M]
cardinality : n
indices : $\{i_1, \ldots i_{n_j}\}, i_k \in [1, \ldots N]$
fonction : $C_j(\mathbf{x}_{i_1}, \ldots, \mathbf{x}_{i_{n_j}}) \rightarrow \mathbb{R}$
stiffness : k $\in$ [0,1]
condition : *equality (C = 0)* or *inequality (C ≥ 0)*



PBD Solver

```
While(True)
    forall vertices i do v_i ← v_i + Δt w_i f_ext

    dampVelocities(v_1, . . . , v_N )

    forall vertices i do p_i ← x_i + Δt v_i

    forall vertices i do generateCollisionConstraints(x_i → p_i)

    loop solverIterations times
        projectConstraints(C_1, . . . ,C_M+Mcoll , p_1, . . . , p_N )
    endloop

    forall vertices i
        v_i ← (p_i − x_i)/Δt
        x_i ← p_i
    endfor
    velocityUpdate(v_1, . . . , v_N )

endloop
```

Compute external interactions

Damp velocities

Compute position

Check collision and create constraints

Project particles to valid positions

Compute new velocity

Set projected position

Fix broken velocities

PBD Solver

While(True)

forall vertices i do $v_i \leftarrow v_i + \Delta t\, w_i\, f_{ext}$

dampVelocities($v_1, \ldots, v_N$)

forall vertices i do $p_i \leftarrow x_i + \Delta t\, v_i$

forall vertices i do generateCollisionConstraints($x_i \rightarrow p_i$)

loop solverIterations times

projectConstraints($C_1, \ldots, C_{M+Mcoll}, p_1, \ldots, p_N$)

endloop

forall vertices i

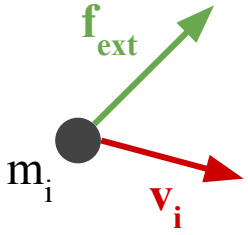
$v_i \leftarrow (p_i - x_i)/\Delta t$

$x_i \leftarrow p_i$

endfor

velocityUpdate($v_1, \ldots, v_N$)

endloop



$w_i = 1 / m_i \quad a_i = f_{ext}\, w_i$

PBD Solver

While(True)

forall vertices i do $v_i \leftarrow v_i + \Delta t\, w_i\, f_{ext}$

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forall vertices i

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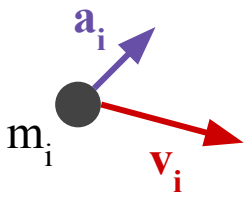
$x_i \leftarrow p_i$

endfor

velocityUpdate($v_1, \ldots, v_N$)

endloop



$w_i = 1 / m_i \qquad a_i = f_{ext}\, w_i$

$v_i = v_i + \Delta t\, a_i$

PBD Solver

While(True)

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forall vertices i

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$x_i \leftarrow p_i$

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velocityUpdate($v_1, \ldots, v_N$)

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forall vertices i

$v_i \leftarrow (p_i - x_i)/\Delta t$

$x_i \leftarrow p_i$

endfor

velocityUpdate($v_1, \ldots, v_N$)

endloop

$v_i = v_i + ???$

$v_i = v_i - k_{damp} v_i$ $\quad k_{damp} \in [0,1]$

PBD Solver

While(True)

forall vertices i do $v_i \leftarrow v_i + \Delta t\, w_i\, f_{ext}$

dampVelocities($v_1, \ldots, v_N$)

forall vertices i do $p_i \leftarrow x_i + \Delta t\, v_i$

forall vertices i do generateCollisionConstraints($x_i \rightarrow p_i$)

loop solverIterations times

projectConstraints($C_1, \ldots, C_{M+Mcoll}, p_1, \ldots, p_N$)

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forall vertices i

$v_i \leftarrow (p_i - x_i)/\Delta t$

$x_i \leftarrow p_i$

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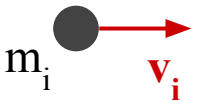
velocityUpdate($v_1, \ldots, v_N$)

endloop



$v_i = v_i + ???$

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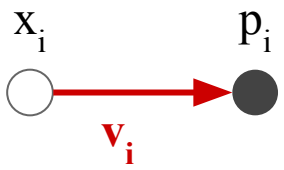

PBD Solver

While(True)

forall vertices i do $v_i \leftarrow v_i + \Delta t\, w_i\, f_{ext}$

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forall vertices i do $p_i \leftarrow x_i + \Delta t\, v_i$

forall vertices i do generateCollisionConstraints($x_i \rightarrow p_i$)

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forall vertices i

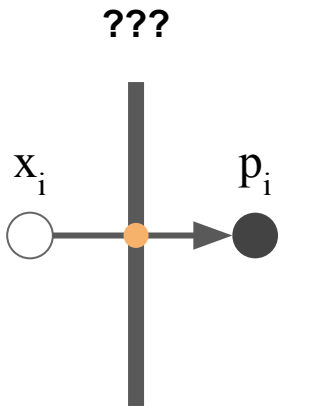
$v_i \leftarrow (p_i - x_i)/\Delta t$

$x_i \leftarrow p_i$

endfor

velocityUpdate($v_1, \ldots, v_N$)

endloop



PBD Solver

While(True)

forall vertices i do $v_i \leftarrow v_i + \Delta t\, w_i\, f_{ext}$

dampVelocities($v_1, \ldots, v_N$)

forall vertices i do $p_i \leftarrow x_i + \Delta t\, v_i$

forall vertices i do generateCollisionConstraints($x_i \rightarrow p_i$)

loop solverIterations times

projectConstraints($C_1, \ldots, C_{M+Mcoll}, p_1, \ldots, p_N$)

endloop

forall vertices i

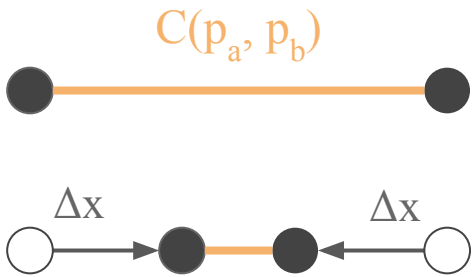
$v_i \leftarrow (p_i - x_i)/\Delta t$

$x_i \leftarrow p_i$

endfor

velocityUpdate($v_1, \ldots, v_N$)

endloop



PBD Solver

While(True)

forall vertices i do $v_i \leftarrow v_i + \Delta t\, w_i\, f_{ext}$

dampVelocities($v_1, \ldots, v_N$)

forall vertices i do $p_i \leftarrow x_i + \Delta t\, v_i$

forall vertices i do generateCollisionConstraints($x_i \rightarrow p_i$)

loop solverIterations times

projectConstraints($C_1, \ldots, C_{M+Mcoll}, p_1, \ldots, p_N$)

endloop

forall vertices i

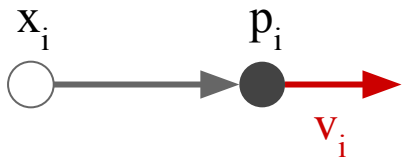
$v_i \leftarrow (p_i - x_i)/\Delta t$

$x_i \leftarrow p_i$

endfor

velocityUpdate($v_1, \ldots, v_N$) ← frictions, collisions and stuff

endloop



PBD Solver

While(True)

forall vertices i do $v_i \leftarrow v_i + \Delta t\, w_i\, f_{ext}$	Compute external interactions
dampVelocities($v_1, \ldots, v_N$)	Damp velocities
forall vertices i do $p_i \leftarrow x_i + \Delta t\, v_i$	Compute position
forall vertices i do generateCollisionConstraints($x_i \rightarrow p_i$)	Check collision and create constraints
loop solverIterations times projectConstraints($C_1, \ldots, C_{M+Mcoll}, p_1, \ldots, p_N$) endloop	Project particles to valid positions
forall vertices i $v_i \leftarrow (p_i - x_i)/\Delta t$ $x_i \leftarrow p_i$ endfor	Compute new velocity Set projected position
velocityUpdate($v_1, \ldots, v_N$)	Fix broken velocities

endloop

Constraints resolution

$C(p + \Delta p) = 0 \qquad \Delta p = ???$

Taylor order 1 :

$f(x) \approx f(a)+f'(a)(x-a)$

$C(x) \approx C(p)+\nabla C(p)(x-p)$

$C(p + \Delta p) \approx C(p) + \nabla C(p) \cdot \Delta p = 0$

$\Delta p = \lambda \nabla C(p)$

$C(p) + \nabla C(p) \cdot (\lambda \nabla C(p)) = 0$

$C(p) + \lambda \|\nabla C(p)\|^2 = 0$

$$\lambda = -\frac{C(p)}{\|\nabla C(p)\|^2}$$

Uniform masses

$$\Delta p_i = -s \nabla_{p_i} C(p_1, \ldots, p_n)$$

$$s = \frac{C(p_1, \ldots, p_n)}{\sum_j \|\nabla_{p_j} C(p_1, \ldots, p_n)\|^2}$$

Different masses

$$\Delta p_i = -s\, w_i \nabla_{p_i} C(p_1, \ldots, p_n)$$

$$s = \frac{C(p_1, \ldots, p_n)}{\sum_j w_j \|\nabla_{p_j} C(p_1, \ldots, p_n)\|^2}$$

Distance Constraint

Distance Constraint

$$C(p_1, p_2) = \| p_1 - p_2 \| - d$$

$$C'(p_1, p_2) = \nabla C(p_1, p_2) = ???$$

$$C(p_1, p_2) = [(p_1 - p_2)^T \cdot (p_1 - p_2)]^{0.5} - d$$

$$\nabla C(p_1, p_2) = \frac{1}{2(\|p_1 - p_2\|^2)^{0.5}} \, 2(p_1 - p_2)$$

$$(a^T.a)' = 2a$$

$$(f \circ g(x))' = f' \circ g(x) \, g'(x)$$

$$(x^n)' = n \, x^{n-1}$$

$$\nabla C(p_1, p_2) = \frac{p_1 - p_2}{\|p_1 - p_2\|}$$

Distance Constraint

$$\Delta p_i = -s\, w_i \nabla_{p_i} C(p_1, \ldots, p_n) \qquad s = \frac{C(p_1, \ldots, p_n)}{\sum_j w_j \|\nabla_{p_j} C(p_1, \ldots, p_n)\|^2}$$

Distance constraint

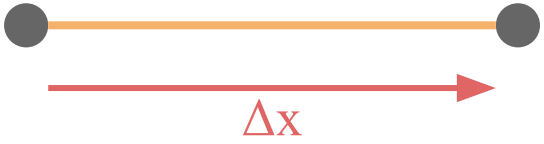
$$C(p_1, p_2) = \| p_1 - p_2 \| - d$$

$$\nabla C(p_1, p_2) = \frac{p_1 - p_2}{\|p_1 - p_2\|}$$

$$\Delta p_1 = -s\, w_1 \frac{p_1 - p_2}{\|p_1 - p_2\|}$$

$$s = \frac{\| p_1 - p_2 \| - d}{w_1 + w_2}$$

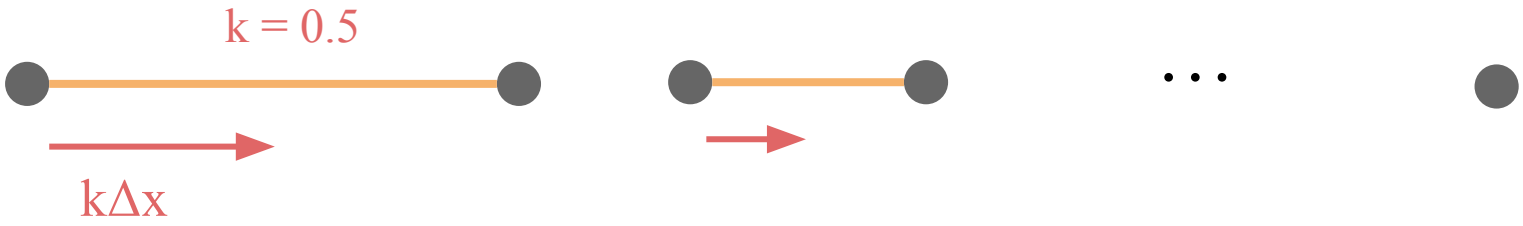
Stiffness $k \in [0, 1]$



Step 1

Step 2

Step N ?



$$k_{fixed} = 1 - (1 - k)^{1/ns}$$

La suite sur le papier :D