Strategy in Geoguessr:
Games of competitive Euclidean geometry

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Geoguessr: what it is ?

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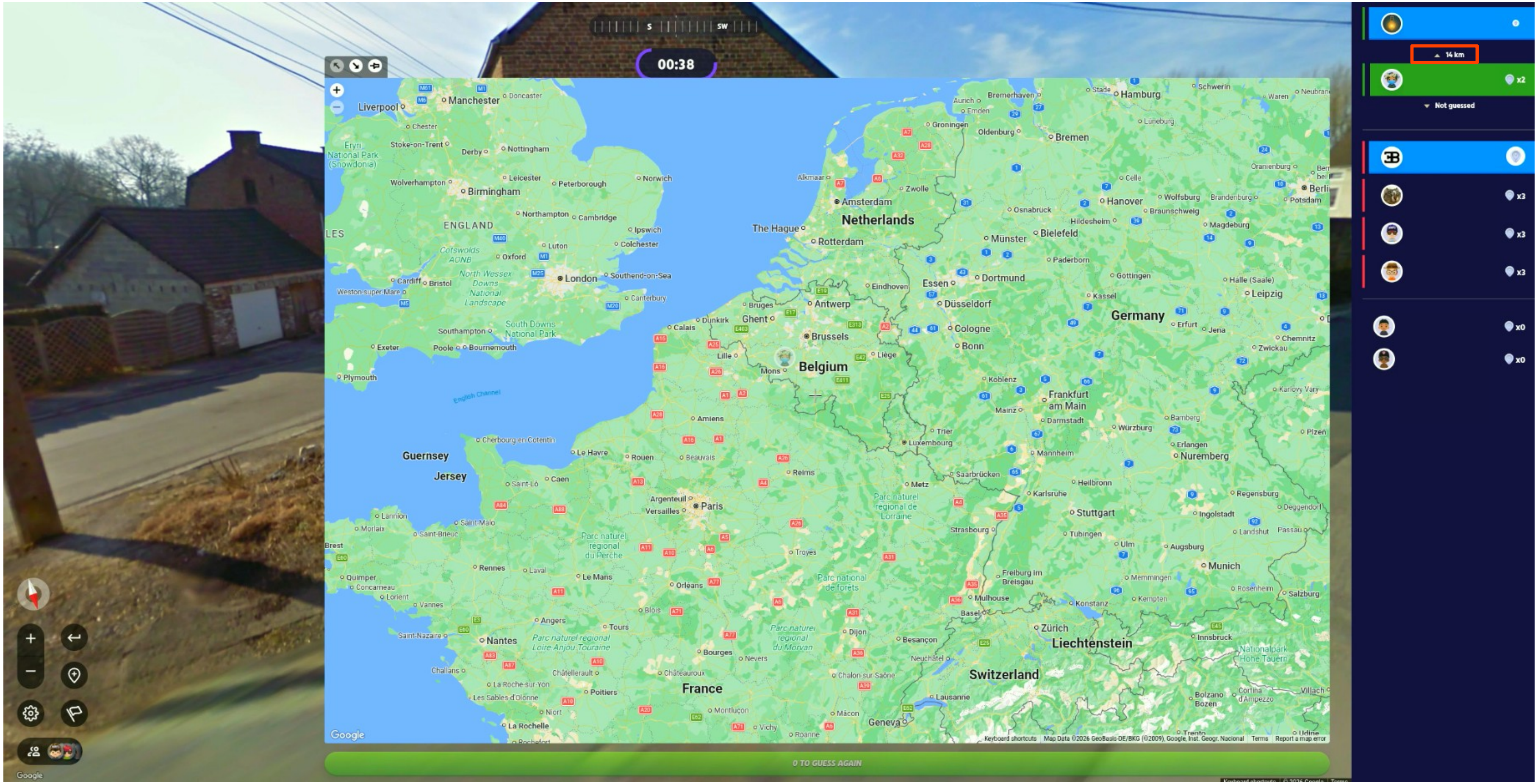


Variant of the game

- Two players A and B
- We suppose the player B already has made all its guesses

Let $S = \{a_1, ..., a_k\}$ be a set of guesses.

$$f(S) = \min_i\{d(a_i, x)\} - d(b, x)$$

Problem 1. Let C, S and f be given. A target x was selected from C, and a guess b ∈ C was made by an adversary. After each of our guesses ak, we can retrieve the value of f ({a1, ..., ak}). The goal is to make a guess an such that f ({a1, ..., an}) ≤ 0. Design an algorithm that returns a set S of guesses {a1, ..., an} ⊆ S such that f (S) ≤ 0 and minimizes n

First points

Claim 1. At least one of a2, a3, a4 is closer to x than a1.

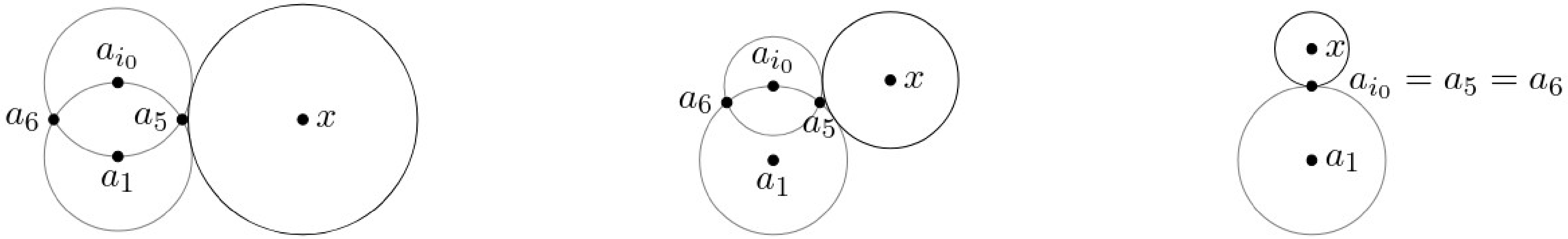
Proof of claim. The Voronoi cell $V_1(\mathbb{R}^2, \{a_1, a_2, a_3, a_4\})$ is contained in $\mathbb{D}(a_1, f(\{a_1\})) \not\ni x$. Since $\bigcup_{i=1}^4 V_i(\mathbb{R}^2, \{a_1, a_2, a_3, a_4\}) = \mathbb{R}^2 \ni x$, the claim falls. $\square$

A5, A6

$$\{a_5, a_6\} = C(a_1, f(\{a_1\})) \cap C(a_{i_0}, f(\{a_1, ..., a_{i_0}\}))$$

Claim 2. At least one of a_5, a_6 is closer to x than a_{i_0}

Proof of claim. We know $0 \leq f(\{a_1, ..., a_{i_0}\}) \leq f(\{a_1\})$

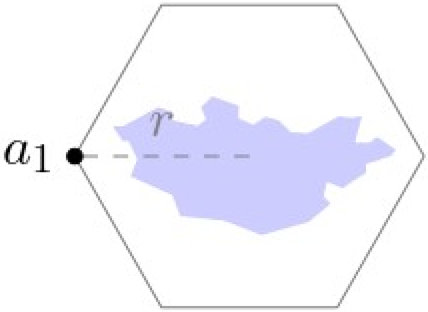


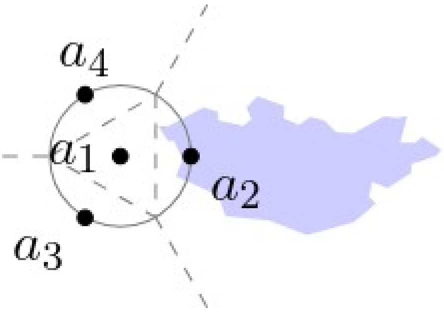
(a) $f(\{a_1, ..., a_{i_0}\}) = f(\{a_1\})$ (b) $0 < f(\{a_1, ..., a_{i_0}\}) < f(\{a_1\})$ (c) $0 = f(\{a_1, ..., a_{i_0}\})$

Last point

CLAIM 3 (Problem of Apollonius). There is a unique ball $B(x,r)$ which is tangent to three given balls $B(c,R)$, $B(c',R')$, $B(c'',R'')$ and that do not intersect them. One can furthermore construct it (in particular finding x).

n <= 5





Expected performance of this strategy is ≤ 4.5

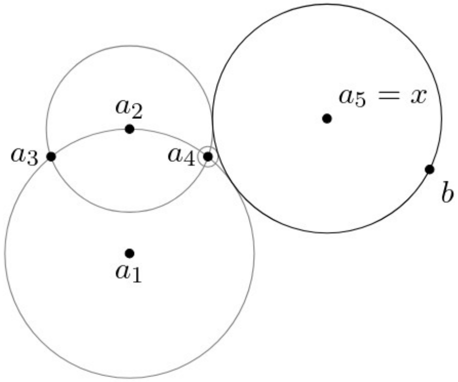


Fig. 4: The **Apollonius** strategy.

Different strategies – Lower expected performance

- Random
- RandCircles $\mathcal{C} - \bigcup_{k=1}^{n} B\big(a_k, f(\{a_1, ..., a_k\})\big)$
- RandVoronoi $V_{i_0}(\mathcal{C}, S)$
- RandVorCir $V_{i_0}(\mathcal{C}, S) - \cup_{a_i \in S} B(a_i, f(\{a_1, ..., a_i\}))$
- Apollonius
- MedoidVorCir
- MedoidVorCirApoll

Strategy	Average n	Largest n
Random	7.58	3526
RandCircles	3.02	336
RandVoronoi	2.69	23
RandVorCir	2.26	13
MedVorCir	1.87	10
MedVorCirApoll	1.80	8
Apollonius7	3.81	7
Apollonius5	3.63	5

Tab. 1: Comparison of strategies. Iterations: $N = 1000$. Country shape: square.

Turn-based version

$$g(S_A, S_B) = \min_i\{d(a_i, x)\} - \min_j\{d(b_j, x)\}.$$

	B.Random	B.RandCircles	B.RandVoronoi	B.RandVorCir	B.MedVorCir
A.Random	−0.02	−0.40	−0.84	−0.84	−0.92
A.RandCircles	0.42	0.12	−0.56	−0.64	−0.78
A.RandVoronoi	0.74	0.68	−0.02	−0.12	−0.34
A.RandVorCir	0.84	0.74	0.06	0.00	−0.30
A.MedVorCir	0.86	0.84	0.30	0.20	0.02

Tab. 2: The expected outcome of a game as described in Problem 2 between Player A's strategy (rows) and Player B's strategy (columns), where 1 is a victory for A and -1 a loss.

Conclusion

- One dimension version
- Spherical region
- Higher dimensions
  - $\mathbb{R}^3$ $4 \cdot 3 + 2 = 14$