Putting Rigid Bodies to Rest

Fig. 11. Our algorithm computes the probability that a given rigid body lands in each possible equilibrium configuration, without performing any dynamical simulation. Here we use it to synthesize a household scene cluttered with toys; for each model we show the most likely configuration.

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Problem formulation

“high-friction scenario, where kinetic energy is rapidly dissipated“

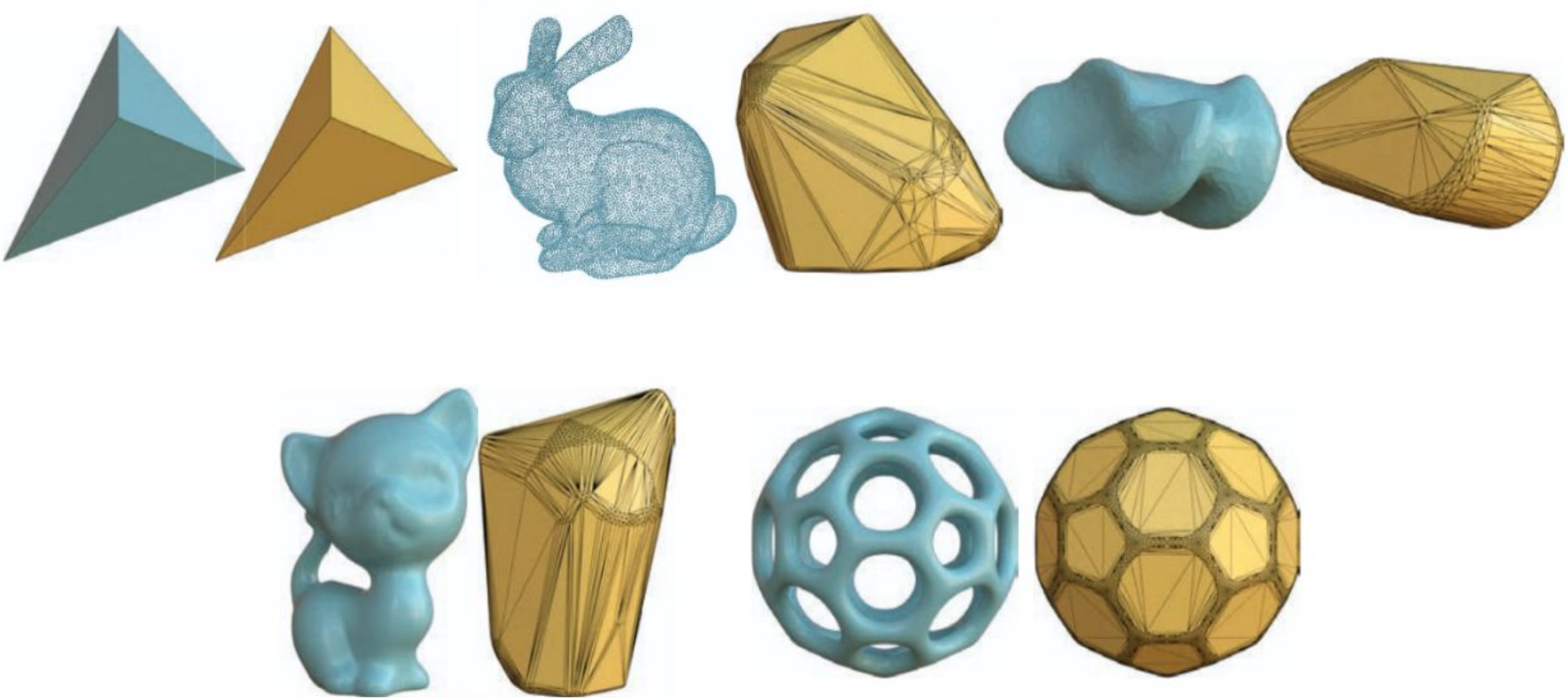
center of mass

height of transformed center of mass $= (Rc + t).z$

transformed center of mass

Local minima correspond to resting positions

Convex Hull



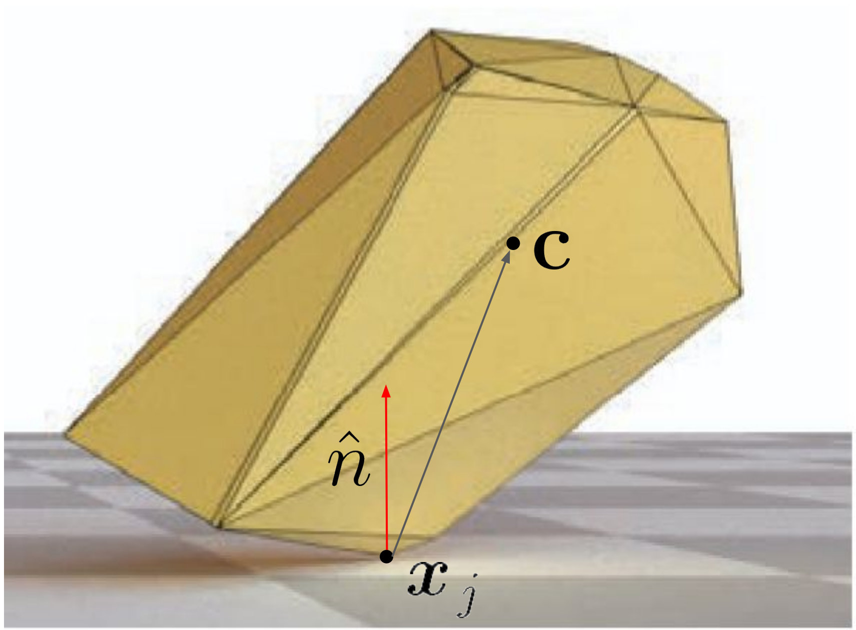
Function to minimize

When a hull vertex with position $\mathbf{x}_j \in \mathbb{R}^3$ is resting on the ground,

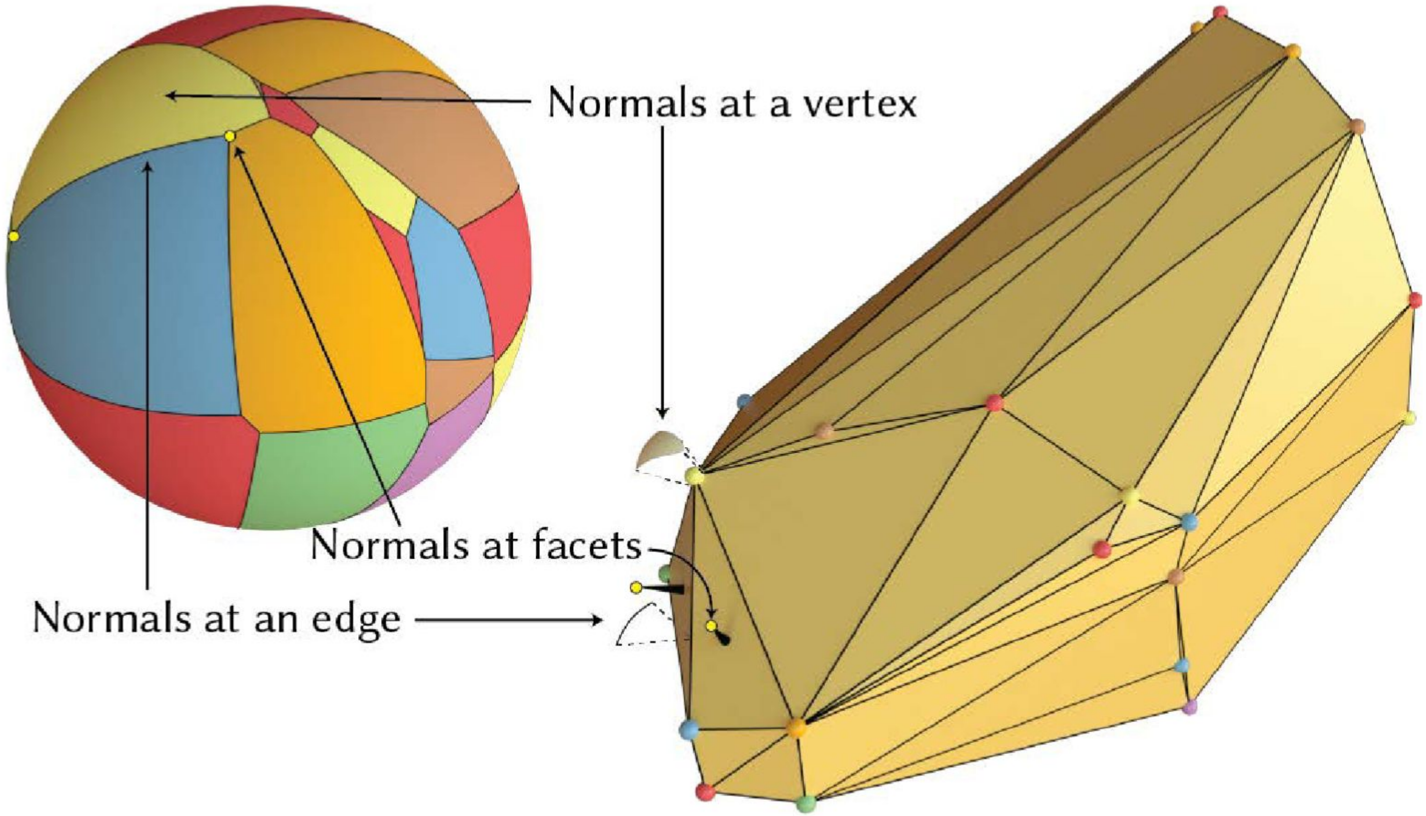
$$U_j(\hat{\mathbf{n}}) = (\mathbf{c} - \mathbf{x}_j) \cdot \hat{\mathbf{n}}$$

$$U(\hat{\mathbf{n}}) = \sum_j \delta_j(\hat{\mathbf{n}}) U_j(\hat{\mathbf{n}})$$

1 if lying on a vertex
½ if lying on an edge
⅓ if lying on a face



Convex Hull and Gauss Map



Height of the center of mass

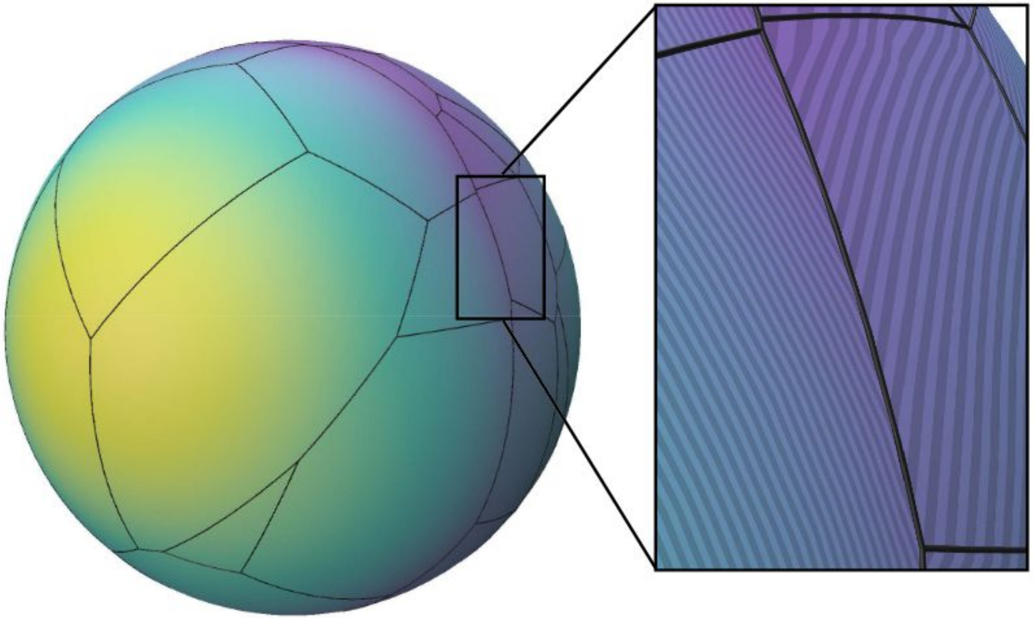
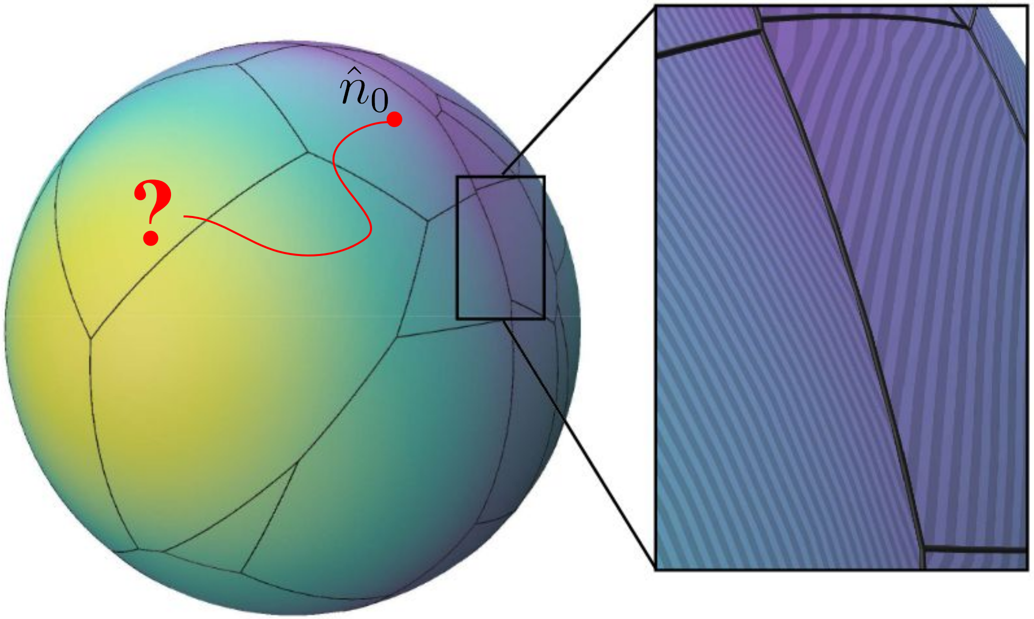


Fig. 4. The height of the center of mass is a continuous function (U) with respect to the downward normal direction. Visualized in pseudocolor over the Gauss map we can see that its gradient $\partial U / \partial \hat{\mathbf{n}}$ can be discontinuous across dual edges.

Finding the resting position

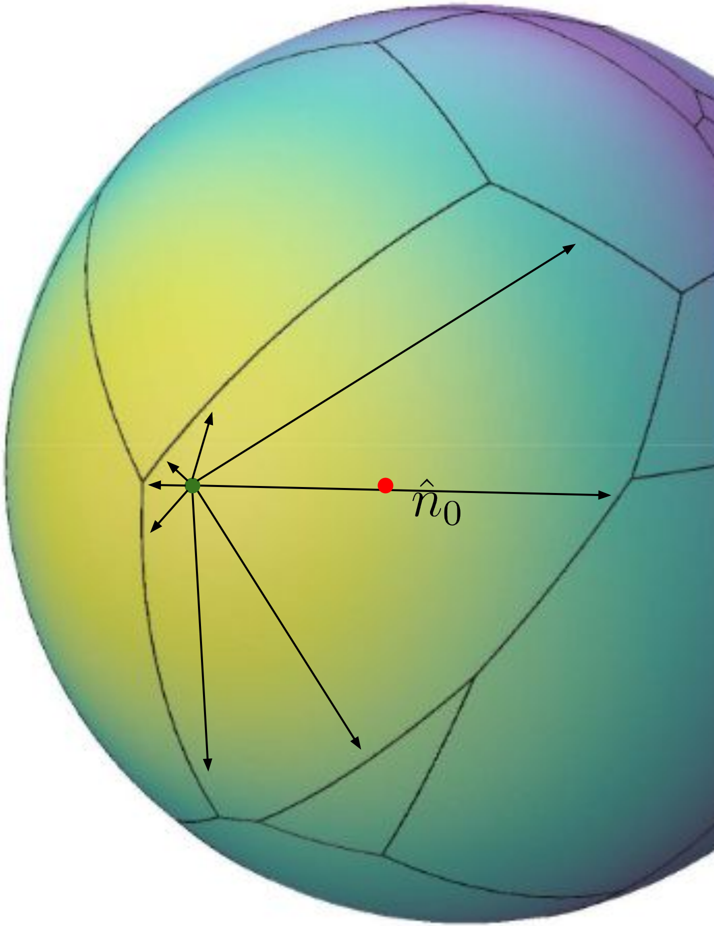


"Given some initial upward orientation $\hat{n}_0$, a momentumless simulation of rolling amounts to rotating in the direction that most decreases U until it is locally minimized, resulting in a stable orientation."

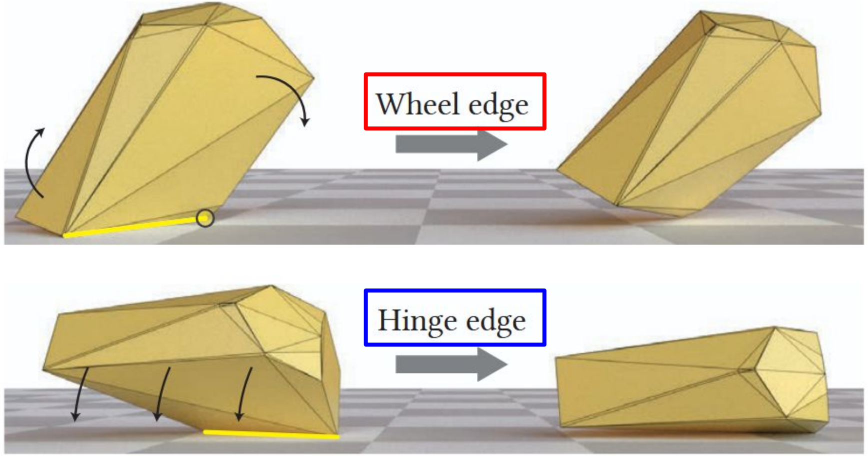
Tracing inside a **vertex**'s image

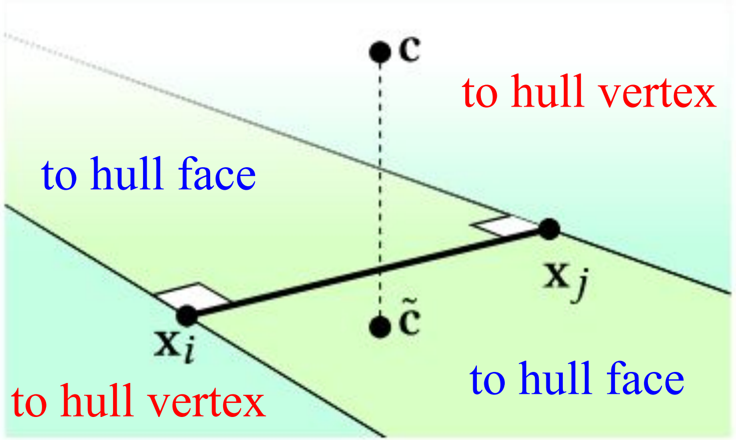
"Pathlines of the $\partial U/\partial n$ vector field inside a vertex *i*'s dual image are great arcs on S^2 emanating from a unique point .”

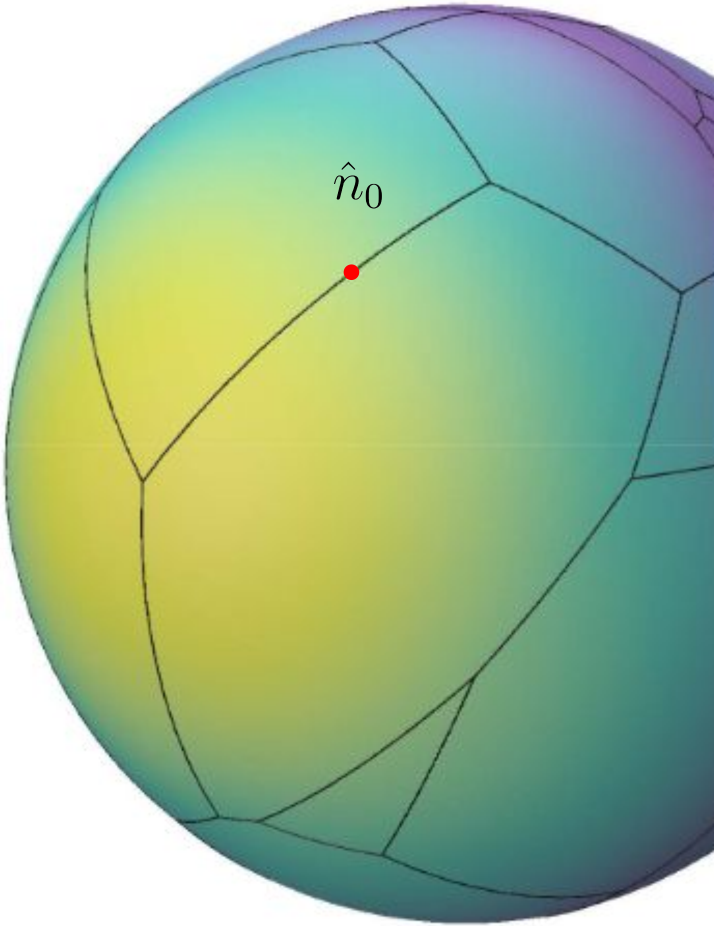
Local maximum of the height of the center of mass



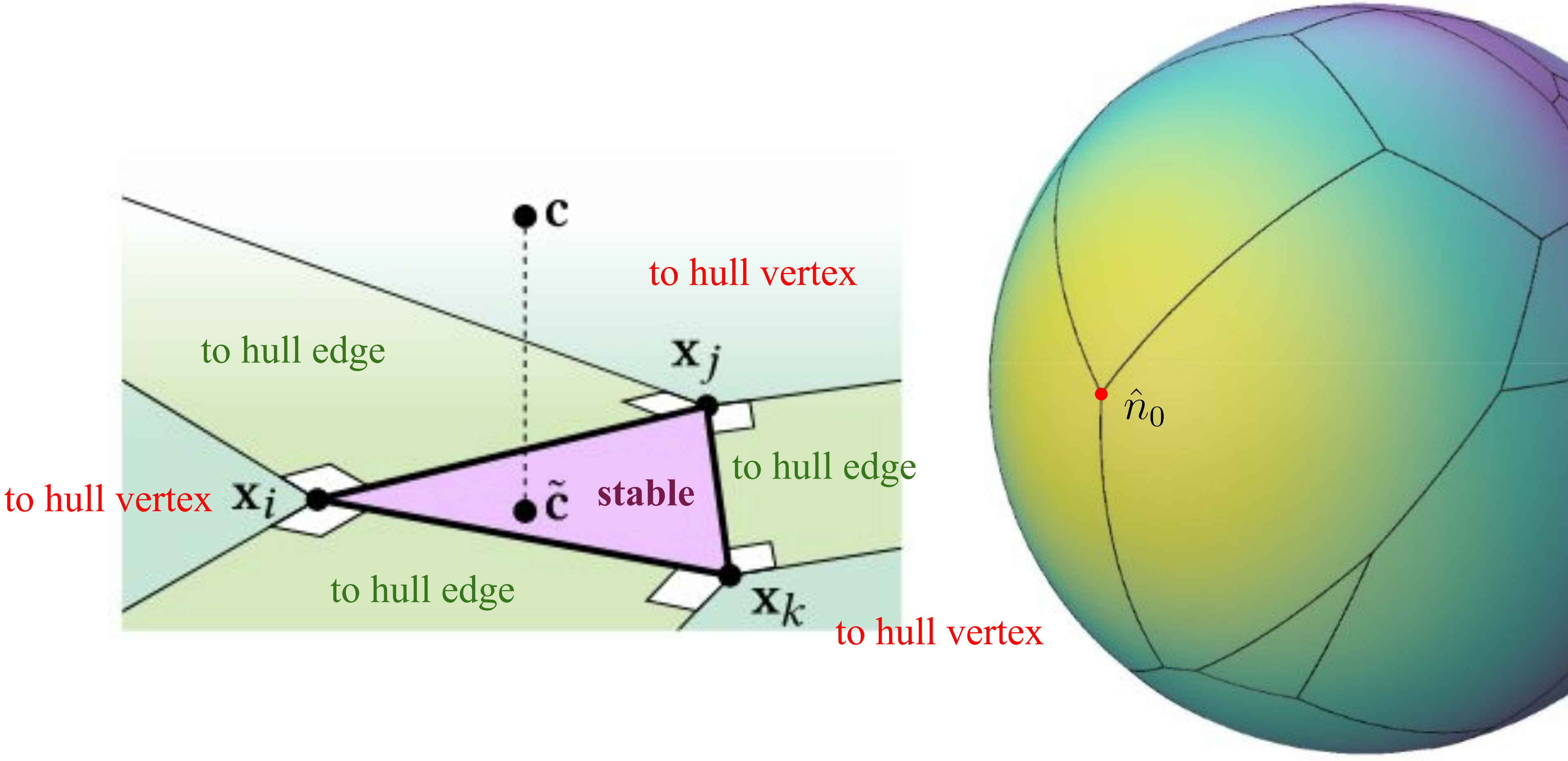
Tracing inside an **edge**'s image







Tracing inside a **face**'s image



Complete trace

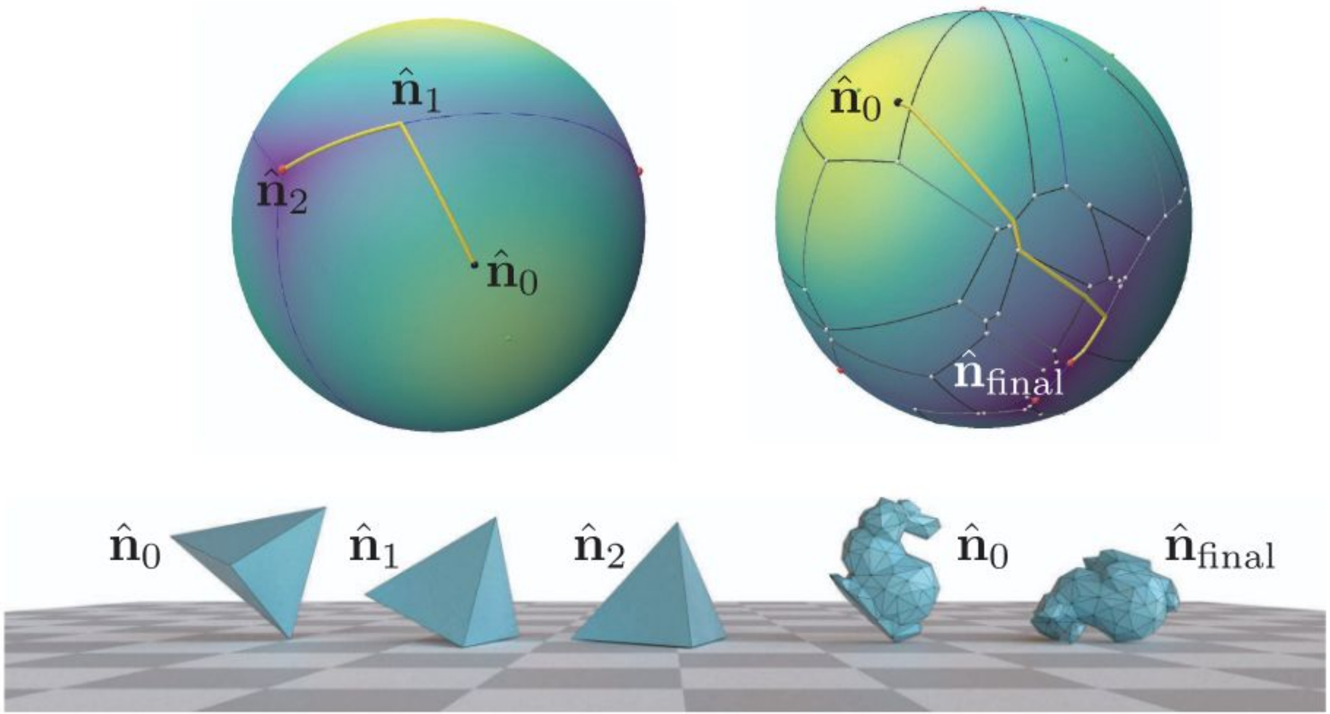


Fig. 7. We follow the gradient of U from an orientation $\hat{\mathbf{n}}_0$ to a local minimum of U for two examples: a tetrahedron and a bunny. Yellow curves show the orientation trajectories: piecewise-great-arc curves on the Gauss image.

Probabilities of resting configurations

known initial position

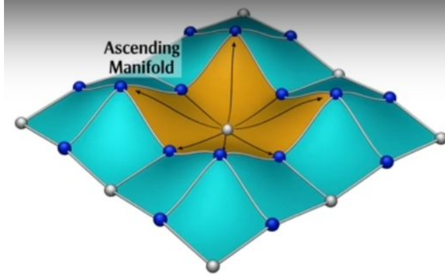
resting position

random initial position

p = ?

p = ?

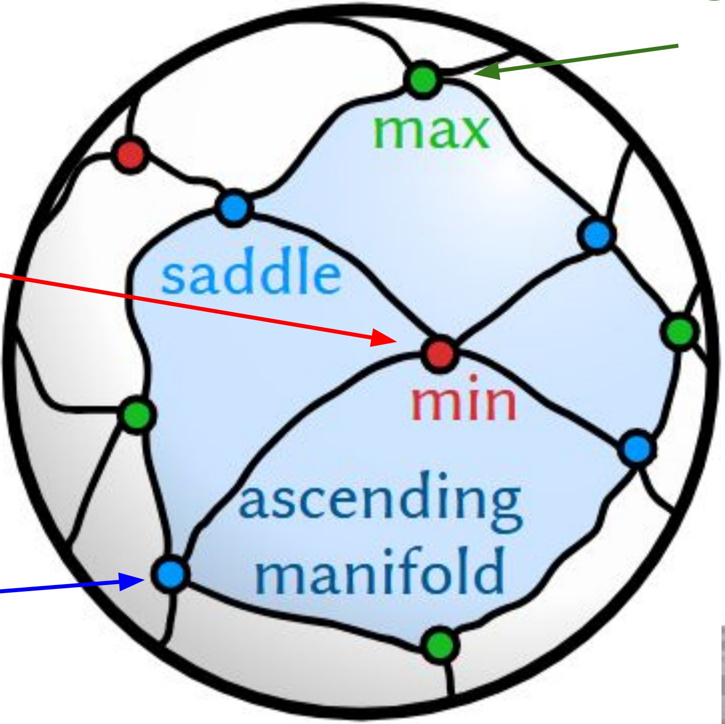
Morse-Smale Complex



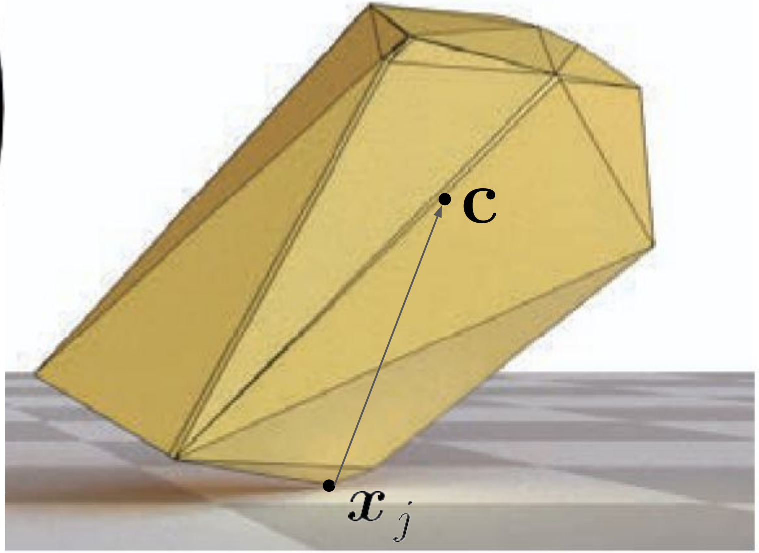
Unstable hull vertices

Stable hull faces

Hinge edges



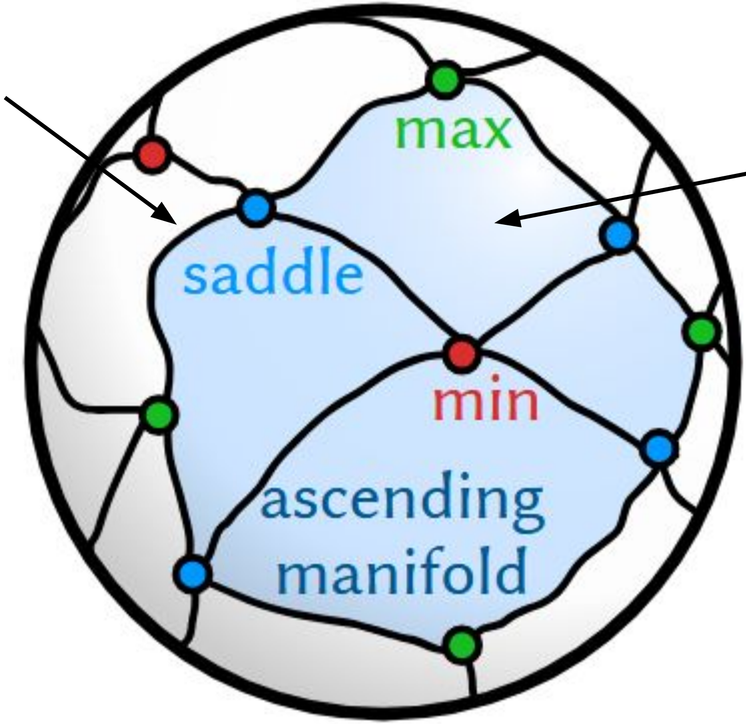
$$\hat{\mathbf{n}}_j^{\star} = \widehat{\mathbf{x}_j - \mathbf{c}}$$



Morse-Smale Complex

Separatrix

(follows gradient)



Probability of resting position = Area of its ascending manifold

Morse-Smale Complex examples

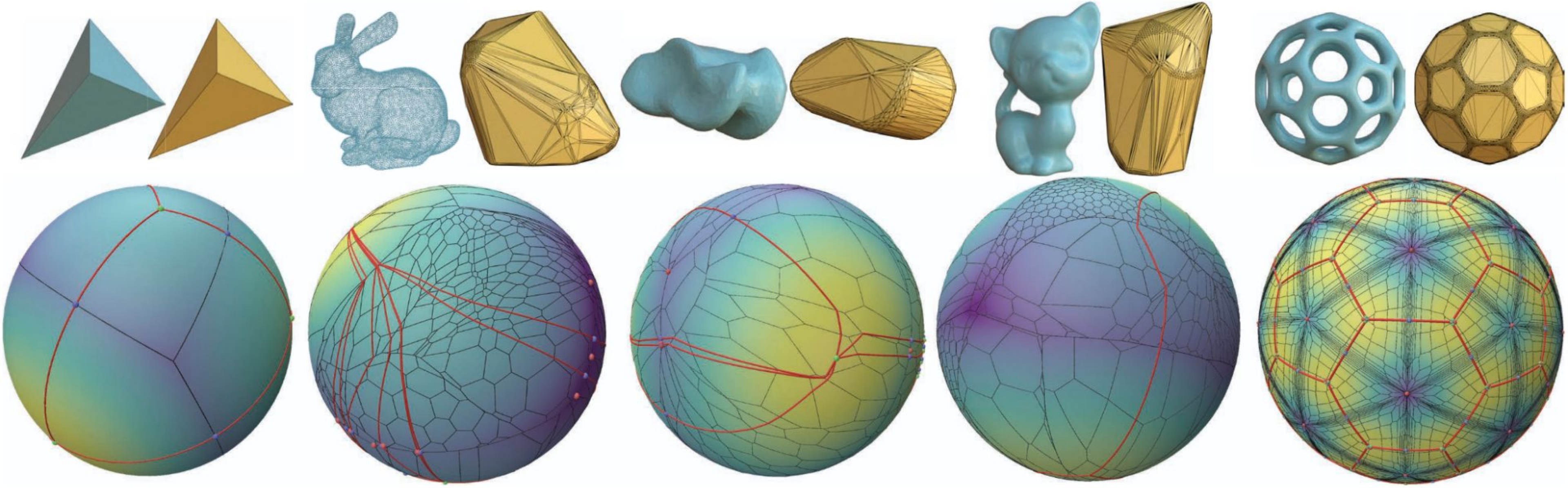


Fig. 8. Morse complex examples (red) drawn on the Gauss map (black) of the convex hull of some models. Stable face normals (red), edge saddle normals (blue), and vertex maximum normals (green) are also shown. For the Bunny example a point cloud is used as input; our method can take any type of input as long as a convex hull of the input can be computed.

Probabilities of resting configurations

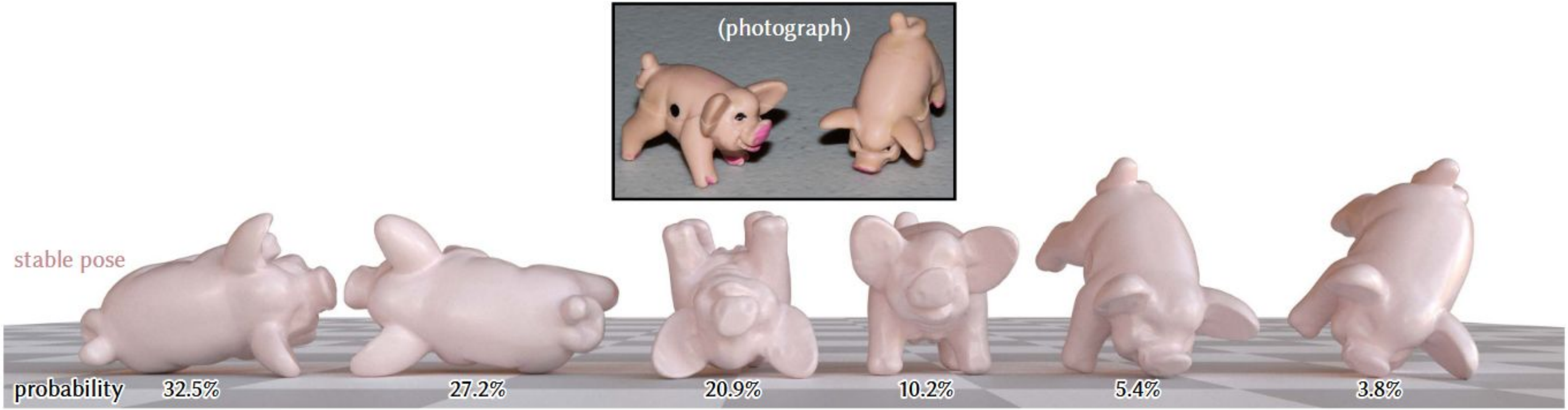
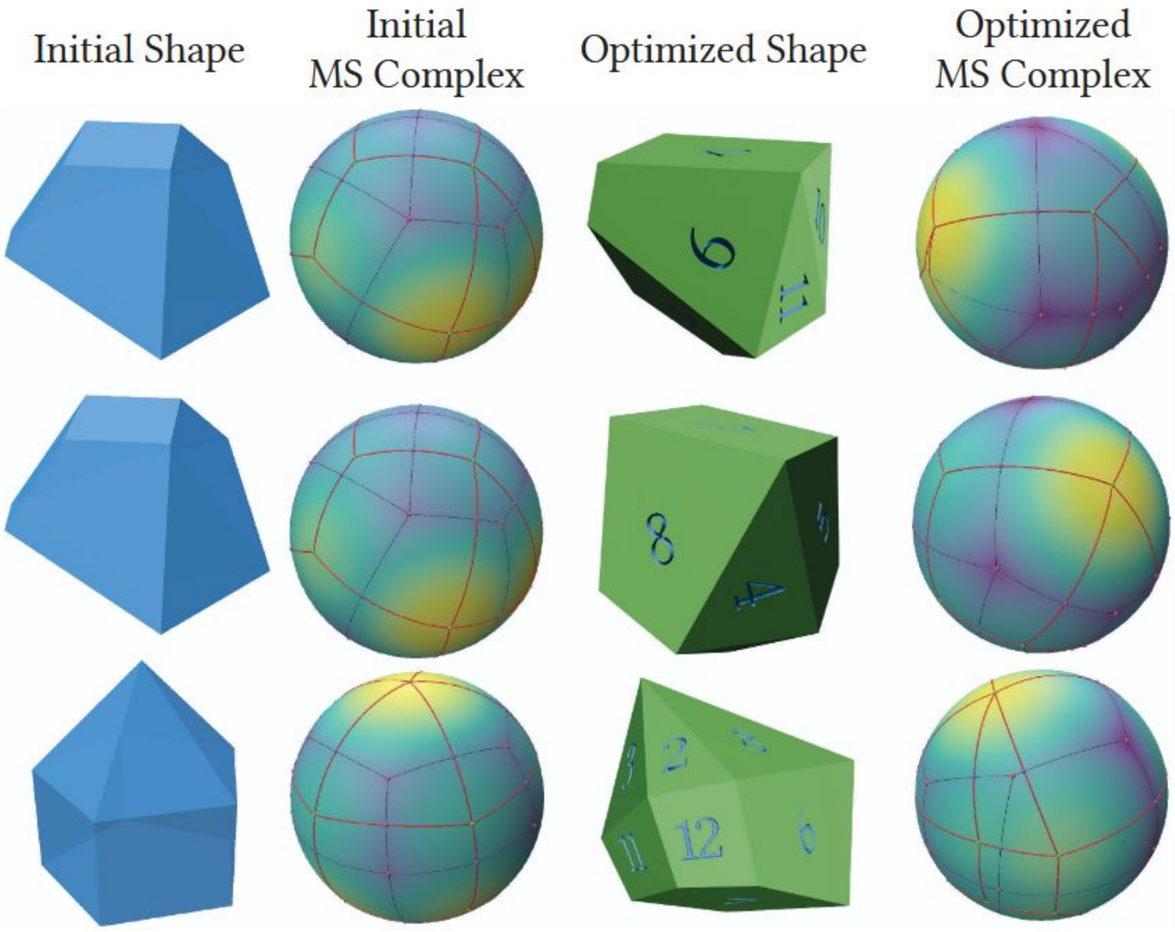
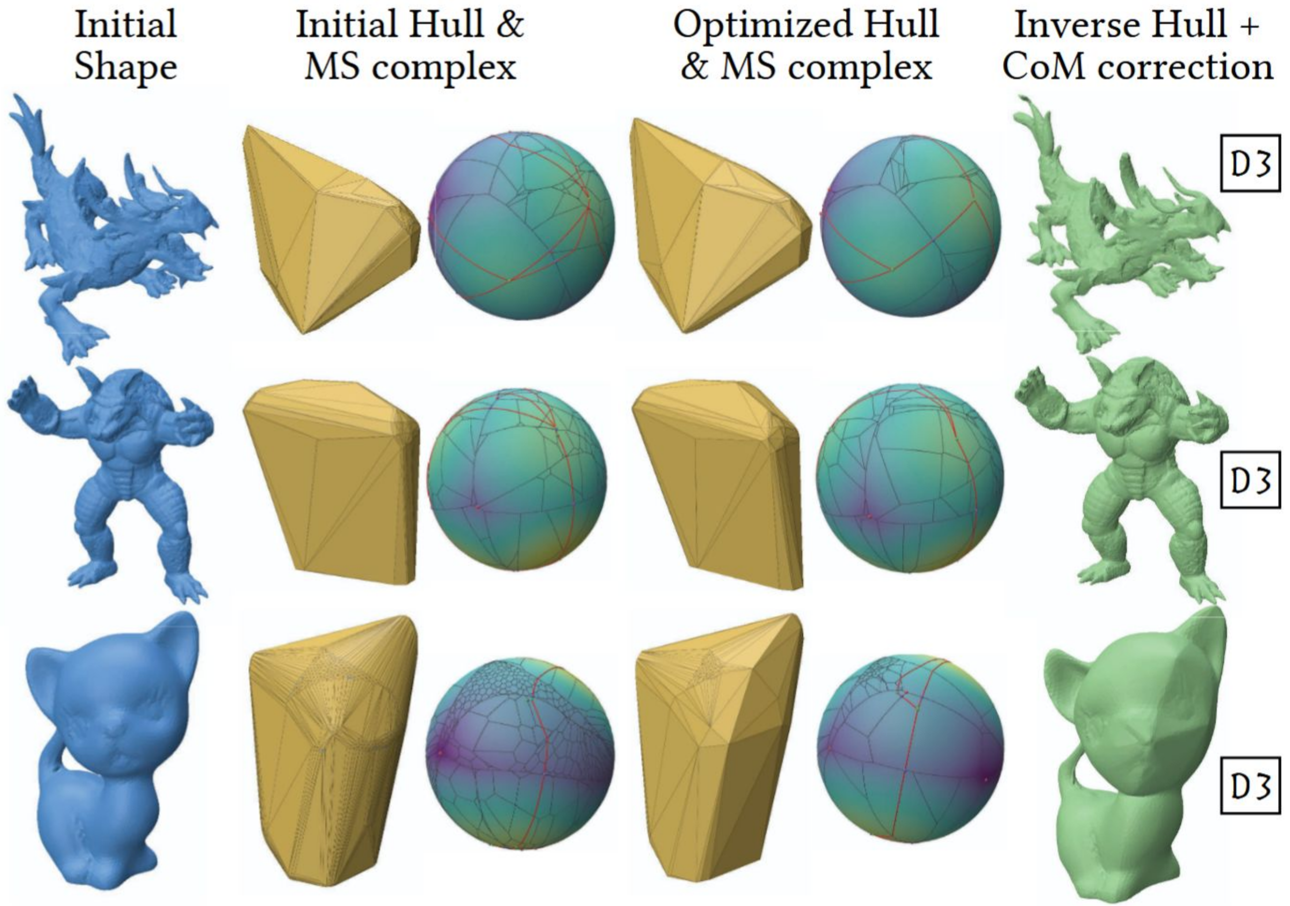


Fig. 2. Our algorithm efficiently and robustly computes the probability of all resting configurations (in 3 *ms*) of the pig model from the popular game "Pass the Piggies" [Moffatt 1977]. In particular, our predictions match the experimental data from [Kern 2006] up to an optimal transport distance of 0.09.

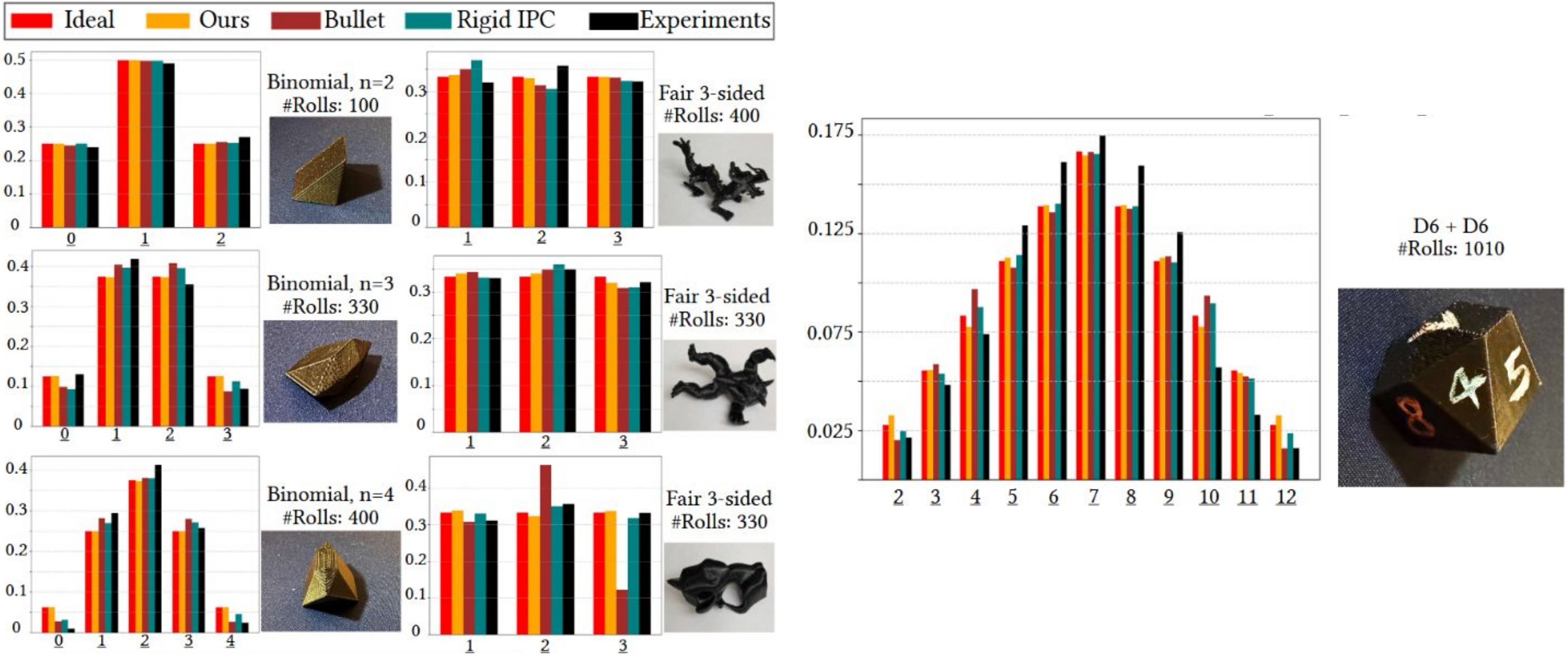
Optimizing for target probabilities - 2D6



Optimizing for target probabilities - D3



Validation



Ideal
Ours
Bullet
Rigid IPC
Experiments
Binomial, n=2
#Rolls: 100
Fair 3-sided
#Rolls: 400
Binomial, n=3
#Rolls: 330
Fair 3-sided
#Rolls: 330
Binomial, n=4
#Rolls: 400
Fair 3-sided
#Rolls: 330
D6 + D6
#Rolls: 1010
0.175
0.125
0.075
0.025
2 3 4 5 6 7 8 9 10 11 12