Dynamic Skeletonization via Variational Medial Axis Sampling

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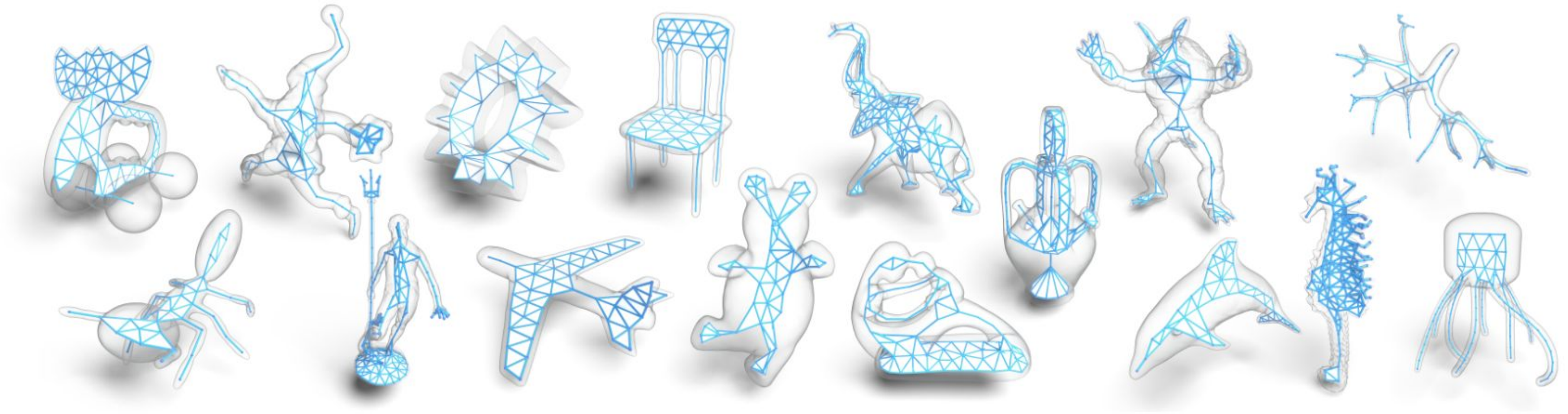
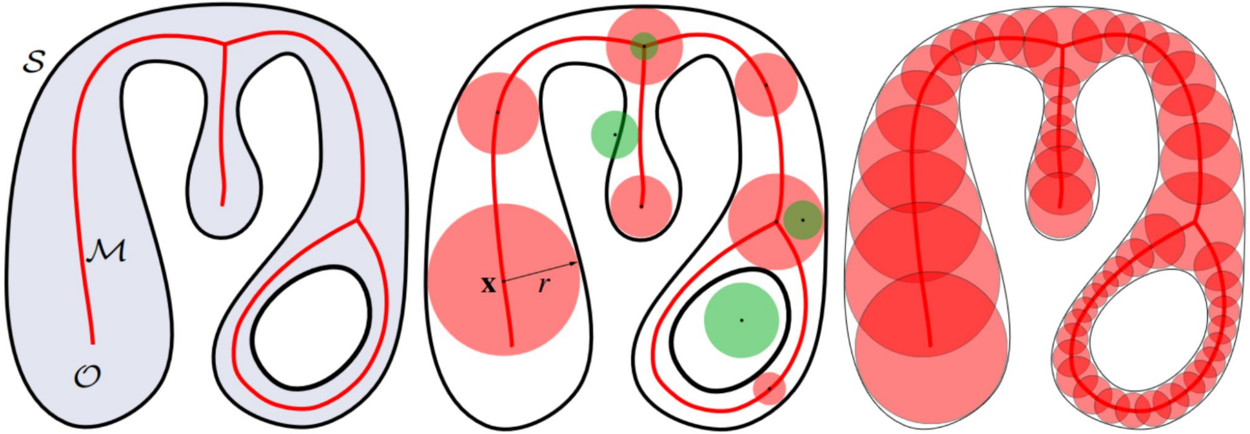
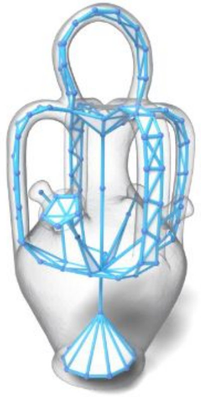


Medial axis



Surface

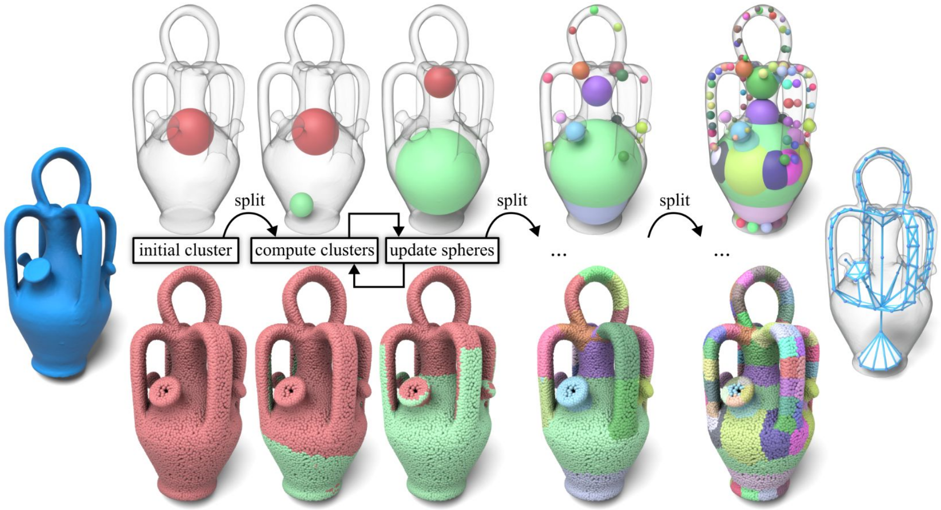
Oriented Point cloud



Discretization of Medial Axis

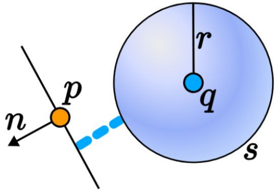
Method overview

1. Find medial spheres
    - determine the cluster of each medial sphere
    - adjust each medial sphere to its cluster
    - correction: project each adjusted medial sphere onto the medial axis
    - split sphere if required
2. Establish connectivity



Distances

- distance of sphere $s = (q, r)$ with a point p : $d_p(s) = (\|p - q\|) - r$
- distance of sphere $s = (q, r)$ with a plane (p, n): $d_{p,n}(s) = n^t \cdot (p - q) - r$



- squared distance sphere – plane: $Q_{p,n}(s) = \frac{1}{2} s^t \cdot A \cdot s - b^t \cdot s + c$

$$A = 2 \left[\begin{array}{c|c} n \cdot n^t & n \\ \hline n^t & 1 \end{array} \right], b = 2\,(n^t \cdot p) \left[\begin{array}{c} n \\ \hline 1 \end{array} \right], c = (n^t \cdot p)^2$$

Distances of a medial sphere *s* to points / tangent planes

Mesh

weighted squared distance from v_i

$$D_{v_i}(s) = \left(\sum_{t_j \in T(v_i)} \frac{\mathcal{A}(t_j)}{3} \right) d_{v_i}(s)^2$$

$$Q_{v_i}(s) = \sum_{t_j \in T(v_i)} \frac{\mathcal{A}(t_j)}{3} Q_{v_i, n_j}$$

weighted squared distance from tangent planes associated with v_i

Oriented point cloud

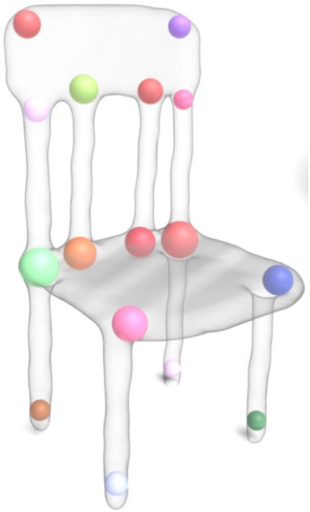
$$D_{v_i}(s) = \mathcal{A}(v_i)\, d_{v_i}(s)^2$$

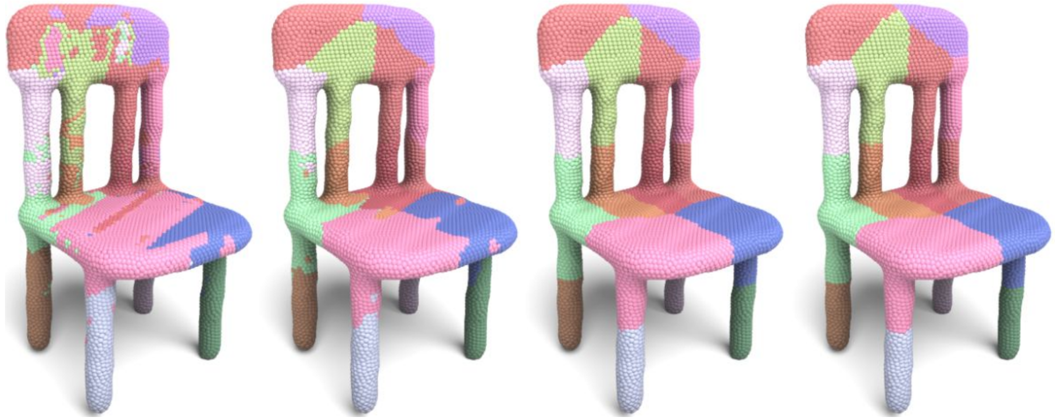
$$Q_{v_i}(s) = \sum_{v_j \in KNN(v_i)} \frac{\mathcal{A}(v_j)}{k} Q_{v_j, n_j}$$

Clusters associated with the medial spheres

- associate vertex v_i with the medial sphere m_j that minimizes the cost:

$$E_{v_i}(m_j) = Q_{v_i}(m_j) + \lambda D_{v_i}(m_j)$$



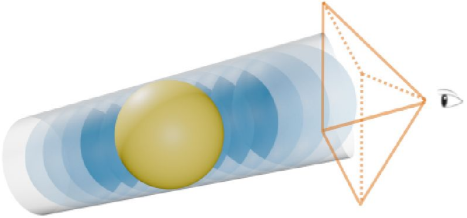


$\lambda = 0$ $\quad$ $\lambda = 0.02$ $\quad$ $\lambda = 0.2$ $\quad$ $\lambda = 1$

Sphere updating

- adjust each medial sphere $s_i = (q_i, r_i)$ to match the geometry of its cluster:

$$(q_i^*, r_i^*) = \arg\min_{q_i, r_i} \left(E_{\mathrm{SQEM}}(C_i) + \lambda E_{\mathrm{euclidean}}(C_i)\right)$$

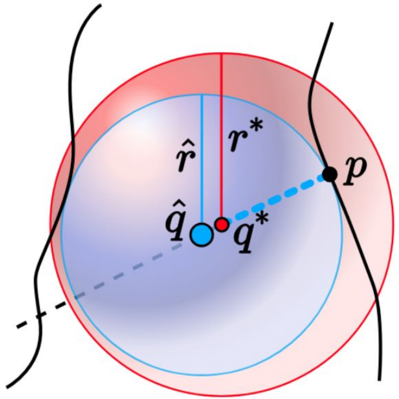


$$E_{\mathrm{SQEM}}(C_i) = \sum_{v_j \in C_i} Q_{v_j}(m_i) \qquad E_{\mathrm{euclidean}}(C_i) = \sum_{v_j \in C_i} D_{v_j}(m_i)$$

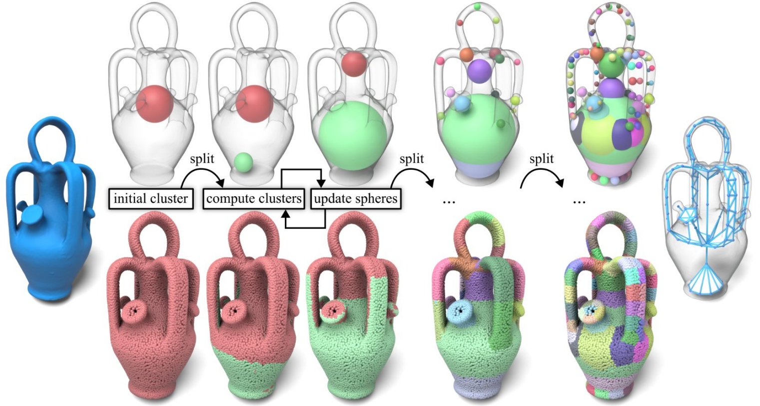
- solved with Gauss-Newton method
- final error by cluster: $E(C_i) = \frac{1}{\mathcal{A}(C_i)} \sum_{v_j \in C_i} E_{v_j}(m_i)$

Sphere correction

- Adjusted medial spheres no more on the medial axis
- Project back each medial sphere $s = (q^*, r^*)$ onto the medial axis
  - p closest point on the shape to q^*
  - direction $d = \frac{q^* - p}{\|q^* - p\|}$
  - make s tangent to p along d
  - iteratively reduce its radius and move its center along d so that it becomes tangent to another point of the shape (shrinking ball algorithm)



Sphere splitting

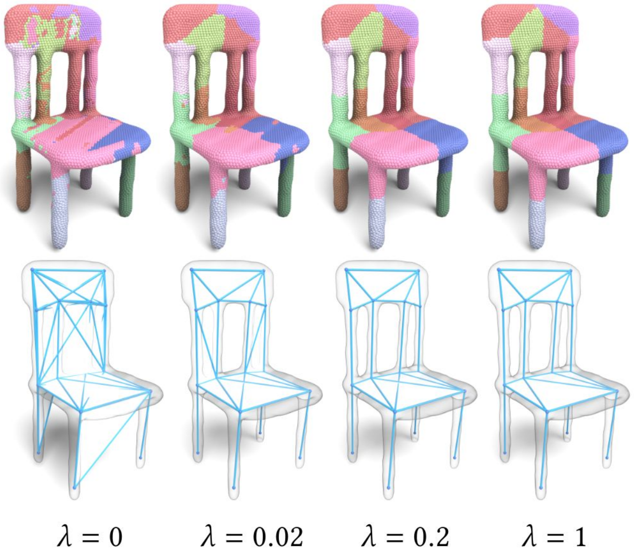


- start with one medial sphere
- at each iteration
  - sort the clusters by error
  - if a cluster has an error above some threshold and its neighbors clusters have not been splitted yet
    - select the vertex with maximal error, define it as a new medial sphere
- stop when all clusters errors are below some threshold (or a number of medial spheres is met)

Connectivity

- Last step: establish the connectivity between medial spheres

  ➡ use the dual of the connectivity of their clusters



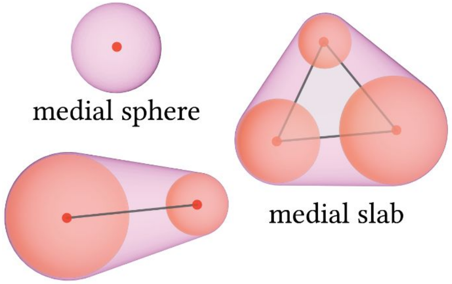
Results (quality)

|V| number of vertices

|M| number of medial spheres

ε: two-sided Hausdorff distance between original and reconstructed shapes

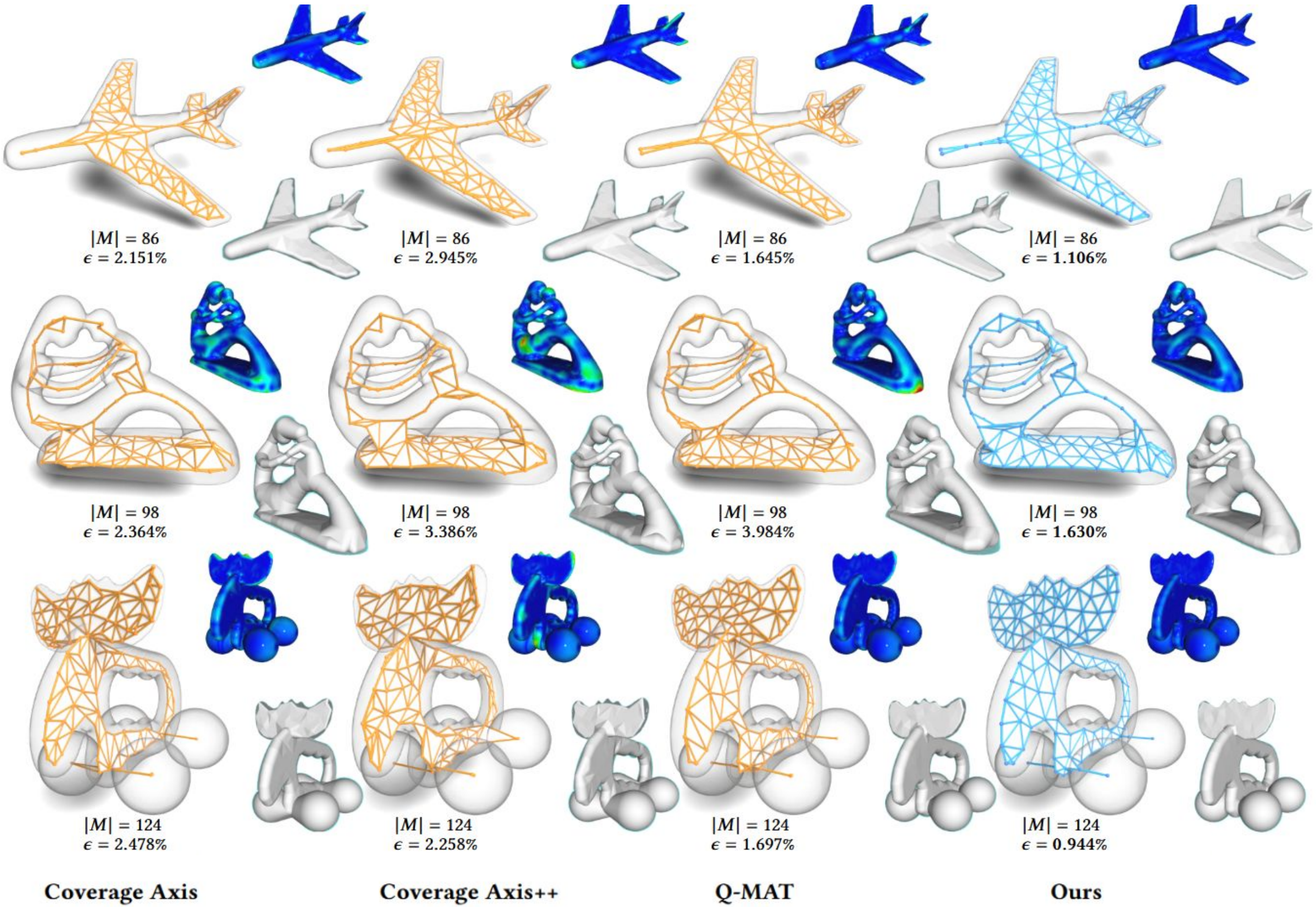
reconstructed shape: interpolation of the medial spheres along the edges and faces of the medial mesh



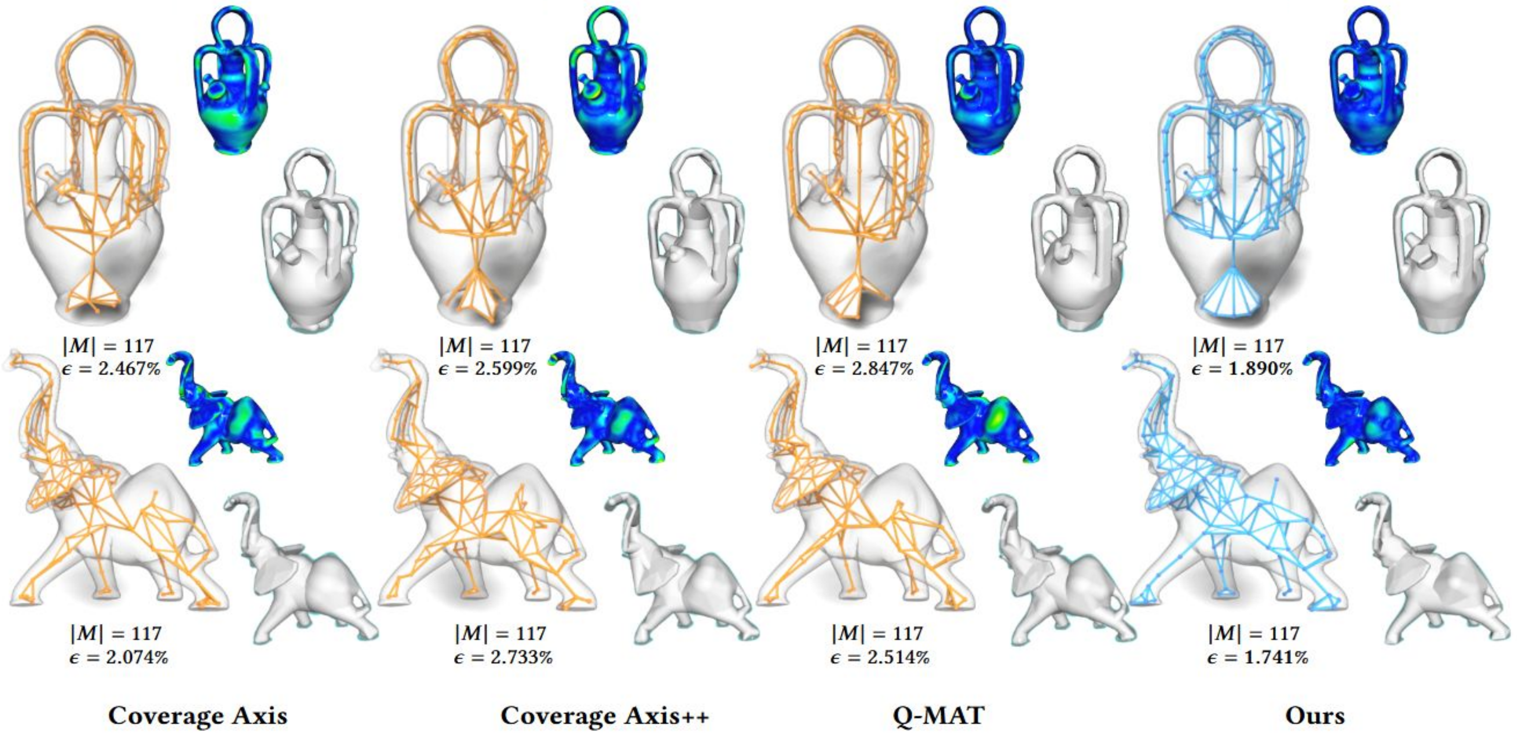
Method		Coverage Axis	Coverage Axis++	Q-MAT	Ours
Model($\|V\|$)	$\|M\|$	ϵ	ϵ	ϵ	ϵ
Armadillo (36725)	83	3.438%	3.905%	3.860%	**2.735%**
Fertility (17827)	98	2.364%	3.386%	3.984%	**1.630%**
Chair (10500)	117	1.892%	2.412%	1.606%	**1.123%**
Vase (14859)	117	2.467%	2.599%	2.847%	**1.890%**
Elephant(24955)	117	2.074%	2.733%	2.514%	**1.741%**
Bug(8640)	75	2.338%	2.429%	2.117%	**1.001%**
Seahorse(20494)	76	3.195%	3.393%	5.088%	**2.967%**
Dove(5519)	76	2.754%	2.885%	2.186%	**0.952%**
Vessel (49698)	99	3.054%	3.033%	**0.818%**	1.716%
Pinion (10369)	230	3.393%	3.319%	2.194%	**2.127%**
Elk (24013)	124	2.478%	2.258%	1.697%	**0.944%**
Neptune (14814)	106	4.063%	3.199%	3.329%	**2.374%**
Dolphin (15100)	51	2.043%	2.446%	1.983%	**1.296%**
Santa (10241)	90	2.011%	2.372%	2.158%	**1.475%**
Bear (10141)	45	3.285%	3.391%	2.204%	**1.659%**
Plane (7651)	86	2.151%	2.945%	1.643%	**1.106%**
Spider (11051)	107	1.692%	1.604%	0.901%	**0.495%**
Venus (10760)	43	**2.162%**	2.701%	3.768%	2.929%

Results (time)

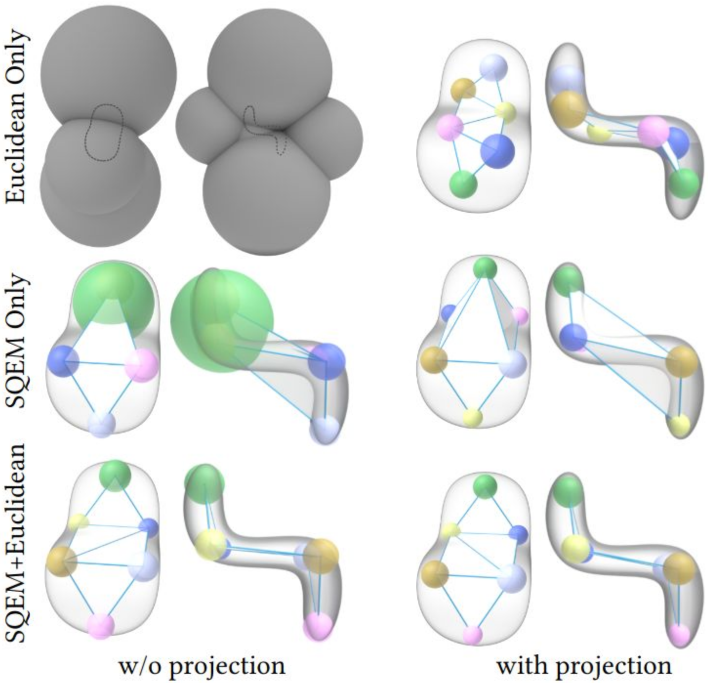
Method			Coverage Axis			Coverage Axis++			Q-MAT		Ours
Model($\|V\|$)	$\|M\|$	MA init	Selection	Connectivity	Total time	Selection	Connectivity	Total time	Simplification	Total time	Time
Armadillo (36725)	83	4.528	3.6	10.476	18.604	0.93	10.753	16.211	10.323	14.851	**2.179**
Fertility (17827)	98	1.811	24.29	5.012	31.113	1.106	4.855	7.772	4.632	6.443	**1.46**
Chair (10500)	117	1.091	509.4	2.481	512.972	1.31	2.347	4.748	2.091	3.182	**0.971**
Vase (14859)	117	1.512	1.3	4.095	6.907	1.3	4.157	6.969	4.256	5.768	**0.9**
Elephant (24955)	117	2.703	3.84	6.933	13.476	1.3	7.359	11.362	6.554	9.257	**2.226**
Bug (8640)	75	0.91	5.1	1.96	7.97	0.9	1.991	3.801	1.848	2.758	**0.429**
Seahorse (20494)	76	2.019	18.85	5.494	26.363	0.9	5.511	8.43	5.074	7.093	**1.035**
Dove (5519)	76	0.53	>1000	1.017	>1000	0.88	1.186	2.596	1.029	1.559	**0.42**
Vessel (49698)	99	5.337	1.2	11.849	18.386	1.127	12.587	19.051	11.185	16.522	**2.495**
Pinion (10369)	230	1.02	2.3	2.295	5.615	2.79	2.289	6.099	2.439	3.459	**2.584**
Elk (24013)	124	2.538	2.62	6.108	11.266	3.485	6.228	12.251	5.746	8.284	**1.94**
Neptune (14814)	106	1.148	5.6	3.478	10.226	1.196	3.562	5.9063	3.221	4.369	**1.017**
Dolphin (15100)	51	1.761	4.47	4.568	10.799	0.7	4.564	7.025	4.31	6.071	**0.751**
Santa (10241)	90	1.07	3.89	2.628	7.588	1.04	2.768	4.878	2.529	3.599	**0.598**
Bear (10141)	45	1.1	2.31	2.76	6.17	0.61	2.803	4.513	2.68	3.78	**0.34**
Plane (7651)	86	0.751	262.1	1.999	264.85	0.973	2.039	3.763	1.51	2.262	**0.529**
Spider (11051)	107	1.176	1.34	2.733	5.249	1.158	2.664	4.998	2.536	3.712	**0.843**
Venus (10760)	43	1.11	3.02	3.023	7.154	0.597	2.973	4.681	2.659	3.77	**0.343**



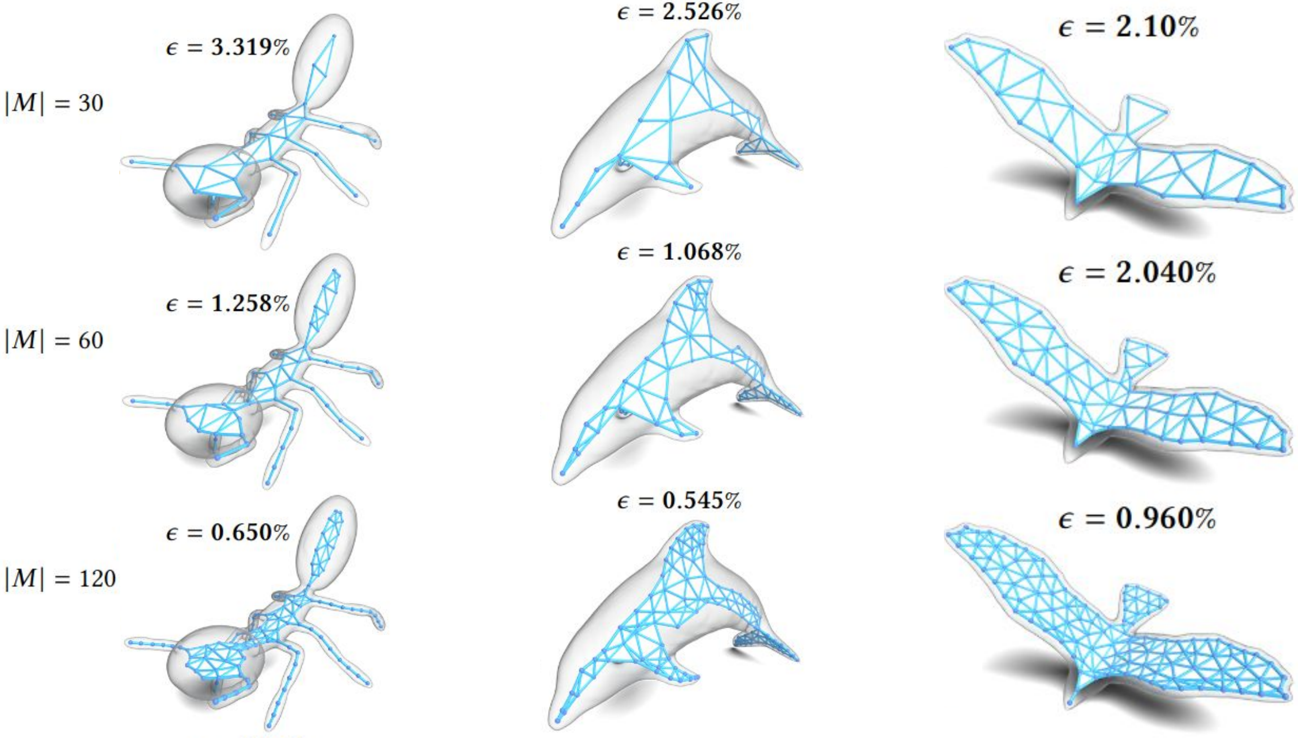
$|M| = 86$
$\epsilon = 2.151\%$
$|M| = 86$
$\epsilon = 2.945\%$
$|M| = 86$
$\epsilon = 1.645\%$
$|M| = 86$
$\epsilon = 1.106\%$
$|M| = 98$
$\epsilon = 2.364\%$
$|M| = 98$
$\epsilon = 3.386\%$
$|M| = 98$
$\epsilon = 3.984\%$
$|M| = 98$
$\epsilon = 1.630\%$
$|M| = 124$
$\epsilon = 2.478\%$
$|M| = 124$
$\epsilon = 2.258\%$
$|M| = 124$
$\epsilon = 1.697\%$
$|M| = 124$
$\epsilon = 0.944\%$
Coverage Axis
Coverage Axis++
Q-MAT
Ours


|M| = 117
ε = 2.467%
|M| = 117
ε = 2.599%
|M| = 117
ε = 2.847%
|M| = 117
ε = 1.890%
|M| = 117
ε = 2.074%
|M| = 117
ε = 2.733%
|M| = 117
ε = 2.514%
|M| = 117
ε = 1.741%
Coverage Axis
Coverage Axis++
Q-MAT
Ours

Ablation

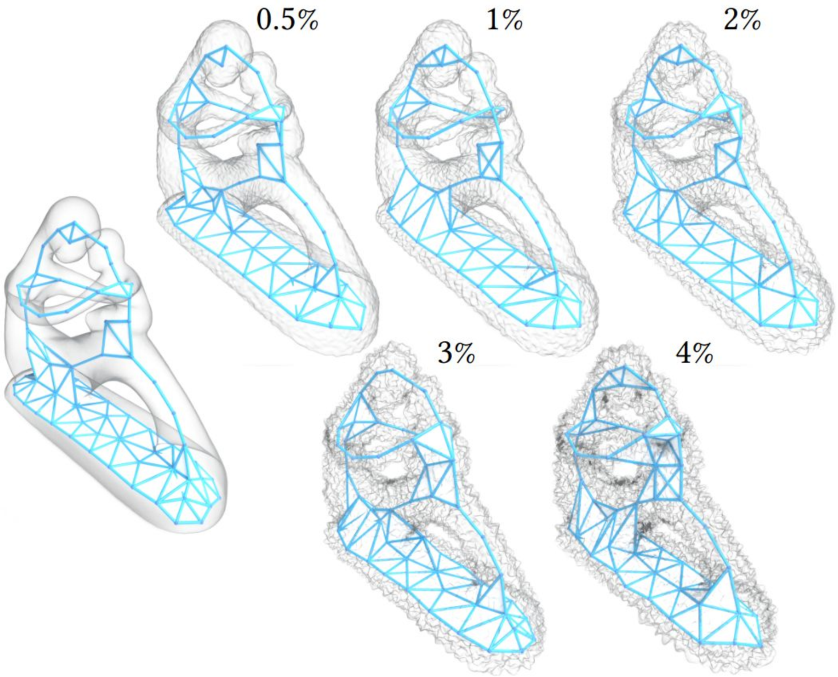

Euclidean Only
SQEM Only
SQEM+Euclidean
w/o projection
with projection

Resolution

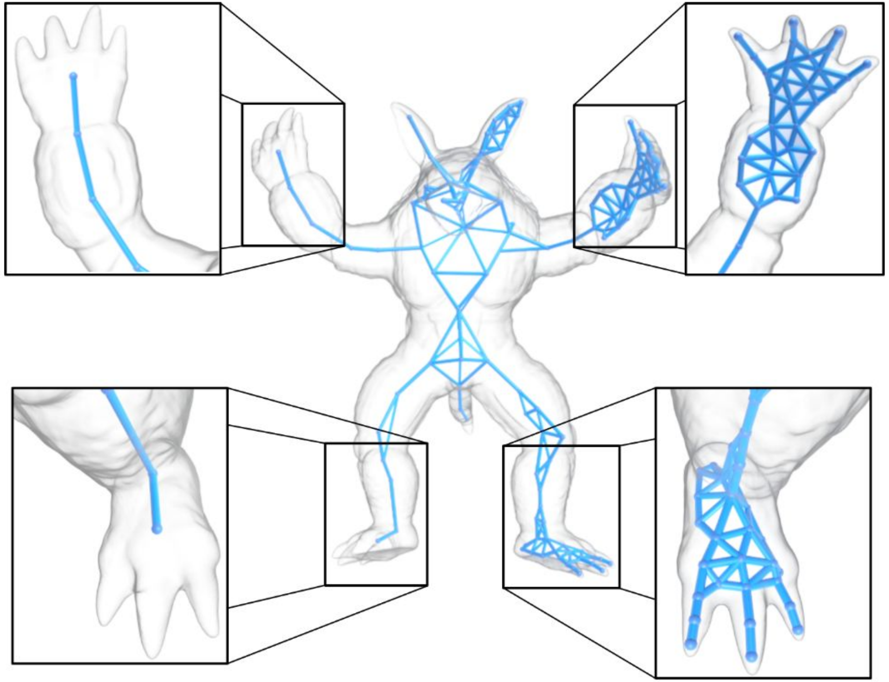


$|M| = 30$
$\epsilon = 3.319\%$
$\epsilon = 2.526\%$
$\epsilon = 2.10\%$
$|M| = 60$
$\epsilon = 1.258\%$
$\epsilon = 1.068\%$
$\epsilon = 2.040\%$
$|M| = 120$
$\epsilon = 0.650\%$
$\epsilon = 0.545\%$
$\epsilon = 0.960\%$

Robustness to noise



Interactivity



Limitations

- Convergence / optimality?
- Topology preservation? → not the same genus
- Connectivity? → overlapping triangles / tetrahedra