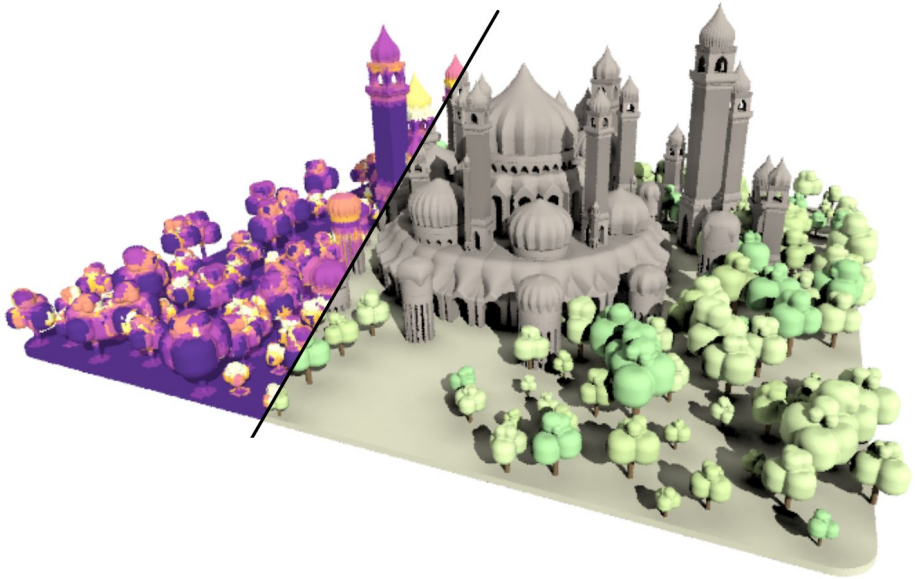
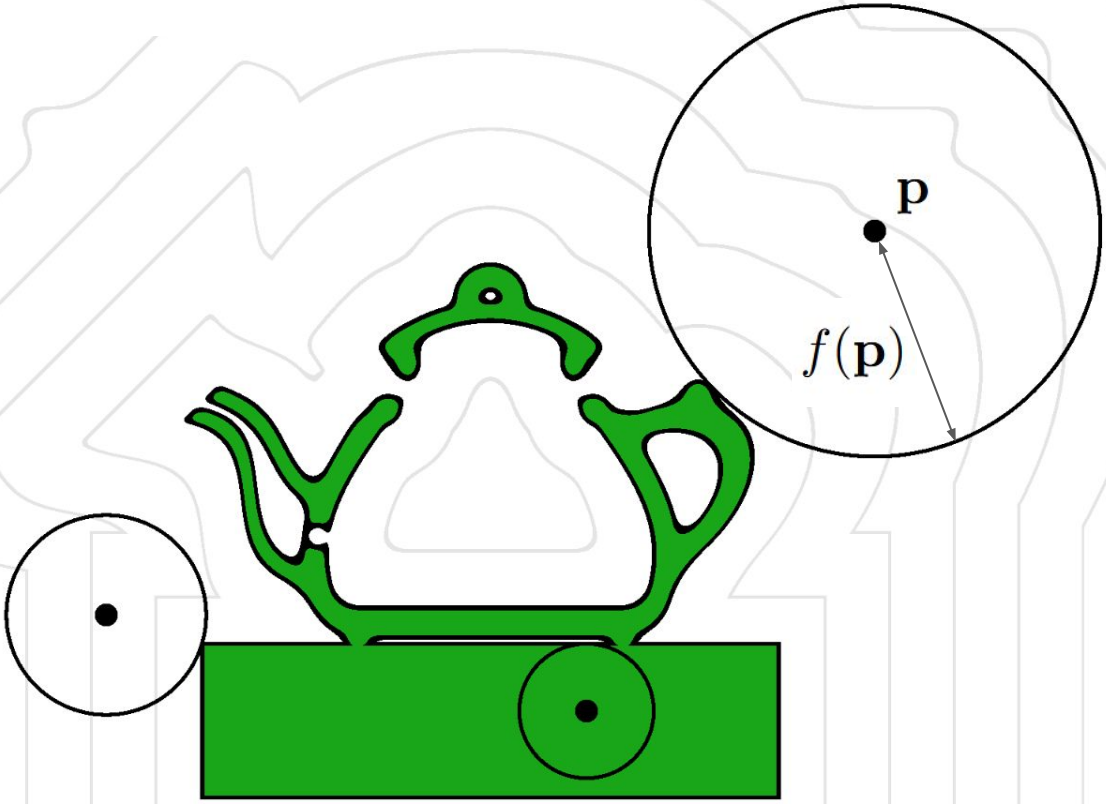
Lipschitz Pruning:
Hierarchical Simplification of Primitive-Based SDFs



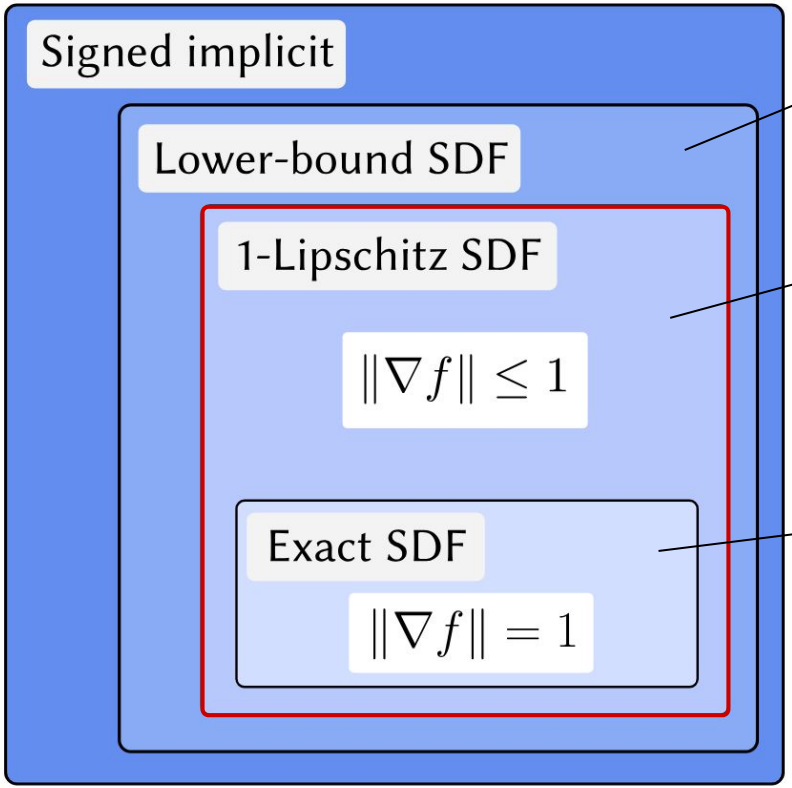
Wilhem Barbier* [1,2] Mathieu Sanchez* [2] Axel Paris [2] Élie Michel [2]
Thibaud Lambert [2] Tamy Boubekeur [2] Mathias Paulin [1] Theo Thonat [2]

[1] IRIT, Université de Toulouse, CNRS, [2] Adobe
* Equal contribution

Signed Distance Fields



Signed Distance Fields

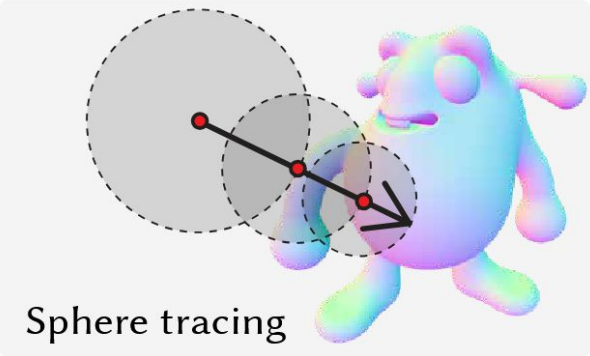


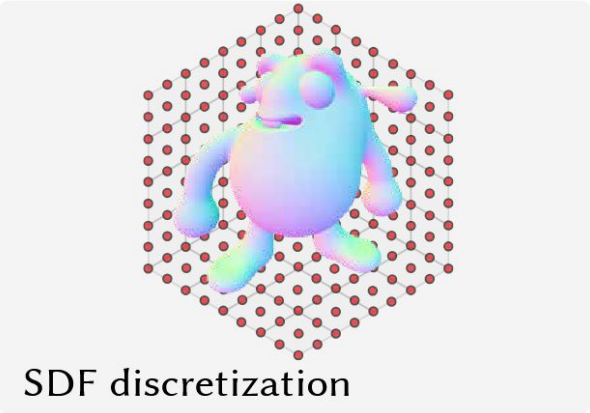
Sphere Tracing
(empty spheres)

Smooth Operators

Reach for the Spheres/Arcs
(spheres are tangent)

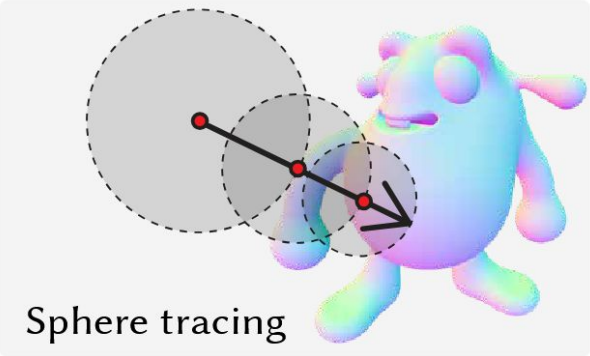
Applications

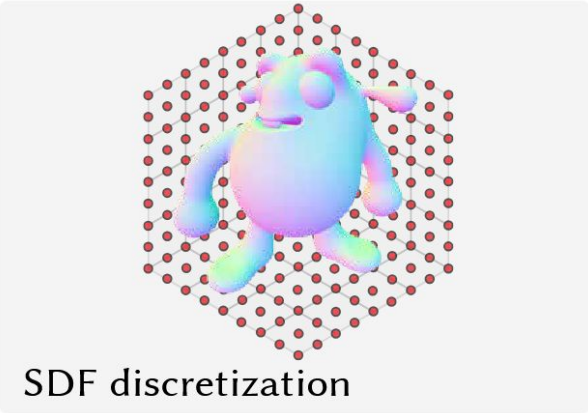




And more ...

Computational Cost





And more ...

Total Cost

≈

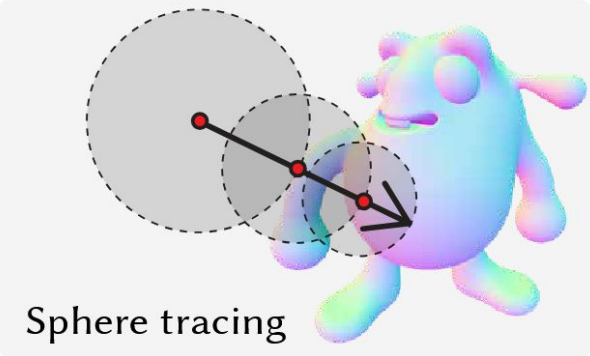
Number of queries

X

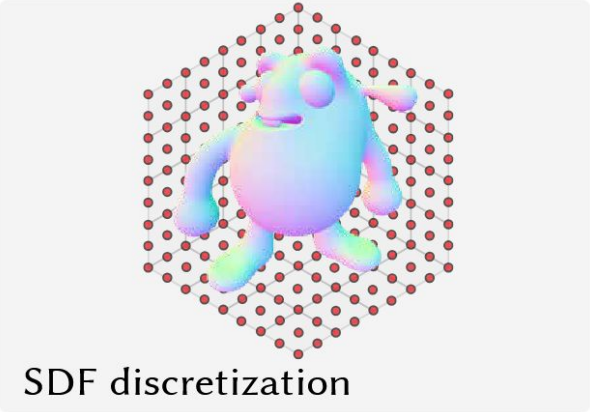
1 query cost

Computational Cost

Number of queries
X
1 query cost



→ Enhanced Sphere Tracing
Segment Tracing
...

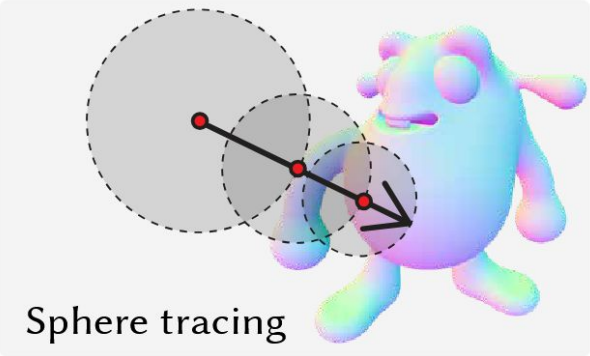


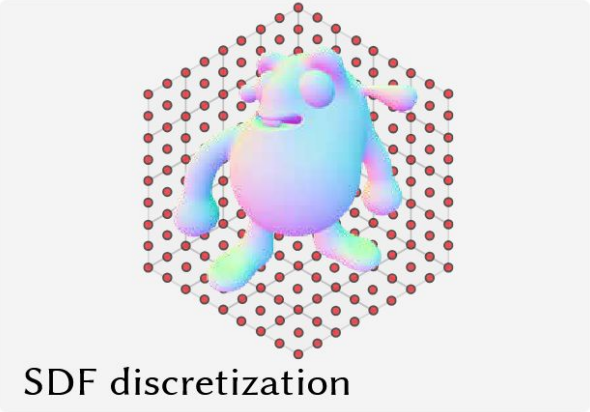
→ Reach for the Spheres
Reach for the Arcs
...

application specific

And more ...

Computational Cost





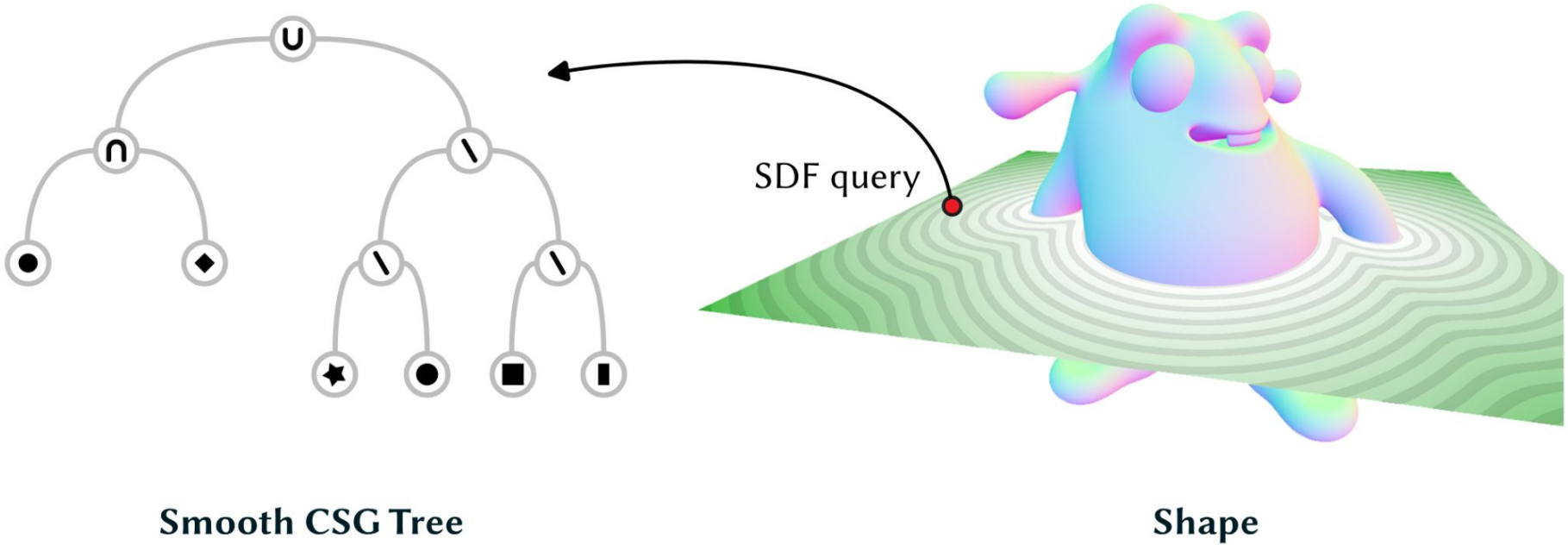
And more …

Number of queries
X
1 query cost

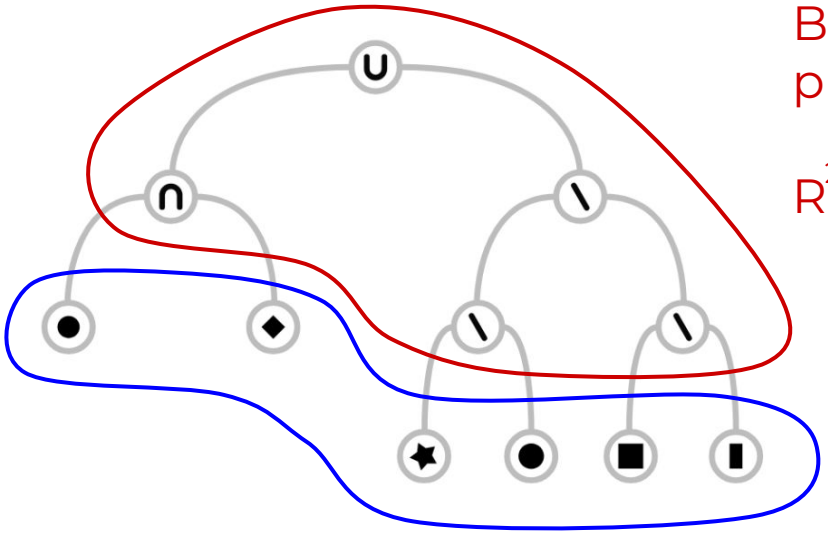
Interval arithmetic
Bounding volumes
Piecewise approximation
…

can be used for
all applications

Construction Tree



Construction Tree



Smooth CSG Tree

Binary operators merging primitives

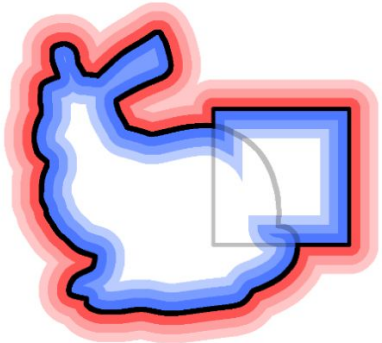
$R^2 \rightarrow R$

Primitives
(= basic shapes)

$R^3 \rightarrow R$

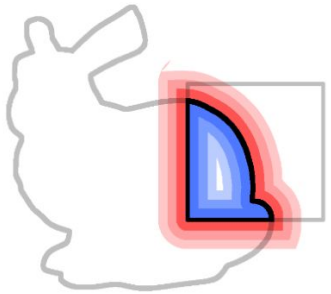
Construction Tree - Operators

Union



$\min(f_1, f_2)$

Intersection



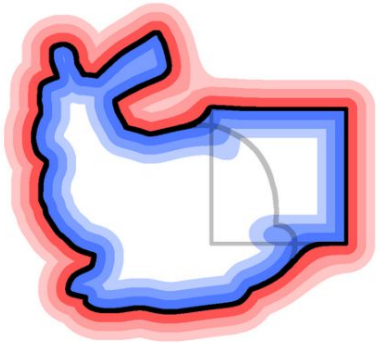
$\max(f_1, f_2)$

Difference

$\max(f_1, -f_2)$

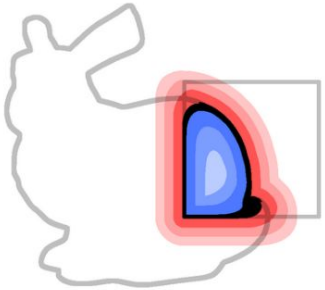
Construction Tree - Smooth Operators

Union



$$\min(f_1, f_2) - \phi(|f_1 - f_2|, k)$$

Intersection



$$\max(f_1, f_2) + \phi(|f_1 - f_2|, k)$$

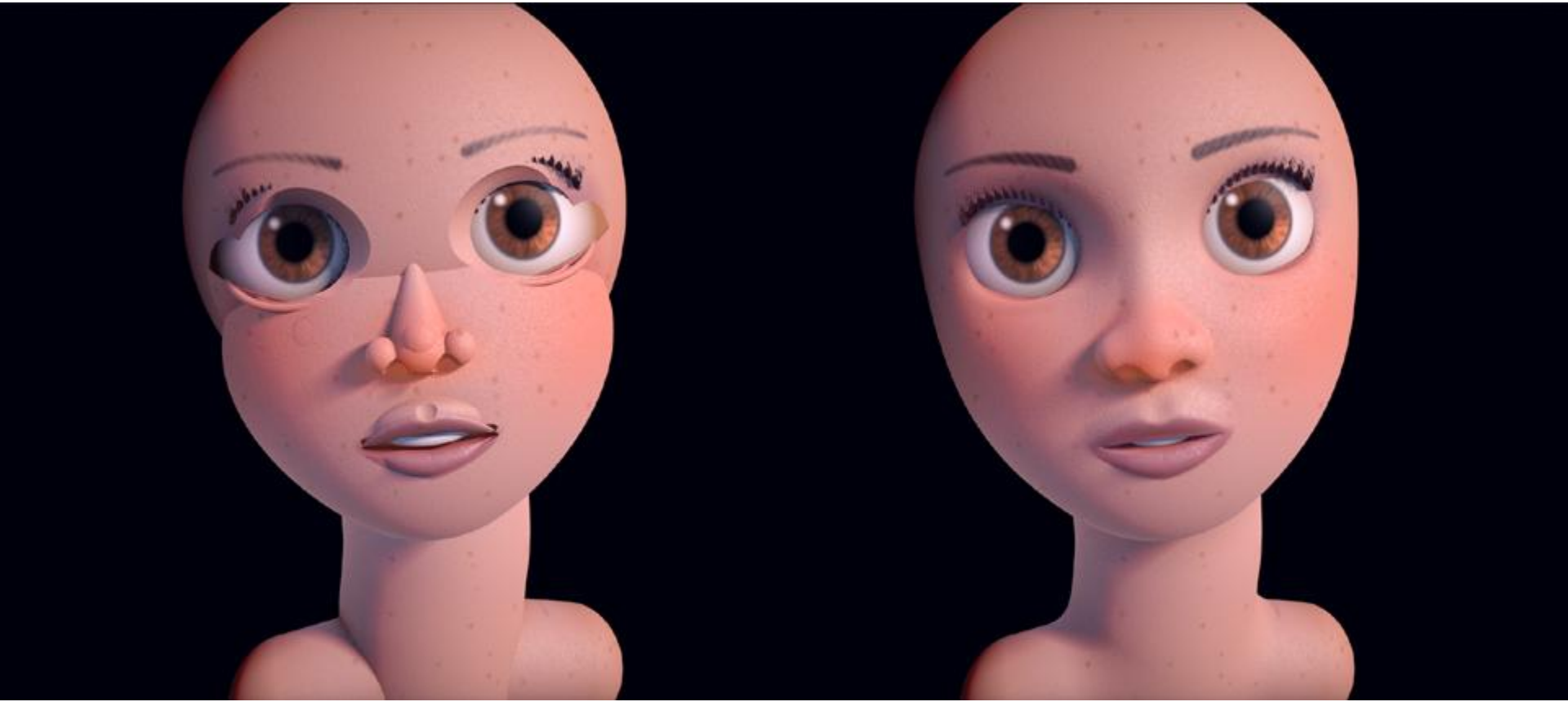
blending kernel

blending radius

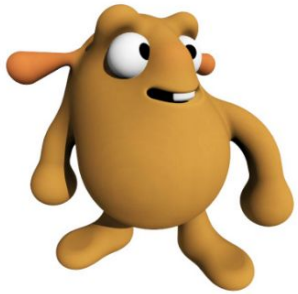
Difference

$$\max(f_1, -f_2) + \phi(|f_1 - f_2|, k)$$

Construction Tree - Smooth Operators



Construction Tree



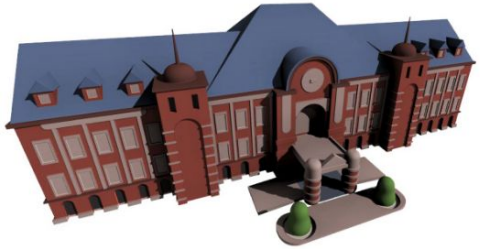
Character $\#N = 43$

Raccoon $\#N = 53$

Console $\#N = 61$

Gameboy $\#N = 65$

Car $\#N = 117$



Train station $\#N = 119$

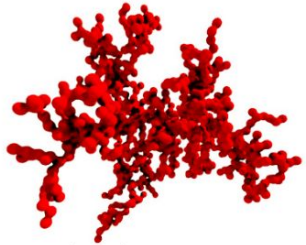
Camera $\#N = 119$

Trees $\#N = 369$

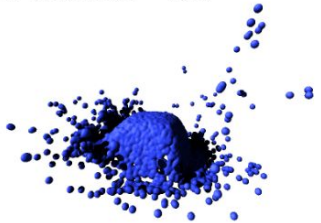
Car chase $\#N = 560$

City $\#N = 691$

Crowd $\#N = 1989$

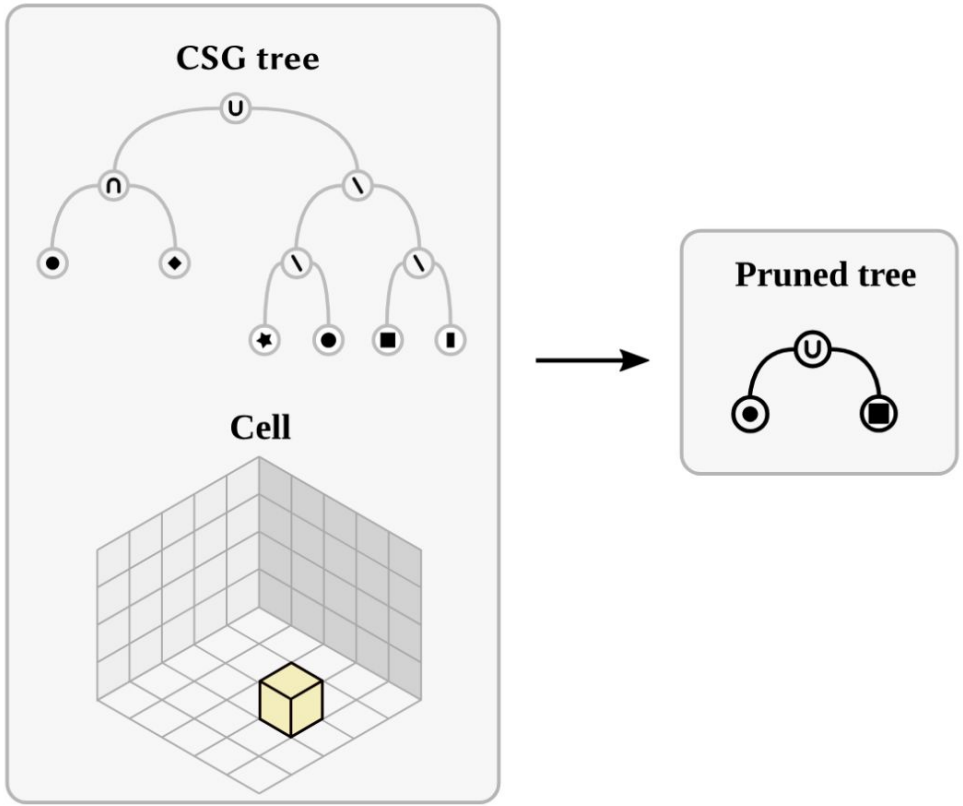


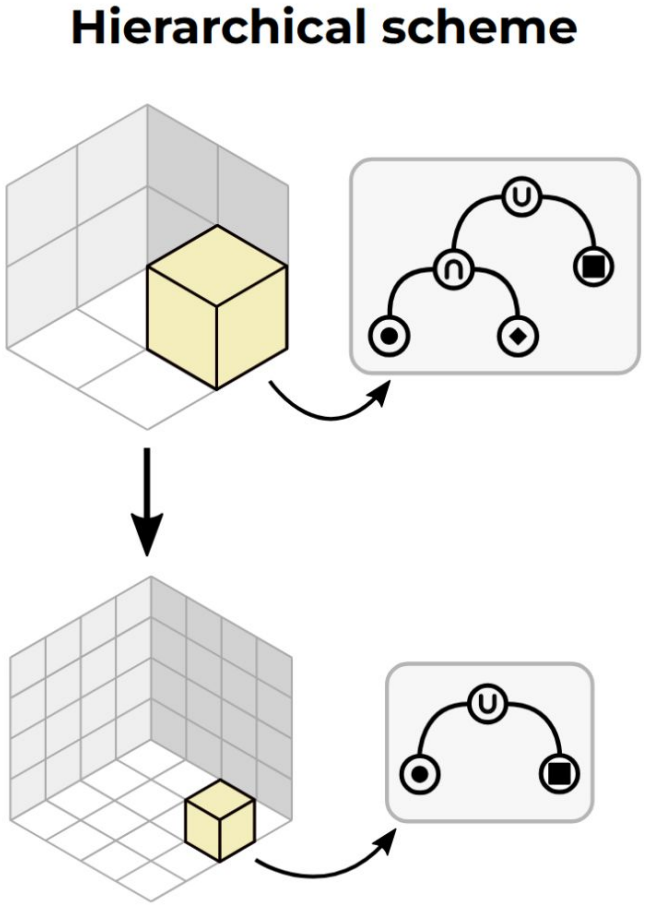
Molecule $\#N = 1999$



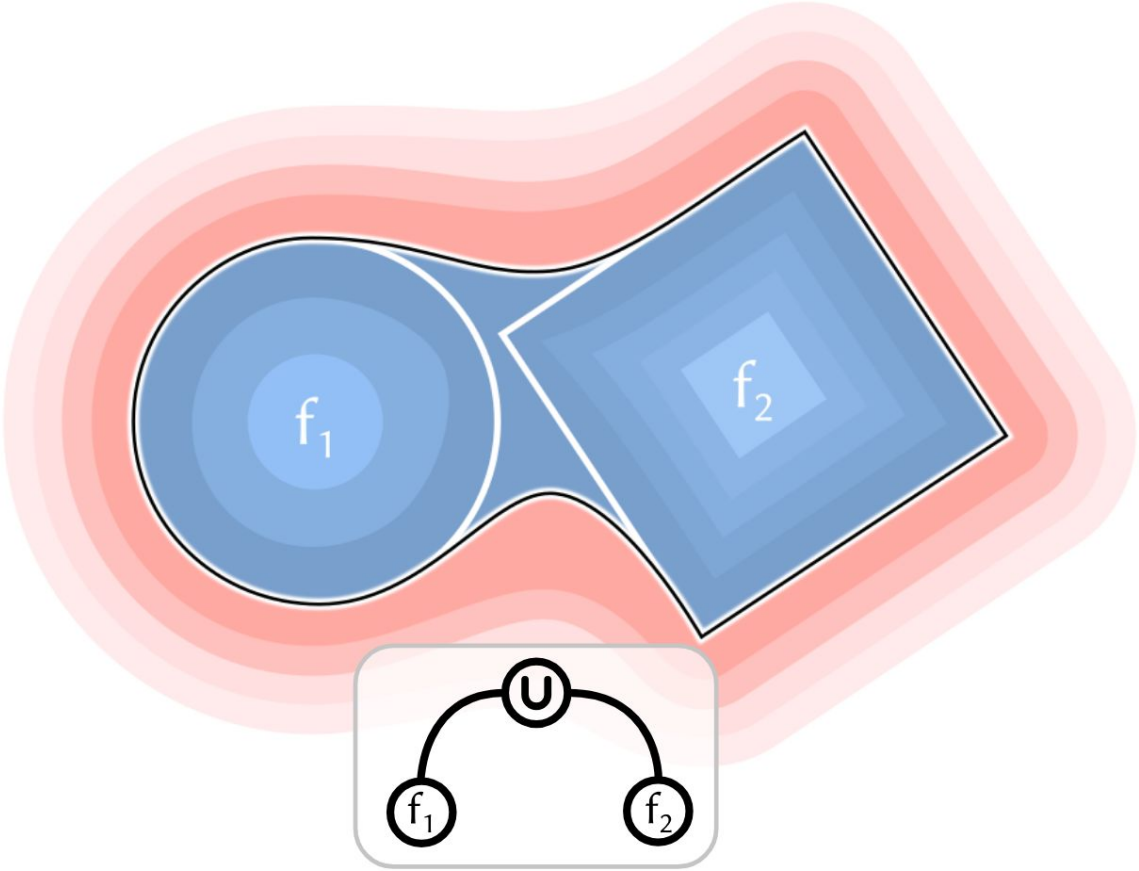
Fluid $\#N = 28k$

Lipschitz Pruning

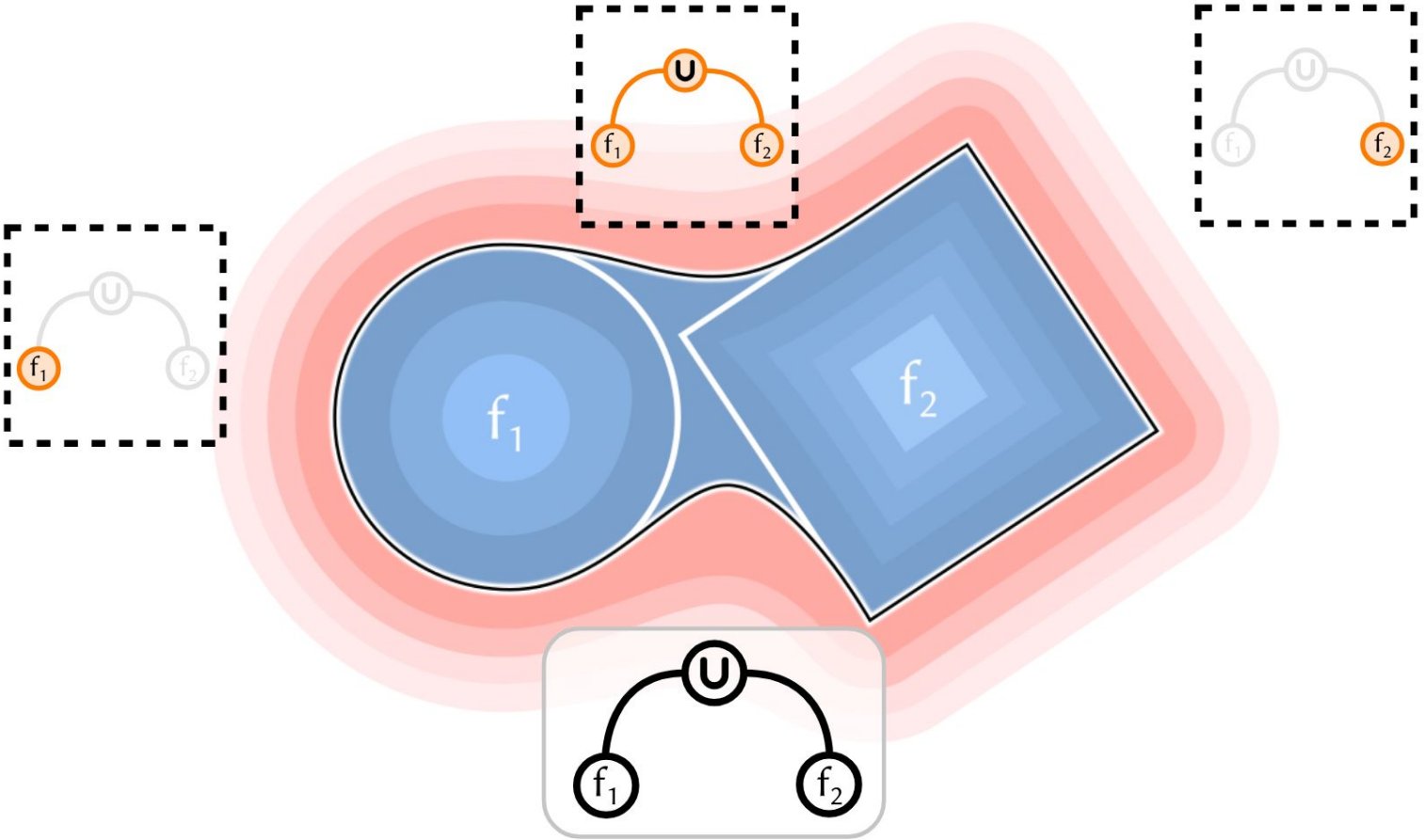




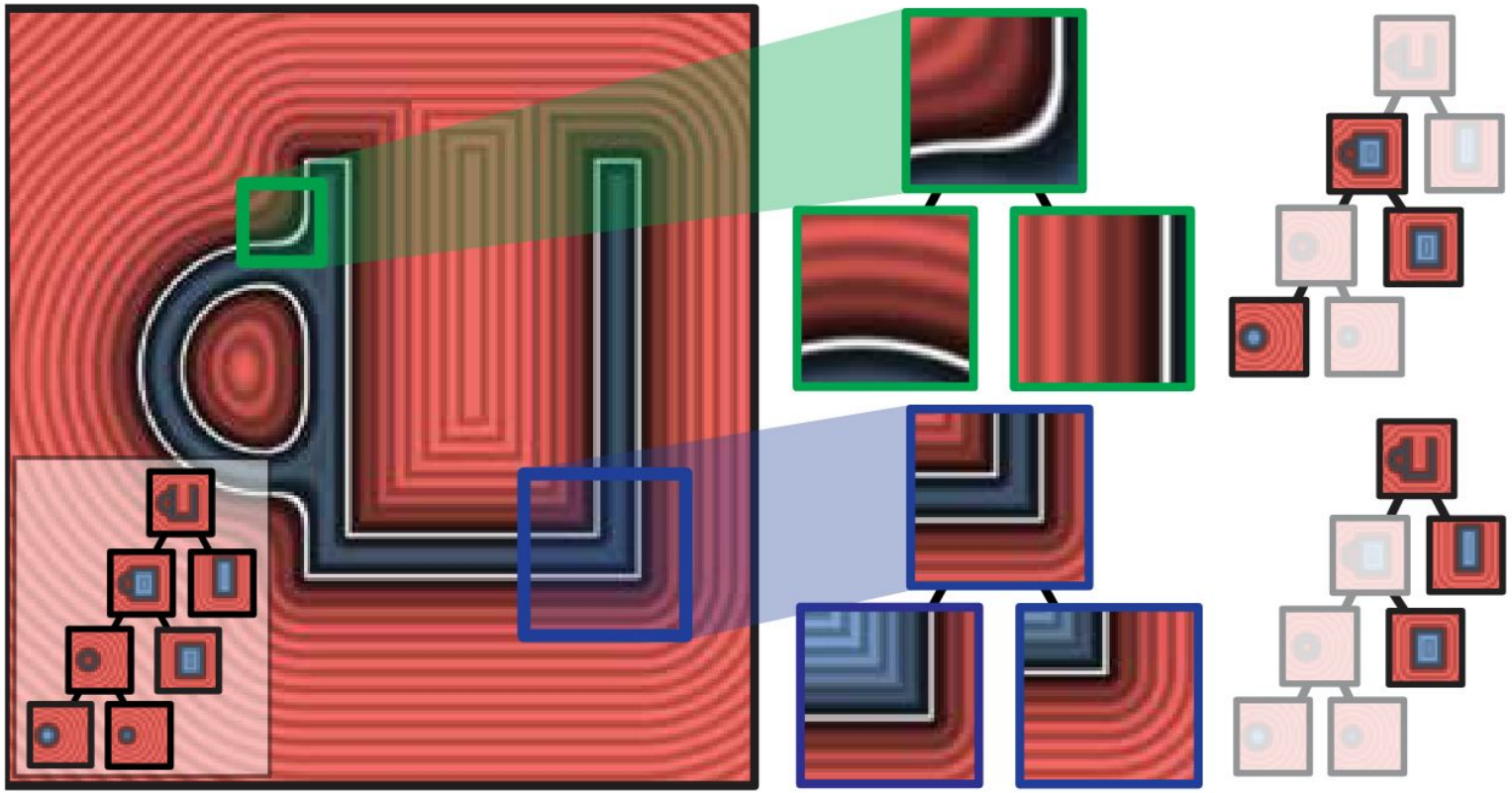
Lipschitz Pruning



Lipschitz Pruning

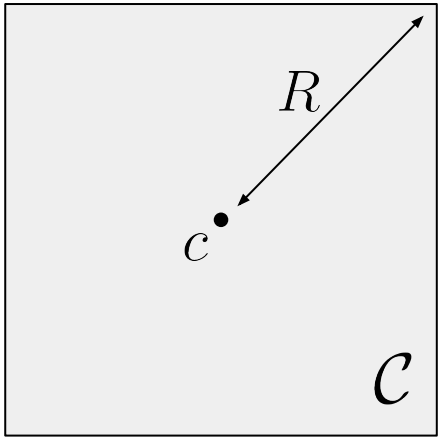


Lipschitz Pruning



(a) Signed Distance Field **(b)** Per-cell pruned tree

Lipschitz Pruning



Union operator: $min(f_1, f_2)$

$f_1(c) = F_1 \qquad f_2(c) = F_2$

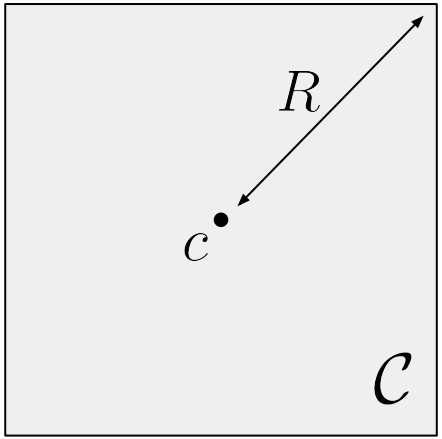
Need a condition on F_1 and F_2 to have :

$$\forall p \in \mathcal{C}, f_1(p) - f_2(p) \geq 0$$

All distance functions are 1-Lipschitz:

$$\forall p \in \mathcal{C}, |f_1(p) - F_1| \leq ||p - c|| \leq R$$

Lipschitz Pruning



$$|f_1(p) - F_1| \leq ||p - c|| \leq R$$

$$\iff -R \leq f_1(p) - F_1 \leq R$$

$$\iff F_1 - R \leq f_1(p) \leq F_1 + R$$

$$F_2 - R \leq f_2(p) \leq F_2 + R$$

$$F_1 - F_2 - 2R \leq f_1(p) - f_2(p) \leq F_1 - F_2 + 2R$$

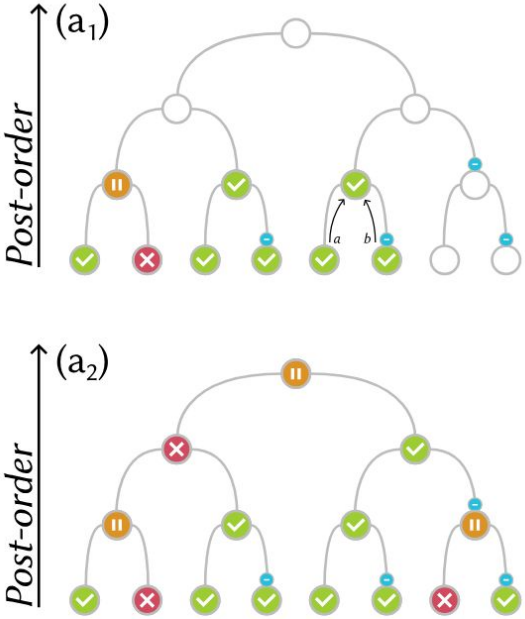
$$0 \leq F_1 - F_2 - 2R$$

$$\boxed{2R \leq F_1 - F_2}$$

If this is verified, f_1 doesn't need to be evaluated in $\mathcal{C}$.

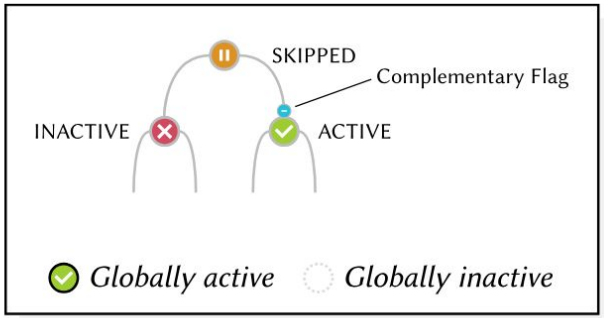
$2R + k \leq F_1 - F_2$ (for smooth operators)

Lipschitz Pruning

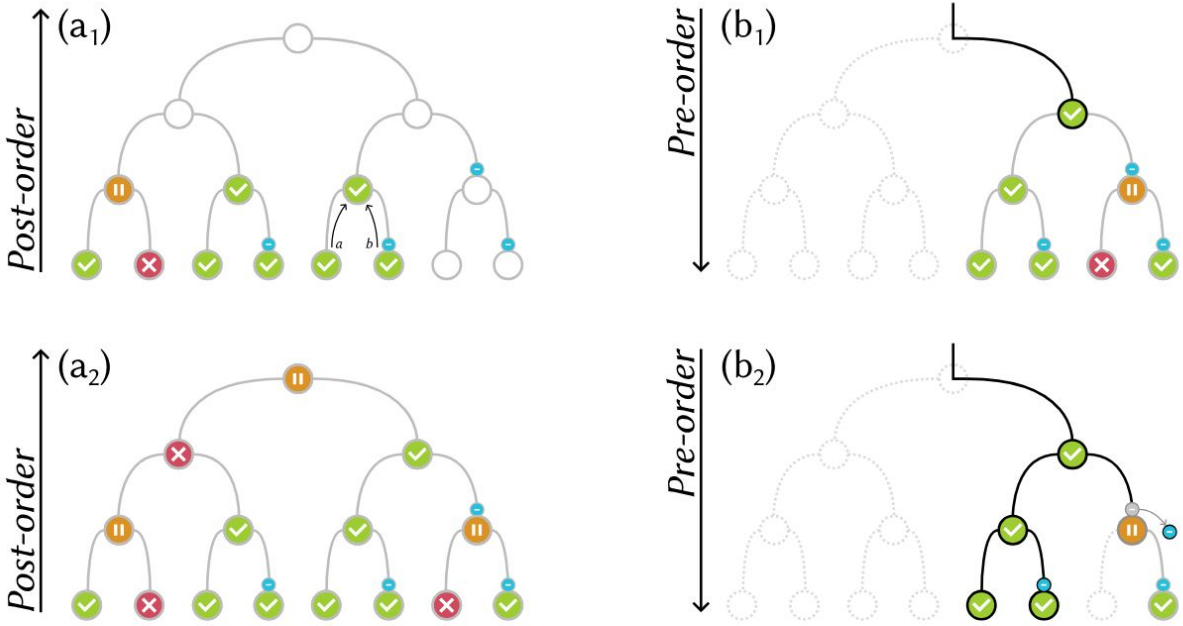


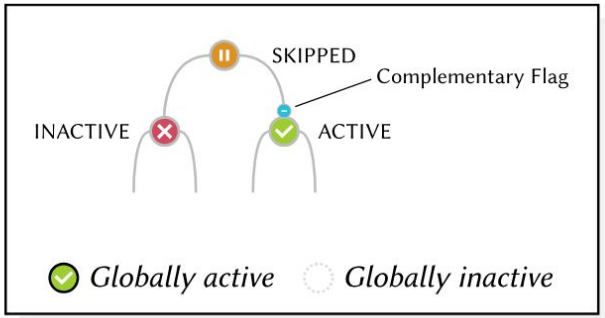
(a) First traversal

Evaluating local state



Lipschitz Pruning





(a) First traversal

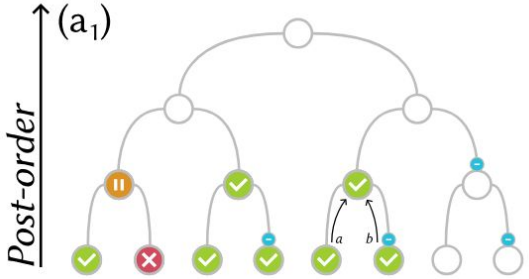
Evaluating local state

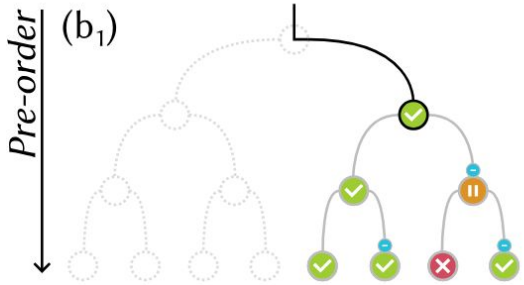
(b) Second traversal

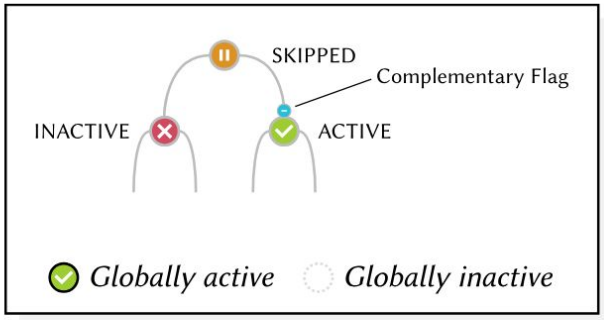
Pruning inactive subtrees and skipped nodes

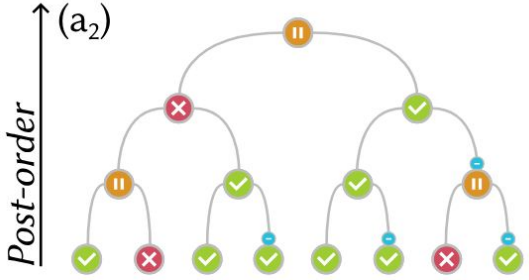
Propagating complementary flags

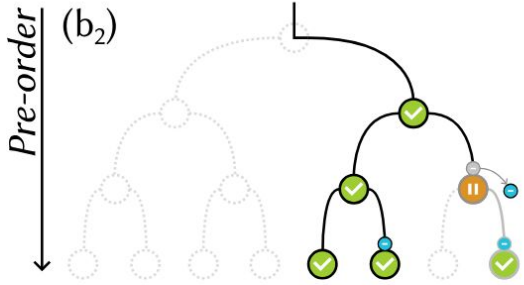
Lipschitz Pruning

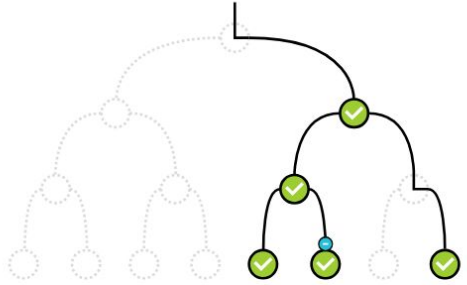












(a) First traversal

Evaluating local state

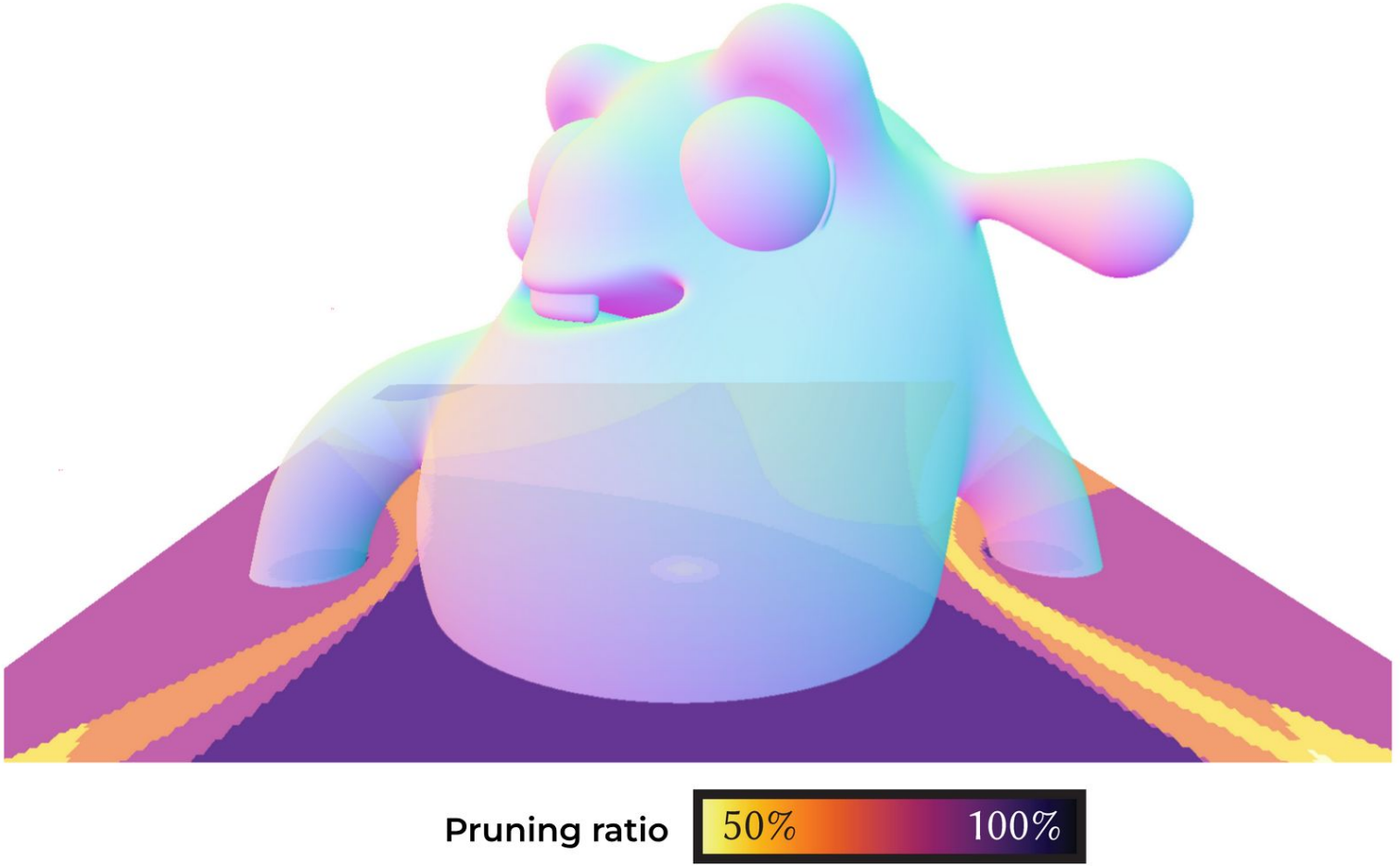
(b) Second traversal

Pruning inactive subtrees and skipped nodes
Propagating complementary flags

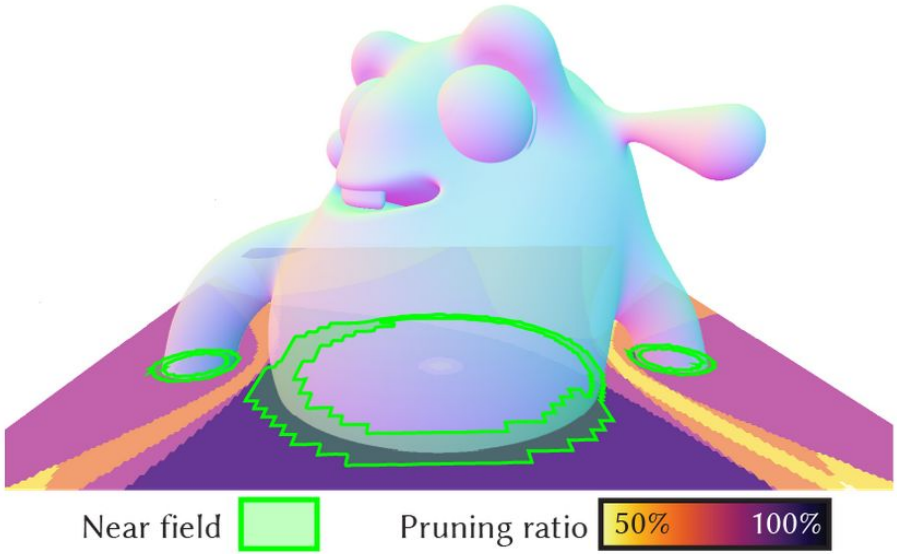
(c) Pruned Smooth CSG tree

New parent assignment
and complementary flag

Lipschitz Pruning - Near Field vs Far Field



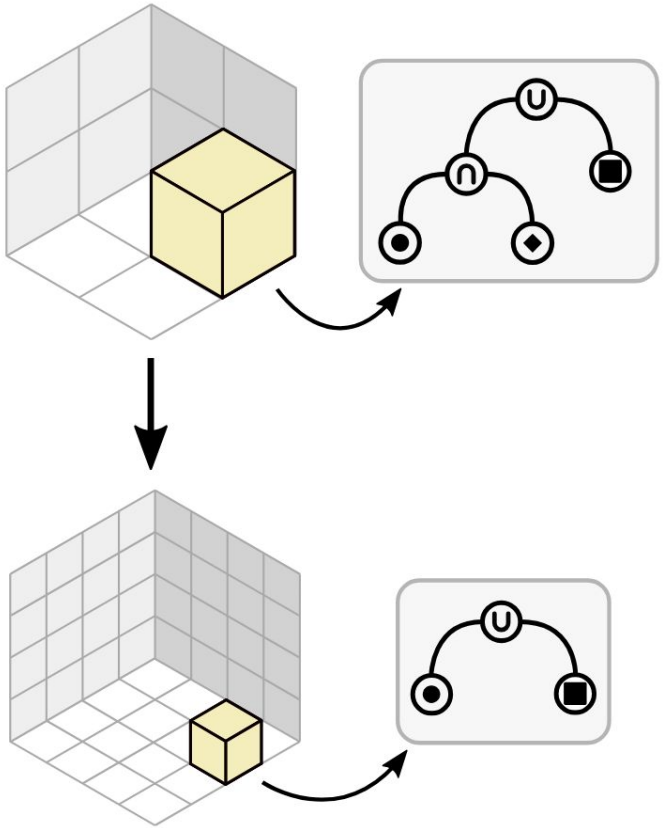
Lipschitz Pruning - Far Field Culling

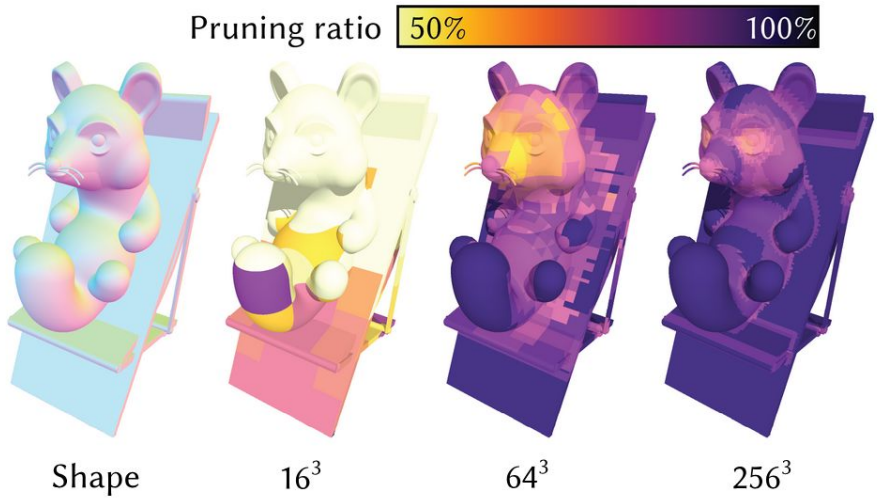


If $|d| > C \cdot R$,

Replace tree
with constant : $sign(d) \cdot (|d| - R)$

Lipschitz Pruning - Hierarchical Scheme





Lipschitz Pruning - Results

	Ours					No far-field culling					No spatial hierarchy					Baseline		
	$\mathcal{T}_p$	$\mathcal{T}_s$	$\mathcal{T}_d$	$\mathcal{M}_p$	$\mathcal{M}_s$	$\mathcal{T}_p$	$\mathcal{T}_s$	$\mathcal{T}_d$	$\mathcal{M}_p$	$\mathcal{M}_s$	$\mathcal{T}_p$	$\mathcal{T}_s$	$\mathcal{T}_d$	$\mathcal{M}_p$	$\mathcal{M}_s$	$\mathcal{T}_s$	$\mathcal{T}_d$	$\mathcal{M}_s$
Camera ($\#\mathcal{N} = 119$)	0.84	**1.47** (x34)	0.91	0.39	0.19	13.84	3.45	4.36	2.28	0.42	-	-	-	11.5*	-	50.31	70.66	0
Car ($\#\mathcal{N} = 117$)	1.17	**2.38** (x23)	1.0	0.46	0.2	4.09	3.32	2.57	1.01	0.32	-	-	-	11.3*	-	56.14	69.07	0
Car Chase ($\#\mathcal{N} = 560$)	2.53	**4.89** (x102)	1.05	0.5	0.2	10.31	8.65	4.4	1.75	0.44	-	-	-	52.8*	-	501.95	334.87	0
Character ($\#\mathcal{N} = 43$)	1.02	**2.79** (x8)	0.99	0.44	0.2	4.76	3.99	3.58	1.23	0.41	25.6	3.6	0.95	4.6	0.20	22.31	26.48	0
City ($\#\mathcal{N} = 691$)	2.52	**5.17** (x150)	1.08	0.52	0.2	12.72	9.7	5.01	2.0	0.45	-	-	-	64.1*	-	778.69	422.7	0
Console ($\#\mathcal{N} = 61$)	0.97	**1.17** (x13)	0.97	0.44	0.2	3.92	1.85	2.56	1.0	0.32	-	-	-	6.1*	-	15.83	36.77	0
Crowd ($\#\mathcal{N} = 1989$)	5.53	**8.35** (x227)	1.02	0.54	0.2	-	-	-	-	-	-	-	-	186*	-	1821	1223	0
Fluid ($\#\mathcal{N} = 27977$)	125.54	**78.52** (x-)	6.99	5.68	0.42	-	-	-	-	-	-	-	-	2625*	-	×	×	0
Gameboy ($\#\mathcal{N} = 65$)	0.72	**2.02** (x15)	0.91	0.38	0.19	10.14	4.39	4.0	1.92	0.43	-	-	-	6.5*	-	31.11	39.09	0
Molecule ($\#\mathcal{N} = 1999$)	8.9	**6.97** (x86)	1.48	1.47	0.21	-	-	-	-	-	-	-	-	188*	-	601.36	841.65	0
Monument ($\#\mathcal{N} = 6023$)	14.29	**15.15** (x629)	1.61	0.96	0.22	-	-	-	-	-	-	-	-	565*	-	9448	3604	0
Raccoon ($\#\mathcal{N} = 53$)	0.85	**2.18** (x9)	0.96	0.41	0.2	3.59	3.03	2.45	0.94	0.31	31.9	2.8	0.95	5.5	0.19	20.64	33.01	0
Train station ($\#\mathcal{N} = 119$)	1.53	**2.81** (x22)	1.12	0.51	0.21	6.94	4.65	3.73	1.38	0.38	-	-	-	11.5*	-	63.9	72.41	0
Trees ($\#\mathcal{N} = 369$)	2.73	**13.01** (x20)	1.24	0.64	0.21	-	-	-	-	-	-	-	-	34.9*	-	266.87	216.56	0

Table 1: *Statistics for the different scenes shown throughout our paper with $\#\mathcal{N}$ nodes, measured on a laptop RTX 4060 with 8GB of GPU memory. We report the pruning time $\mathcal{T}_p$, sphere tracing time $\mathcal{T}_s$, discretization time over a 256^3 grid $\mathcal{T}_d$, memory usage during pruning $\mathcal{M}_p$, and memory usage during tracing $\mathcal{M}_s$. All timings are in milliseconds (**ms**) and memory usage is in gigabytes (**GB**). We report numbers for our method with both spatial hierarchy and far-field culling, and also ablations without these additions, and compare against a naive SDF evaluation baseline. For sphere tracing, we use one primary and one shadow ray over 1920×1080 pixels. Empty cells marked with a dash (−) correspond to tests that overflowed the maximum temporary buffer size (5.7GB), and empty cells marked with a cross (×) correspond to tests where rendering or discretization did not finish after a few minutes. Cells reporting memory marked with a star (*) give only a theoretical lower-bound memory usage that would be required for the pruning without spatial hierarchy.*

Lipschitz Pruning - Actives Nodes Statistics

	Ours		**No far-field culling**		**Baseline**
	Avg $\pm$ std. dev.	Max	Avg $\pm$ std. dev.	Max	$\#\mathcal{N}$
Camera	**1.0** $\pm$ **0.01**	**7.0**	2.36 $\pm$ 2.93	15.0	119
Car	**1.03** $\pm$ **0.05**	**6.0**	1.51 $\pm$ 0.5	8.0	117
Car Chase	**1.06** $\pm$ **0.15**	**33.0**	2.52 $\pm$ 3.68	46.0	560
Character	**1.03** $\pm$ **0.06**	**8.0**	2.27 $\pm$ 1.39	8.0	43
City	**1.05** $\pm$ **0.14**	**14.0**	2.56 $\pm$ 2.82	24.0	691
Console	**1.02** $\pm$ **0.02**	**5.0**	1.56 $\pm$ 0.89	11.0	61
Crowd	**1.03** $\pm$ **0.17**	**17.0**	-	-	1989
Fluid	**2.83** $\pm$ **776.07**	**1202.0**	-	-	27 977
Gameboy	**1.0** $\pm$ **0.0**	**9.0**	2.44 $\pm$ 1.75	15.0	65
Molecule	**1.12** $\pm$ **0.98**	**62.0**	-	-	1999
Monument	**1.17** $\pm$ **0.81**	**45.0**	-	-	6023
Raccoon	**1.01** $\pm$ **0.02**	**8.0**	1.43 $\pm$ 0.87	10.0	53
Train station	**1.05** $\pm$ **0.11**	**10.0**	1.98 $\pm$ 1.99	16.0	119
Trees	**1.1** $\pm$ **0.97**	**36.0**	-	-	369