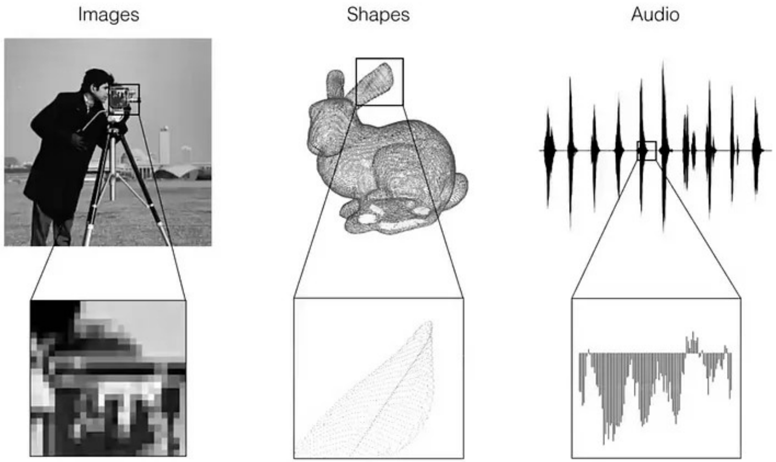
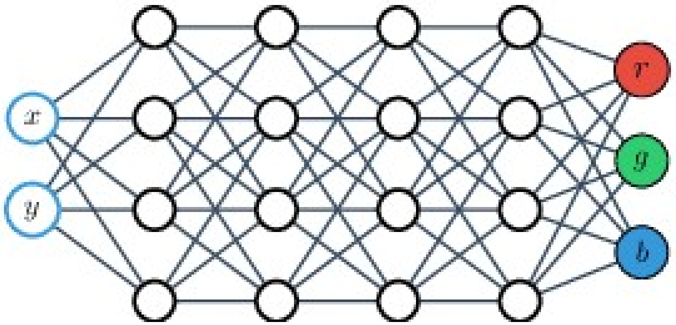
Groupe de lecture :

Multiplicative Filter Networks

Buonomo Camille,
4/12/2024,

1. Implicit neural representations

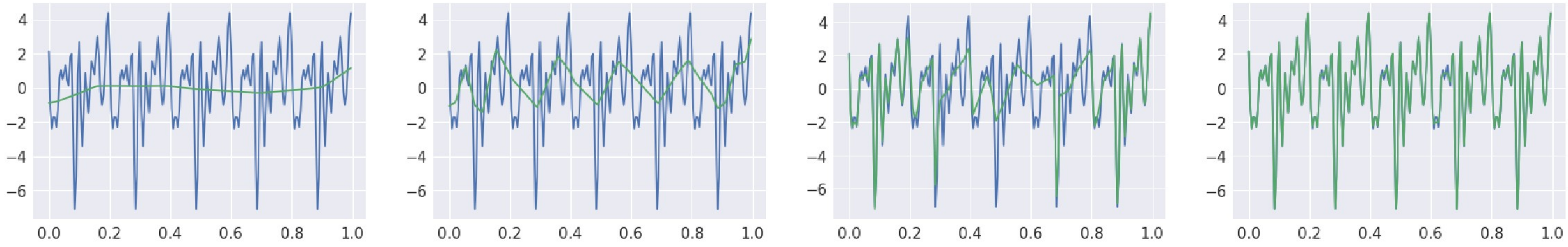




Représentation discrète

Représentation continue

2.Initial limitations



(a) Iteration 100 (b) Iteration 1000 (c) Iteration 10000 (d) Iteration 80000

[Rahaman et al, On the Spectral Bias of Neural Networks, 2019]

---Ground Truth
---Model

→ Achitectures usuelles biaisées vers les basses fréquences
→ Reste à apprendre : les hautes fréquences

3. Positional encoding

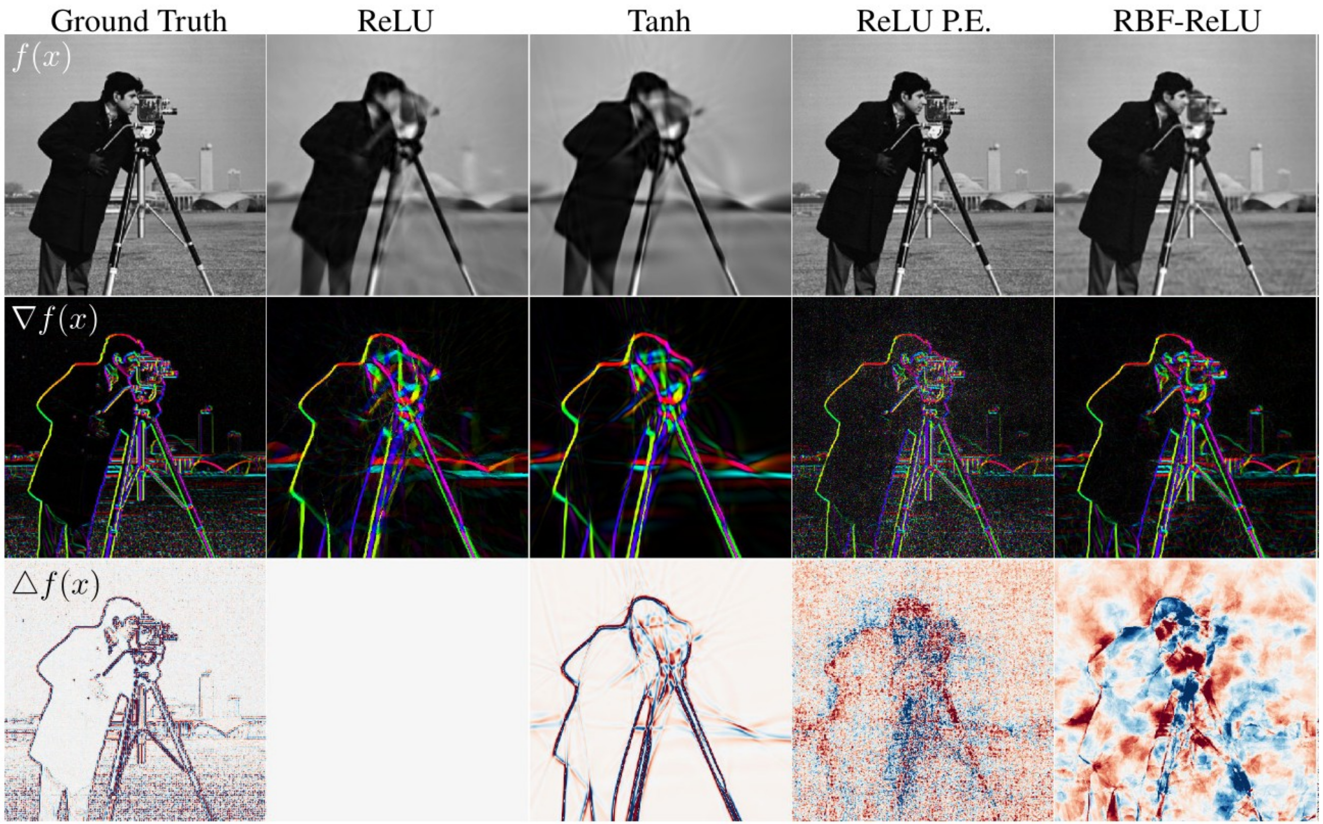
$$\gamma(\mathbf{v}) = \left[a_1 \cos(2\pi \mathbf{b}_1^{\mathrm{T}} \mathbf{v}), a_1 \sin(2\pi \mathbf{b}_1^{\mathrm{T}} \mathbf{v}), \ldots, a_m \cos(2\pi \mathbf{b}_m^{\mathrm{T}} \mathbf{v}), a_m \sin(2\pi \mathbf{b}_m^{\mathrm{T}} \mathbf{v})\right]^{\mathrm{T}}$$

$$\gamma(p) = \left(\sin\left(2^0 \pi p\right), \cos\left(2^0 \pi p\right), \cdots, \sin\left(2^{L-1} \pi p\right), \cos\left(2^{L-1} \pi p\right) \right)$$

$$\gamma_i(\mathbf{x}) = \frac{1}{\sqrt{2L+1}} \left(x_i, \frac{\cos(2^0 \pi \mathbf{x}_i)}{2^0 \pi}, \frac{\sin(2^0 \pi \mathbf{x}_i)}{2^0 \pi}, \ldots, \frac{\cos(2^L \pi \mathbf{x}_i)}{2^L \pi}, \frac{\sin(2^L \pi \mathbf{x}_i)}{2^L \pi} \right)$$

...

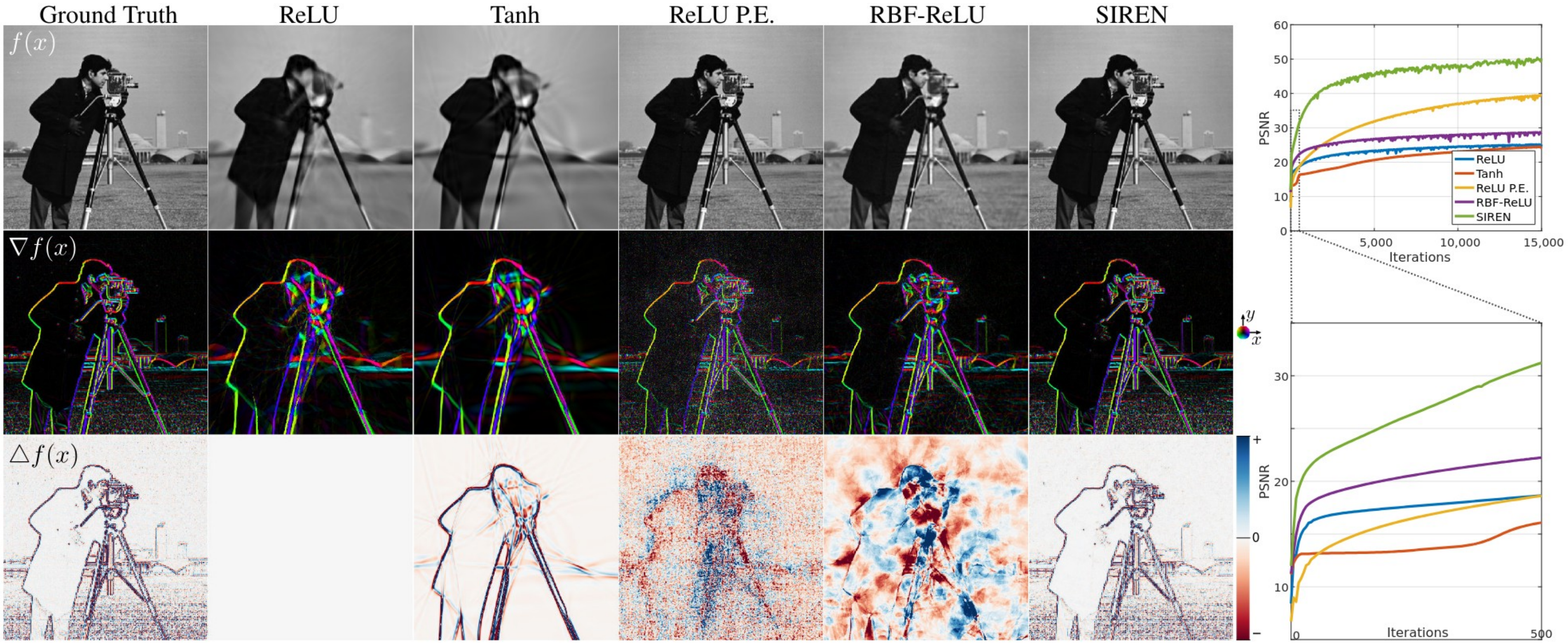
3. Positional encoding - limitations



[Sitzmann et al, Implicit Neural Representations with Periodic Activation Functions,2020]

4. Periodic Activation Functions - SIREN

σ(x) = sin(ω)



5. Multiplicative Filter Network - Architecture

Traditional NN :

$$z^{(1)} = x$$
$$z^{(i+1)} = \sigma\left(W^{(i)} z^{(i)} + b^{(i)}\right), \ i = 1, \ldots, k-1$$
$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$

MFN :

$$z^{(1)} = g\left(x; \theta^{(1)}\right)$$
$$z^{(i+1)} = \left(W^{(i)} z^{(i)} + b^{(i)}\right) \circ g\left(x; \theta^{(i+1)}\right), \ i = 1, \ldots, k-1$$
$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$

5. Multiplicative Filter Network – Choix de g

$$g(x;\theta^{(i)}) = \sin(\omega^{(i)}x + \phi^{(i)})$$ → FourrierNet

$$g_j(x;\theta^{(i)}) = \exp\left(-\frac{\gamma_j^{(i)}}{2}\left\|x - \mu_j^{(i)}\right\|_2^2\right)\sin\left(\omega_j^{(i)}x + \phi_j^{(i)}\right)$$ → GaborNet

Pourquoi ?

5. Multiplicative Filter Network – Choix de g

Propriété voulu : Somme multiplicative

$$g(x;\theta^{(1)}) \circ g(x;\theta^{(2)}) = \sum_{i=1}^{N} \beta_i g(x;\bar{\theta}^{(i)})$$

Sinus : $\sin(a)\sin(b)=\frac{\cos(a-b)-\cos(a+b)}{2}$

Gabor : $\sin(a)e^{c}\sin(b)e^{d}=\frac{\cos(a-b)-\cos(a+b)}{2}e^{b+d}$

$$a(x-b)^2\cdot c(x-d)^2=ac\frac{(b-d)^2}{a+c}+(a+c)\left(x-\frac{ab+cd}{a+c}\right)^2$$

5. Multiplicative Filter Network – Representation

Hypothèse : $$z_j^{(i-1)} = \sum_{t=1}^{N} \alpha_t^j g(x; \tau_j^{(t)})$$

Récurence : $$\begin{aligned} z_j^{(i)} &= \left(\sum_{p=1}^{d_i} W_{jp}^{(i)} z_p^{(i-1)} + b^{(i)} \right) g(x; \theta_j^{(i)}) \\ &= \left(\sum_{p=1}^{d_i} W_{jp}^{(i)} \sum_{t=1}^{T} \alpha_t^p g(x; \tau_p^{(t)}) + b_j^{(i)} \right) g(x; \theta_j^{(i)}) \\ &= \sum_{p=1}^{d_i} \sum_{t=1}^{T} W_{jp}^{(i)} \alpha_t^p g(x; \tau_p^{(t)}) g(x; \theta_j^{(i)}) + b_j^{(i)} g(x; \theta_j^{(i)}) \\ &= \sum_{p=1}^{d_i} \sum_{t=1}^{T} W_{jp}^{(i)} \alpha_t^p \sum_{q=1}^{N} \beta_q g(x; \bar{\tau}_p^{(t,q)}) g(x; \theta_j^{(i)}) + b_j^{(i)} g(x; \theta_j^{(i)}) \\ &= \sum_{t=1}^{T'} \bar{\alpha}_t g(x; \bar{\tau}_j^{(t)}) \end{aligned}$$

$$f_j(x) = \sum_{t=1}^{T} \alpha_t g(x; \tau_j^{(t)}) + c.$$

5. Multiplicative Filter Network – Representation

$$f_j(x) = \sum_{t=1}^{T} \alpha_t g(x; \tau_j^{(t)}) + c.$$

→ Approximation de la décomposition de Fourier/en ondeletttes de la fonction.

Intérêt: T = $O(2^{k-1})$

5. Multiplicative Filter Network – Benchmark

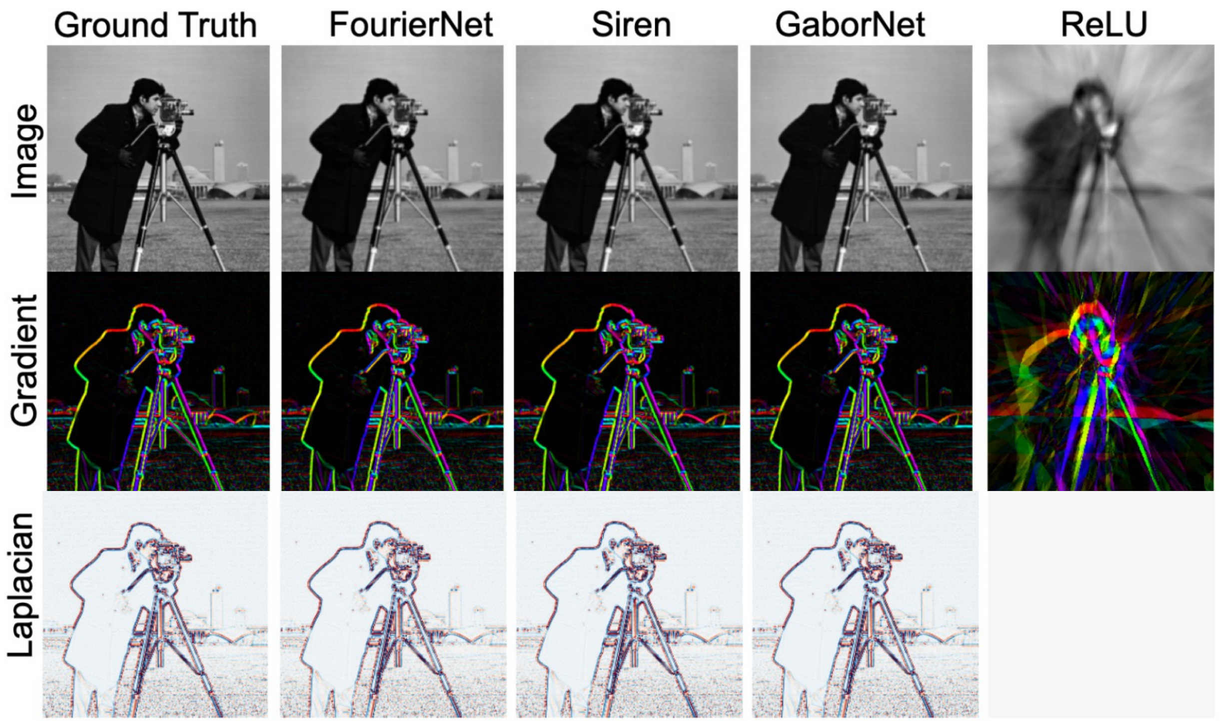
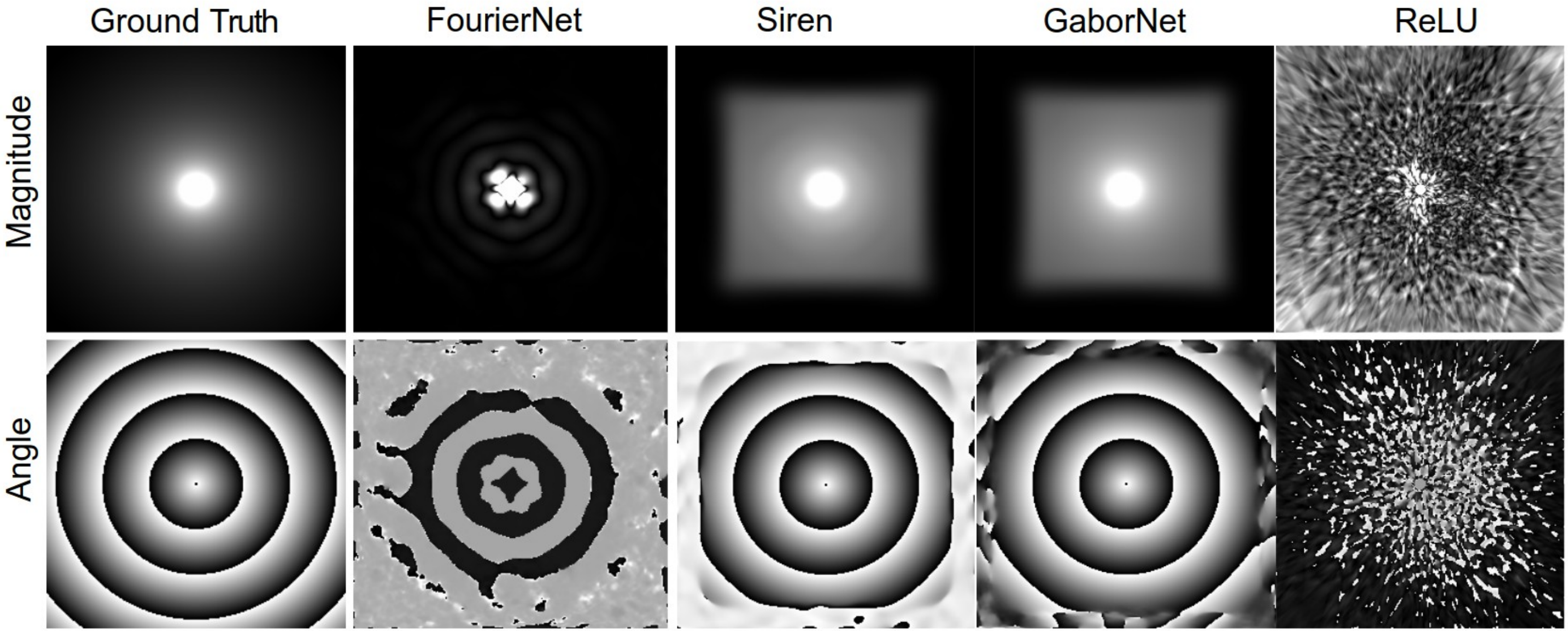


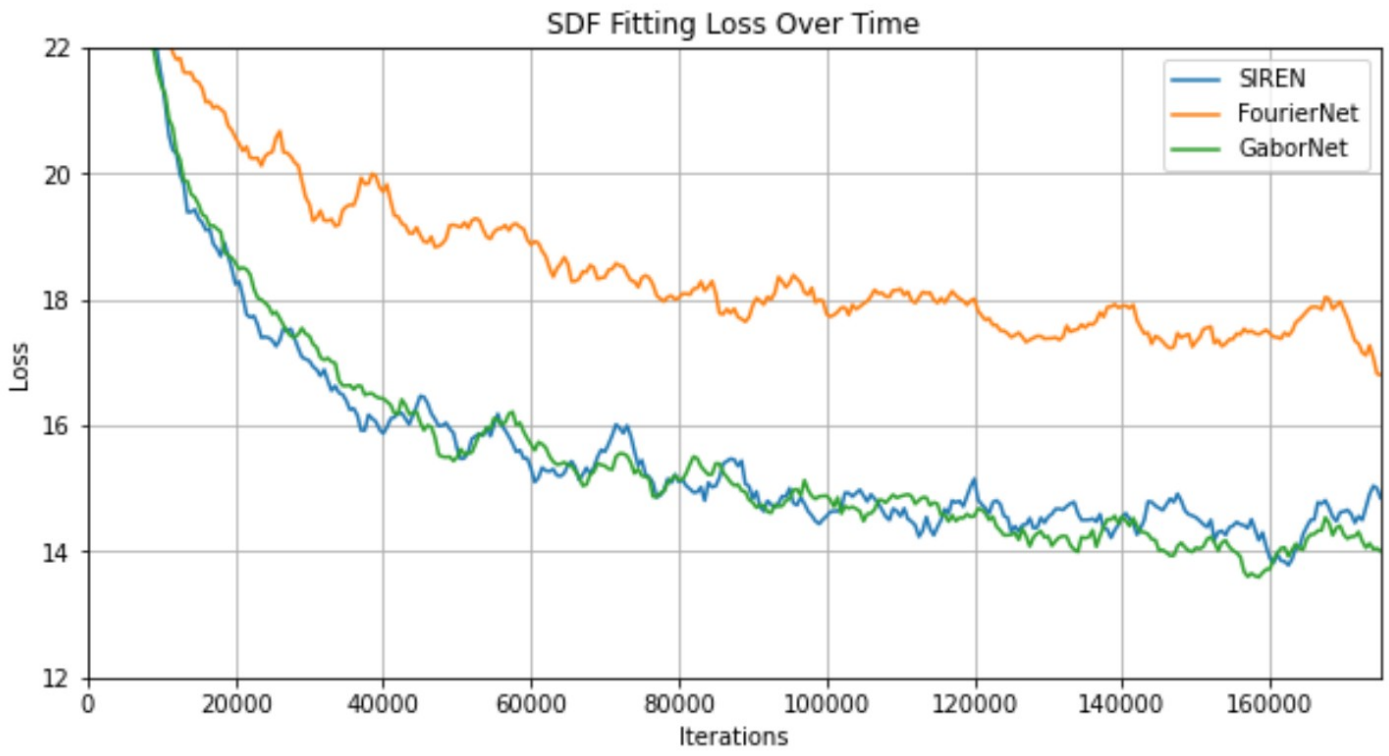
Image reconstruction

5. Multiplicative Filter Network – Benchmark



PDE

5. Multiplicative Filter Network – Benchmark



5. Multiplicative Filter Network – Benchmark