Sphere partitioning

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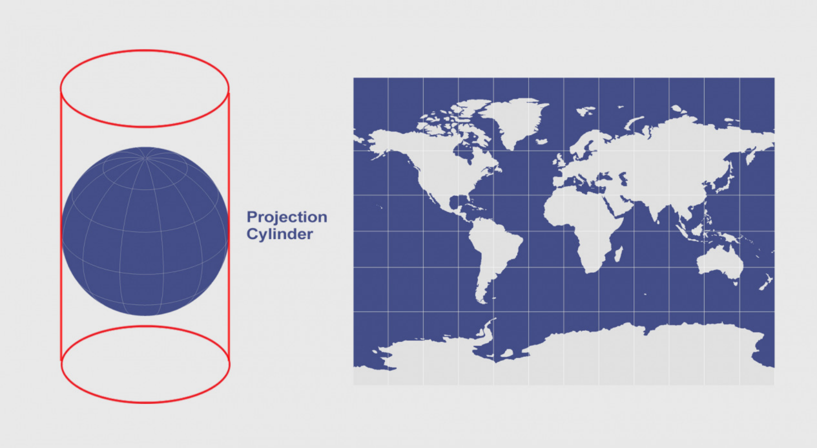
Methods list

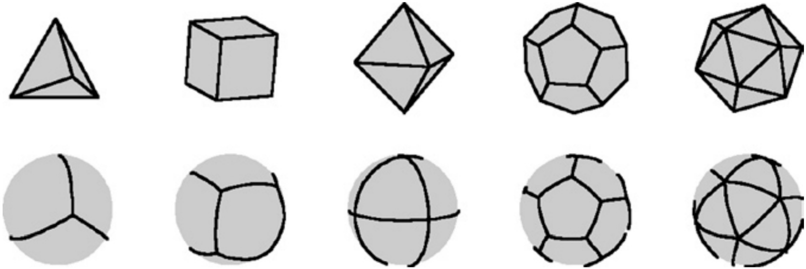
Algorithm	simple	equal area	Cell form
Cylindrical projection	Very	Awful	squares
Geo bisector triangles	Very	Bad	triangle
Geo bisector hexagons	Yes	good	hexagonal
SAC	Nieh	very good	triangle
EQ	Nieh	perfect	squares

Cylindrical projection[1]



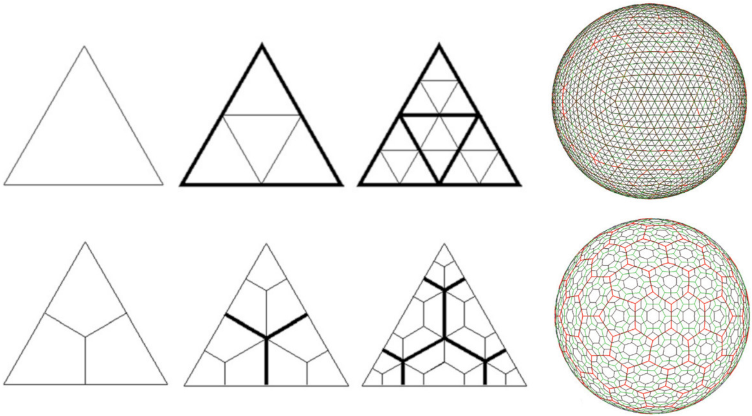
The cylindrical projection of the Earth

Geo bisector[4]



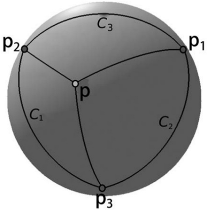
The five platonic solids and there projections on the sphere

Geo bisector[4]

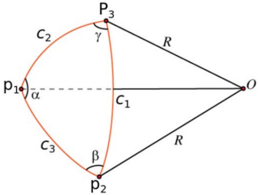


The subdivision of a triangle to form either triangular or hexagonal cells

Spherical Area Coordinates (SAC)[2]



(**a**) The basis of spherical area coordinates (SACs): a point $\boldsymbol{p}$ in the spherical reference triangle.

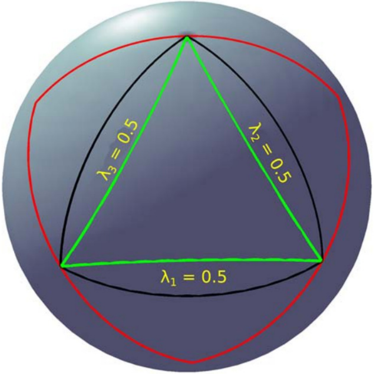


(**b**) The notation of a spherical triangle.

$$\lambda_1 = \frac{S_1}{S} = \frac{\alpha_1+\beta_1+\gamma_1-\pi}{\alpha+\beta+\gamma-\pi} \quad \lambda_2 = \frac{S_2}{S} = \frac{\alpha_2+\beta_2+\gamma_2-\pi}{\alpha+\beta+\gamma-\pi}$$

$$\lambda_3 = \frac{S_3}{S} = \frac{\alpha_3+\beta_3+\gamma_3-\pi}{\alpha+\beta+\gamma-\pi}$$

Spherical Area Coordinates (SAC)[2]



Comparison between geo bisector (Black) and SAC (green) in the first subdivision of a face of a tetrahedron (red)

Spherical Area Coordinates (SAC)[2]

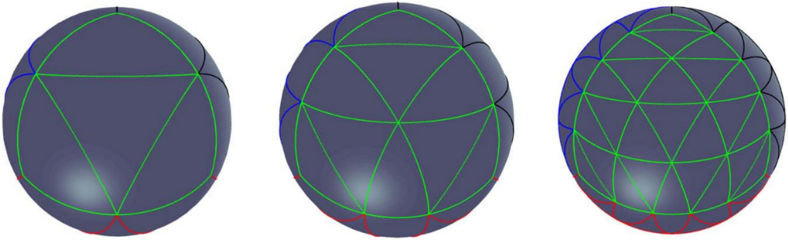
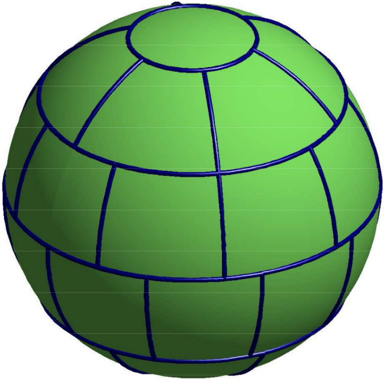


Figure 9. The UV-isogrid $\Delta\lambda = 1/2, 1/3, 1/5$.

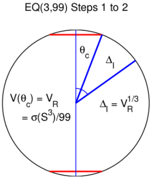
Grid on the sphere using the barycentric coordinate

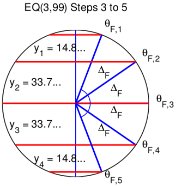
recursive zonal EQual area (EQ)[3]

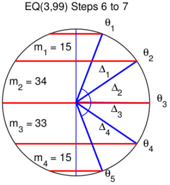


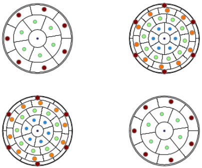
The sphere divided into 33 part of equal area

recursive zonal EQual area (EQ)[3] Algorithm









1. Determine the colatitudes of polar caps
2. Determine an ideal collar angle
3. Determine an ideal number of collars
4. Determine the actual number of collars
5. Create a list of the ideal number of regions in each collar
6. Create a list of the actual number of regions in each collar
7. Create a list of colatitudes of each zone

Annexes, points on platonic solid

Points on a tetrahedron:

- $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
- $(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$
- $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$
- $(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

Points on an octahedron:

- $(\pm 1, 0, 0)$
- $(0, \pm 1, 0)$
- $(0, 0, \pm 1)$

Points on a tetrahedron:

- $(\pm\varphi, \pm 1, 0)$
- $(0, \pm\varphi, \pm 1)$
- $(\pm 1, 0, \pm\varphi)$

Sources

[1] GISGeography.
Cylindrical projection.

[2] Lei, K., Qi, D., and Tian, X.
A new coordinate system for constructing spherical grid systems.
Applied Sciences 10, 2 (2020), 655.

[3] Leopardi, P.
A partition of the unit sphere into regions of equal area and small diameter.
Electronic Transactions on Numerical Analysis 25, 12 (2006), 309–327.

[4] Sahr, K., White, D., and Kimerling, A. J.
Geodesic discrete global grid systems.
Cartography and Geographic Information Science 30, 2 (2003),