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M. Skouras and M. Livesu
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# Medial Axis Aware Learning of Signed Distance Functions

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[1]University of Bonn, Germany

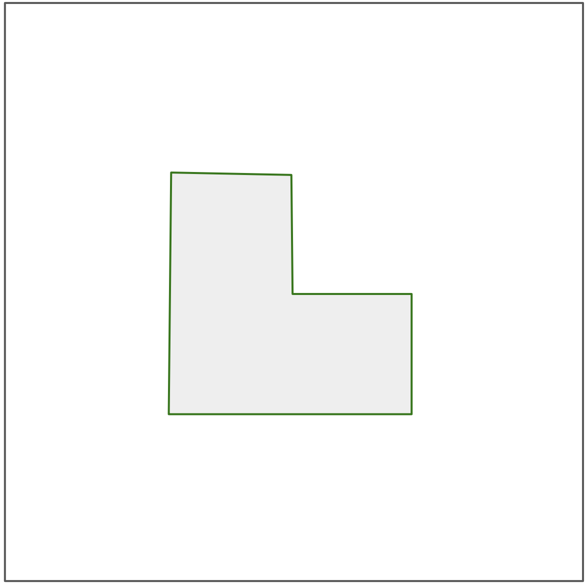
# Introduction: What's the deal with SDFs

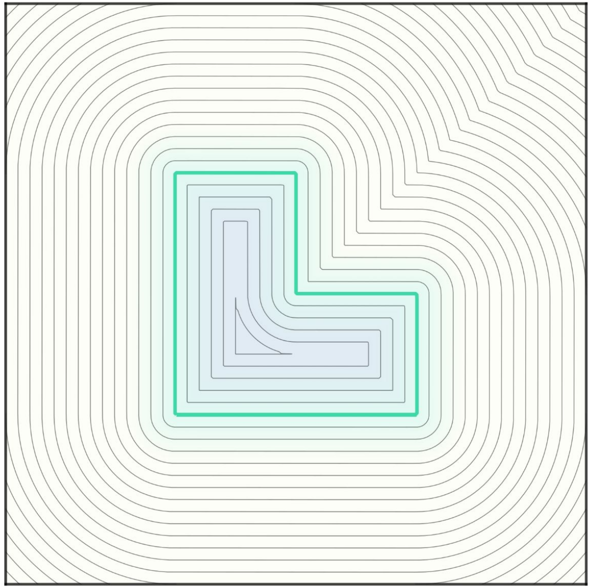
$$\mathrm{sgndist}(x,\mathcal{S}) := \begin{cases} \mathrm{dist}(x,\mathcal{S}) & \text{, if } x \notin \omega \\ -\mathrm{dist}(x,\mathcal{S}) & \text{, if } x \in \omega. \end{cases}$$

$$\|\nabla\phi\| = 1 \quad \text{a.e. in } \Omega.$$

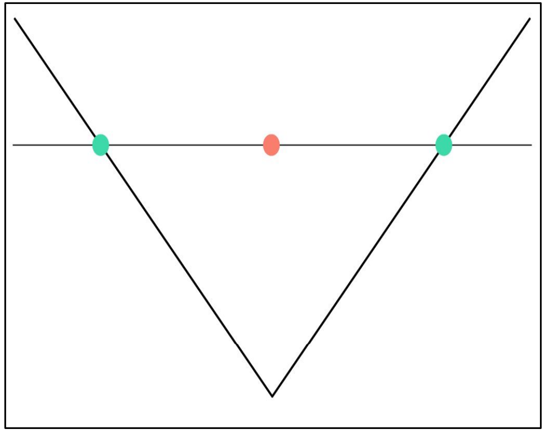
$$D^2\phi\,\nabla\phi = 0$$

$\mathcal{S} = \partial\omega$ est la frontière d'un ouvert (Lipschitz) $\omega \subset \Omega$ un fermé de l'espace euclidien 2D ou 3D.

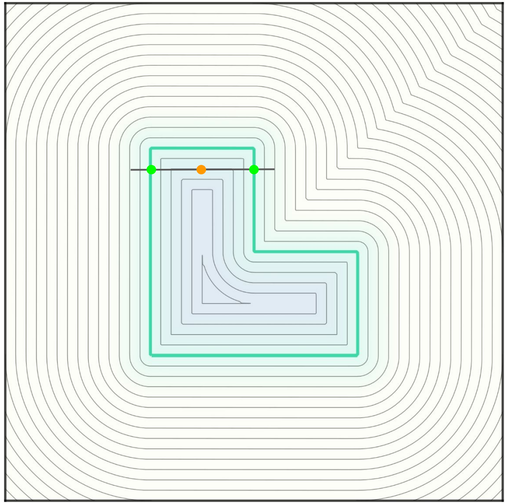




# Introduction: What's the deal with SDF



Zoom sur sur l'intervalle

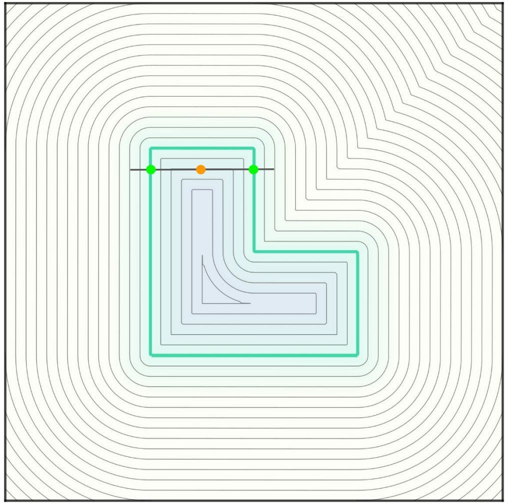


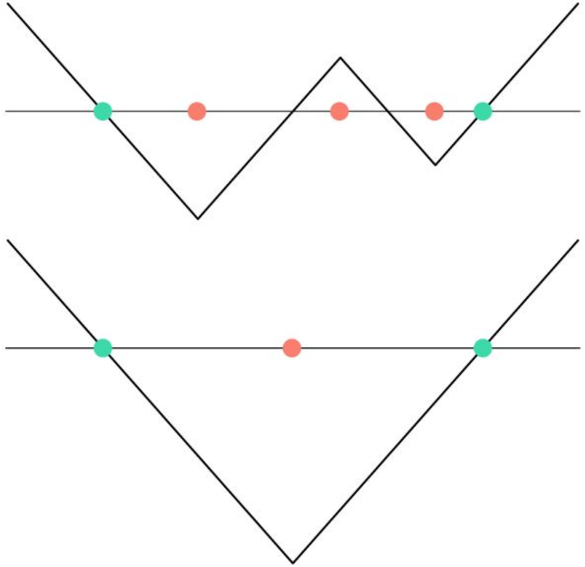
Solutions continues et différentiables

Solutions localement Lipschitz → Différentiable presque partout

Premier

$$\|\nabla\phi\| = 1 \quad \text{a.e. in } \Omega. \Longrightarrow D^2\phi\,\nabla\phi = 0$$





L'ensemble de solution Diff pp est trop grand

# Solution de viscosité

**Loss functionals:**

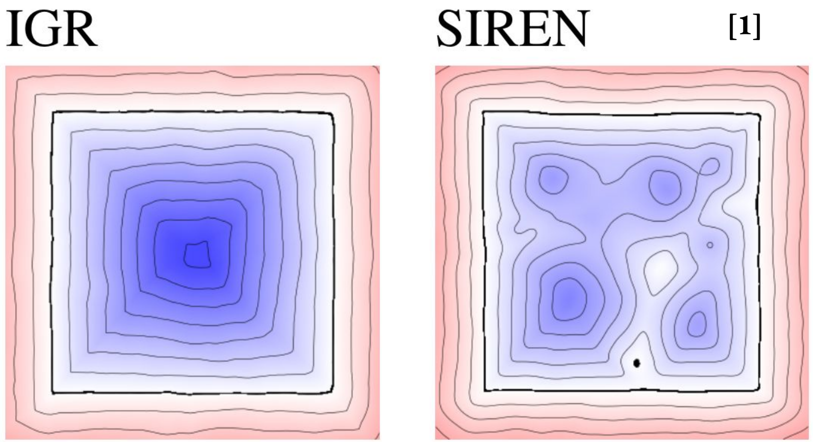
- **Surface:** $\mathcal{L}_{\text{fit}}(\phi) := \int_S \phi^2 \, da$
- **Eikonal:** $\mathcal{L}_{\text{eik}}(\phi) := \int_\Omega \left| \|\nabla \phi\|^2 - 1 \right|^2 \, dx$
- **Viscosity:** $\mathcal{L}_{\exp}(\phi) := \int_\Omega \exp\left(-\alpha |\phi|^2\right) \, dx$

} Standard SDF losses

Les losses sont définies de manière discrète sur le gradient → Aucune propriété explicite du second ordre

# Solution de viscosité

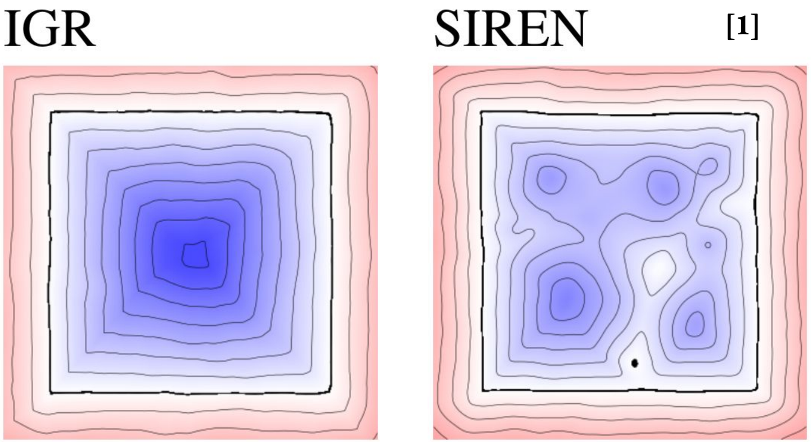
♭ Les losses sont définies de manière discrète sur le gradient → Aucune propriété explicite du second ordre!

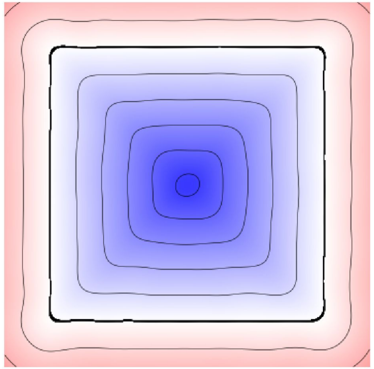



[1] Neural skeleton: Implicit neural representation away from the surface, Clémot, Digne, SMI 2023

# Solution de viscosité

Les losses sont définies de manière discrète sur le gradient $\rightarrow$ Aucune propriété explicite du second ordre!

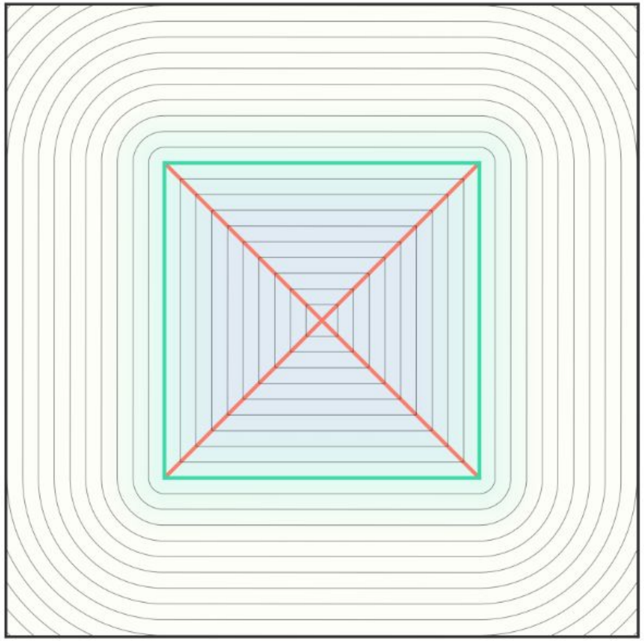


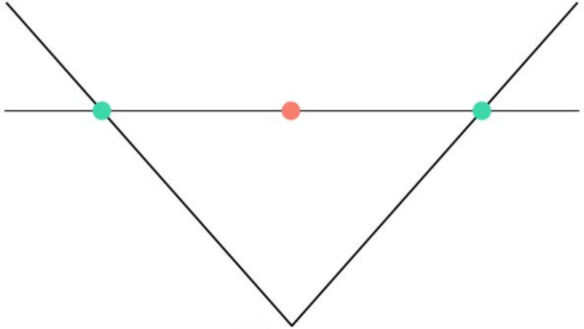
Propriété du second ordre: $\|D^2\phi\,\nabla\phi\|^2\,\mathrm{d}x$

[1] Neural skeleton: Implicit neural representation away from the surface, Clémot, Digne, SMI 2023

# Un $\flat$ additionnel



La solution est différentiable pp **MAIS** son gradient saute (discontinue) sur le medial axis!



$J_{\nabla \phi}$

# Solution de viscosité

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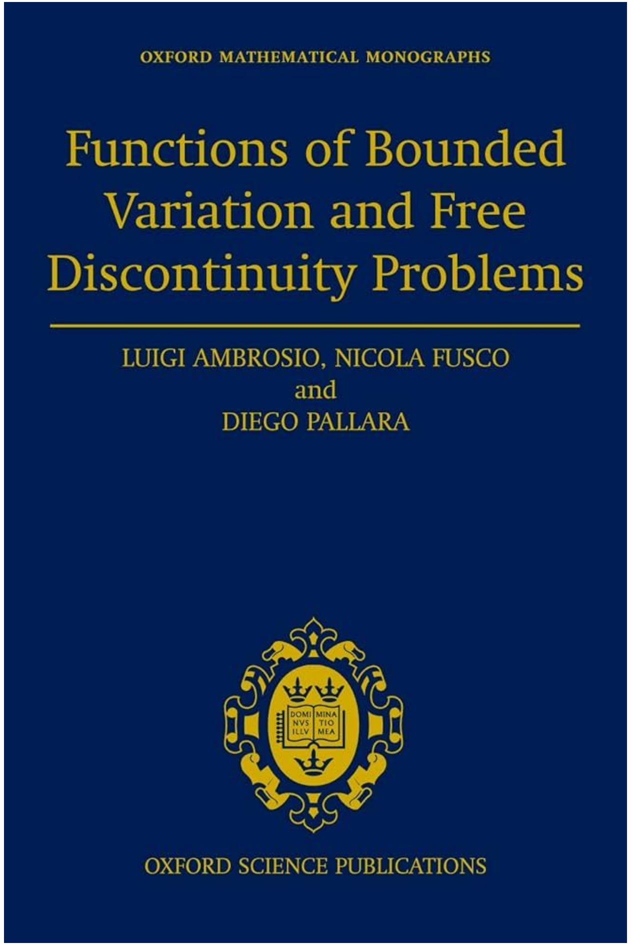
$\Bigg\}$ Standard SDF losses

- **Higher order:** $\displaystyle\int_{\Omega \setminus J_{\nabla\phi}} \|D^2\phi \, \nabla\phi\|^2 \, dx$

On ne connaît pas l'ensemble de discontinuité $J_{\nabla\phi}$ au préalable!
$\rightarrow$ Existe-il une approche variationnel pour régulariser la SDF et cet ensemble?

# Problème de discontinuité libre




# Problème de discontinuité libre classique

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Quelle est la meilleure approximation lisse **par morceaux** ?

[1] Real-Time Minimization of the Piecewise Smooth Mumford-Shah Functional, E. Strekalovskiy, D. Cremers, ECCV, 2014.

# Problème de discontinuité libre classique

L'image est lisse dans cette zone sans pour autant être lisse globalement

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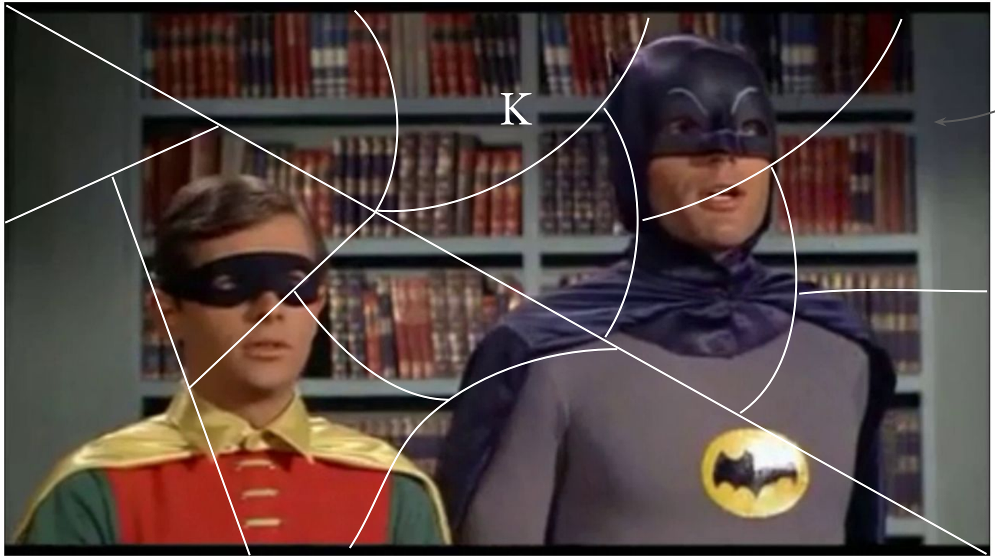


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# Problème de discontinuité libre classique

$$\min_{u,K} \left\{ \boxed{\int_{\Omega} |u - f|^2 \, dx} + \alpha \boxed{\int_{\Omega \setminus K} |\nabla u|^2 \, dx} + \boxed{\lambda \, |K|} \right\}$$

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# Problème de discontinuité libre classique

$$\min_{u,K}\left\{\boxed{\int_{\Omega}|u-f|^2\,dx}+\alpha\boxed{\int_{\Omega\setminus K}|\nabla u|^2\,dx}+\boxed{\lambda\,|K|}\right\}$$

$$\min_{u,s}\boxed{\int_{\Omega}|u-f|^2\,dx}+\alpha\boxed{\int_{\Omega}(1-s)^2|\nabla u|^2\,dx}+\lambda\boxed{\int_{\Omega}\left(\varepsilon|\nabla s|^2+\frac{1}{4\varepsilon}s^2\right)dx}$$

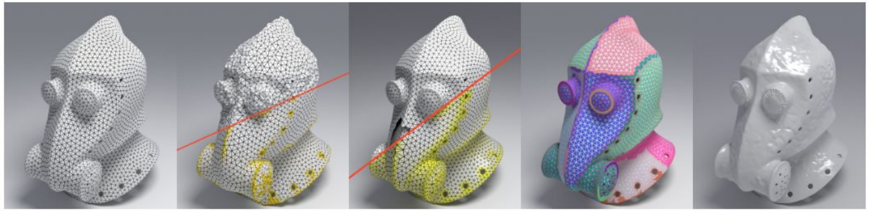
[1] Real-Time Minimization of the Piecewise Smooth Mumford-Shah Functional, E. Strekalovskiy, D. Cremers, ECCV, 2014.

# Cela a été appliqué en GP, pour optimiser des SDF ....

**Mumford-Shah Mesh Processing using the Ambrosio-Tortorelli Functional**

Nicolas Bonneel[*,1], David Coeurjolly[*,1], Pierre Gueth[*,2], Jacques-Olivier Lachaud[*,3]

[*]joint first authors [1]CNRS, Univ. Lyon [2]Arskan [3]Université Savoie Mont Blanc



(a) Original (b) Denoising (c) Inpainting (d) Segmentation (e) Embossing

**Phase Transitions, Distance Functions, and Implicit Neural Representations**

Yaron Lipman [1 2]

**Abstract**

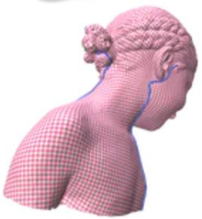
Representing surfaces as zero level sets of neural networks recently emerged as a powerful modeling paradigm, named Implicit Neural Representations (INRs), serving numerous downstream applications in geometric deep learning and 3D vision. Training INRs previously required choosing between occupancy and distance function representation and different losses with unknown limit behavior and/or bias. In this paper we draw inspi-

**Joint optimization of distortion and cut location for mesh parameterization using an Ambrosio-Tortorelli functional**

Colin Weill–Duflos, David Coeurjolly, Fernando de Goes, Jacques-Olivier Lachaud

*Computer Aided Geometric Design (Proc. of GMP 2023)*, July, 2023

PDF Cite Code Slides DOI PDF (lowres)



# De retour à NeatSDF

**Loss functionals:**

- **Surface:** $\mathcal{L}_{\text{fit}}(\phi) := \int_S \phi^2 \, da$
- **Eikonal:** $\mathcal{L}_{\text{eik}}(\phi) := \int_\Omega \left| \|\nabla \phi\|^2 - 1 \right|^2 \, dx$
- **Viscosity:** $\mathcal{L}_{\text{exp}}(\phi) := \int_\Omega \exp\left(-\alpha |\phi|^2\right) \, dx$

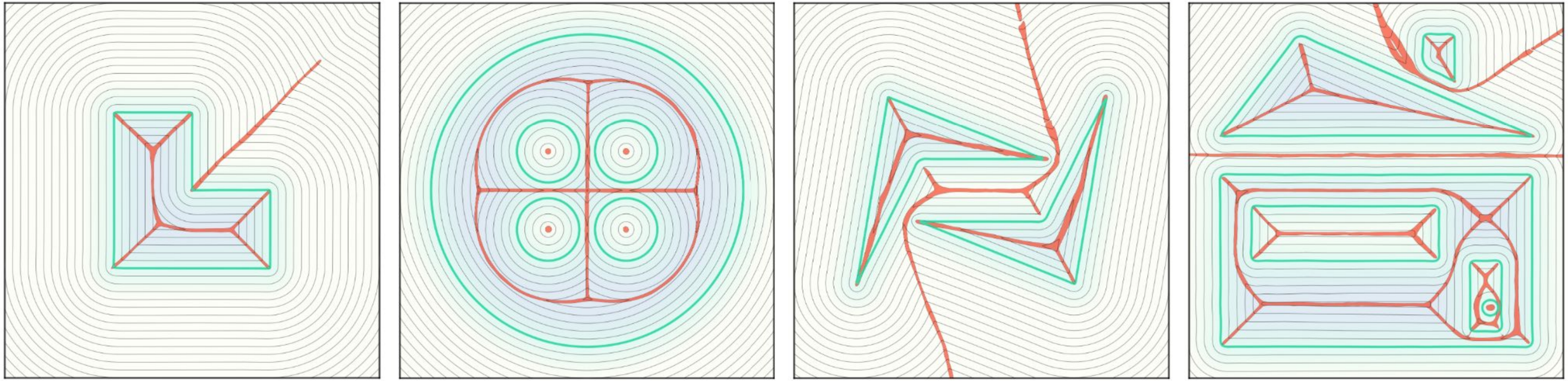
Standard SDF losses

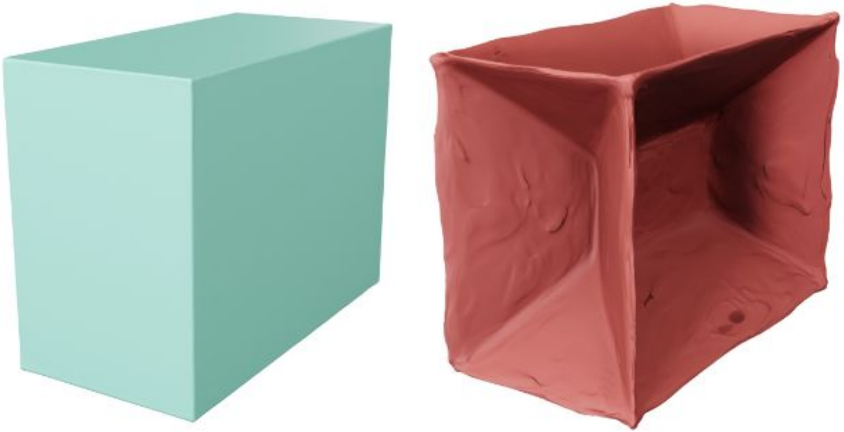
- **Higher order:** $\mathcal{L}_{\text{HO}}(\phi, v) := \int_\Omega v^2 \|D^2\phi \nabla \phi\|^2 + \epsilon^2 \|D^2 \phi\|^2 \, dx$
- **Ambrosio-Tortorelli:** $\mathcal{L}_{\text{AT}}(v) := \int_\Omega \epsilon \|\nabla v\|^2 + \frac{1}{4\epsilon}(v-1)^2 \, dx$

**Total loss.**

Let $\epsilon > 0$, then define

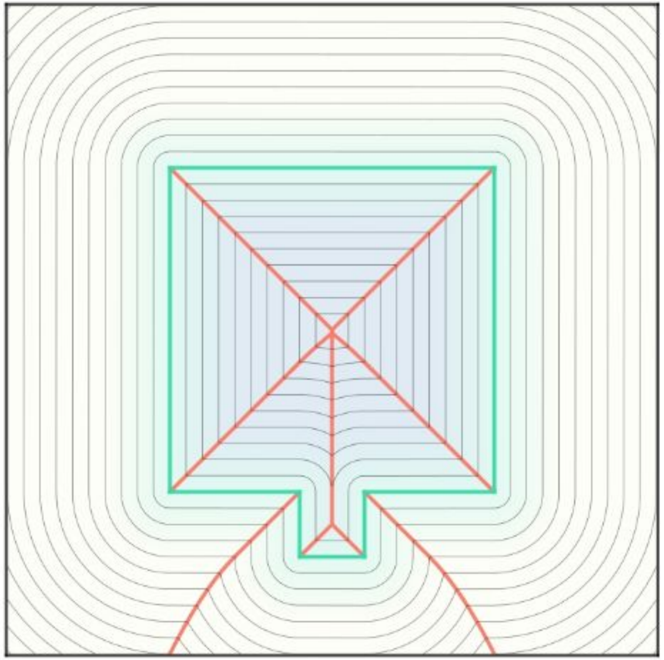
$$\mathcal{L}_{\text{total}}(\phi, v) = \lambda_{\text{fit}} \mathcal{L}_{\text{fit}}(\phi) + \lambda_{\text{eik}} \mathcal{L}_{\text{eik}}(\phi) + \lambda_{\text{exp}} \mathcal{L}_{\text{exp}}(\phi) + \lambda_{\text{HO}} \mathcal{L}_{\text{HO}}(\phi, v) + \lambda_{\text{AT}} \mathcal{L}_{\text{AT}}(v)$$



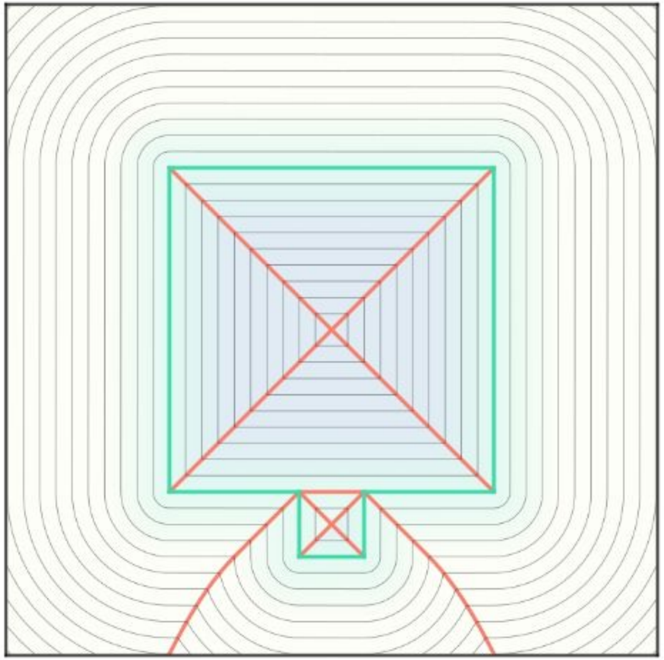


# Petit point d'ordre

Solution de viscosité



Solution de périmètre minimal

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