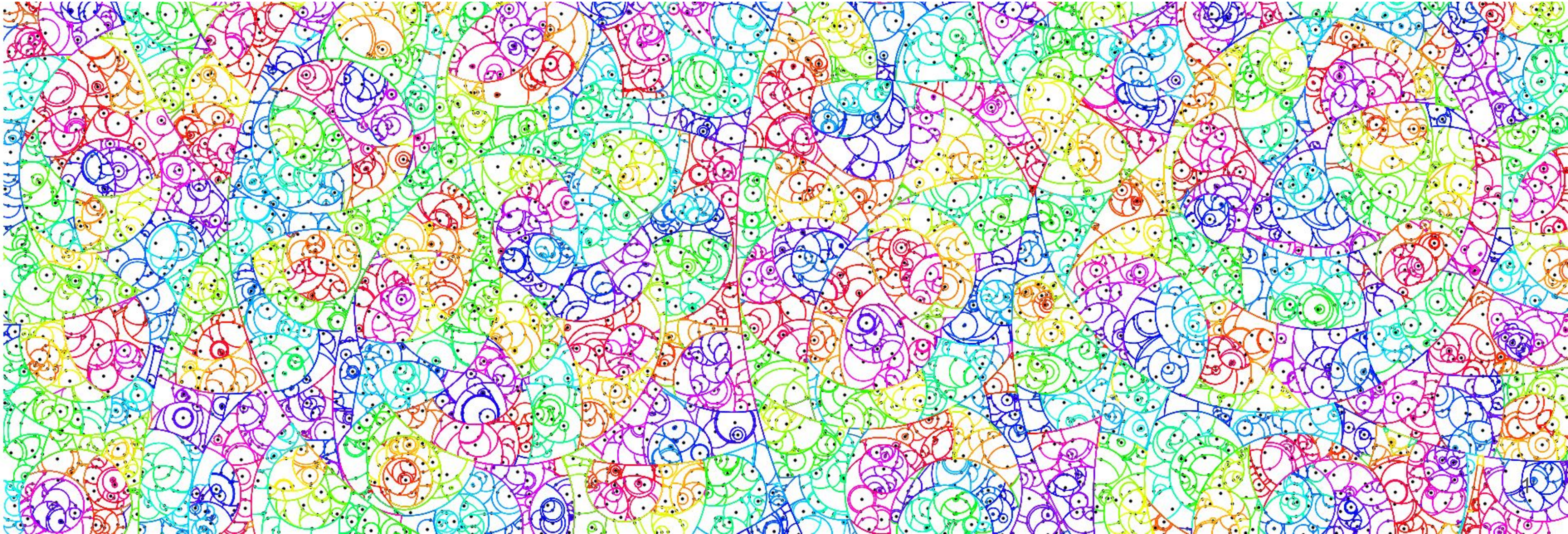
Vantage Point Trees

Based on “Data Structures and Algorithms for Nearest Neighbor Search in General Metric Spaces”, Peter N. Yianilos (1993) - http://algorithmics.lsi.upc.edu/docs/practicas/p311-yianilos.pdf

(Image from https://fribbels.github.io/vptree/writeup)



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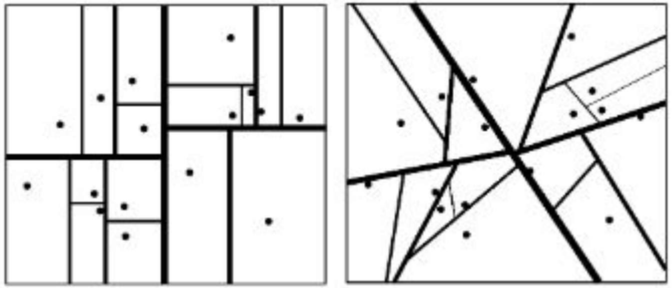

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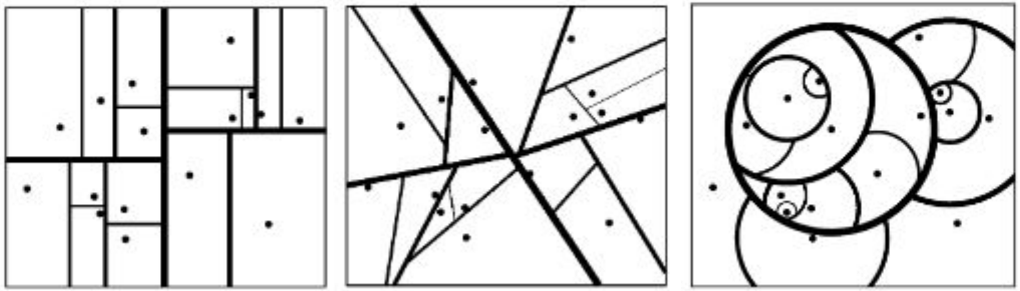
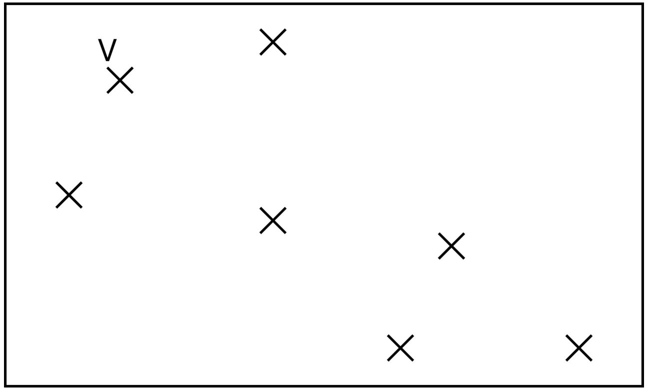
    - **Vantage Point Trees**
      - O(N log(N)) to build it
      - O(log(N)) for a NN Search



(Images from https://link.springer.com/content/pdf/10.1007/978-3-540-88688-4_27.pdf)

Vantage Point Tree: Construction

1. Select a vantage point v in X (eg. following a uniform distribution);



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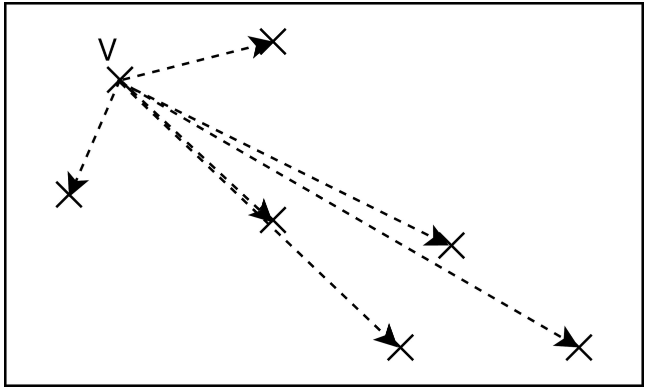
2. Compute the distances d(v, xi) between v and each point xi in X;

Euclidean in this example but works with any metric respecting the triangle inequality!



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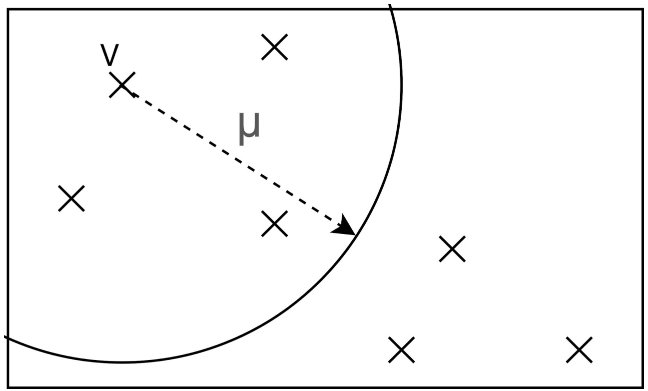

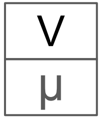

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    a. X_{left} the set of points closest to v is put at the left
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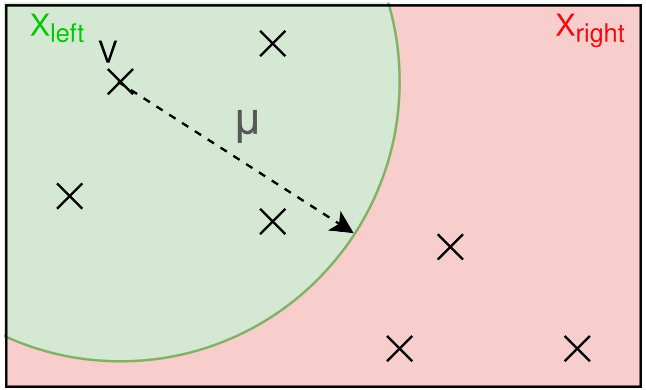

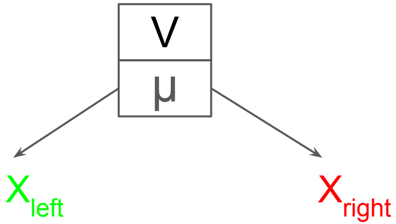

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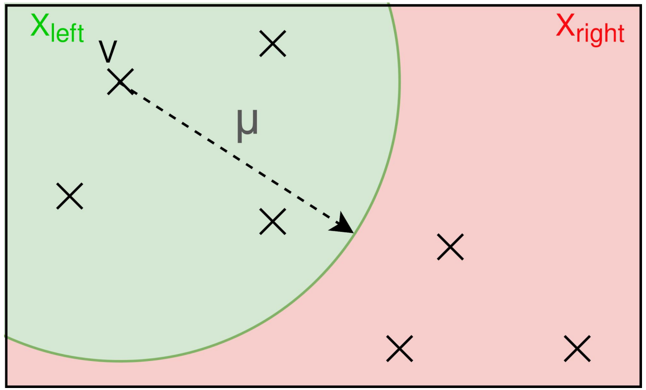

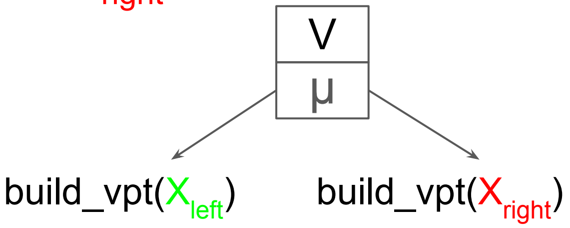

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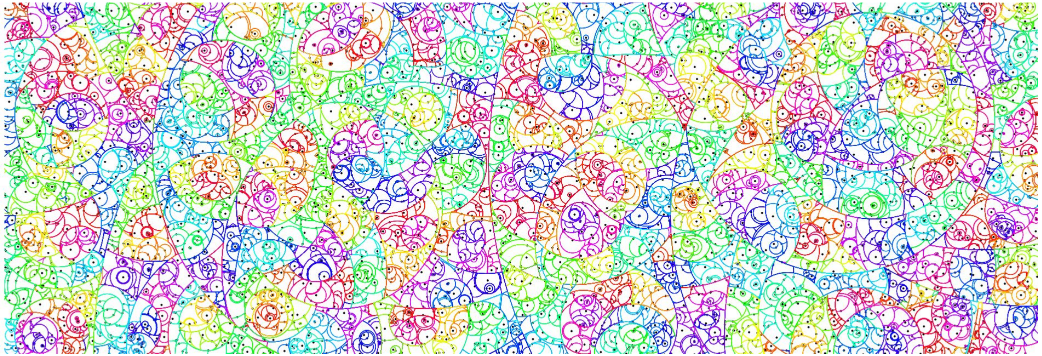

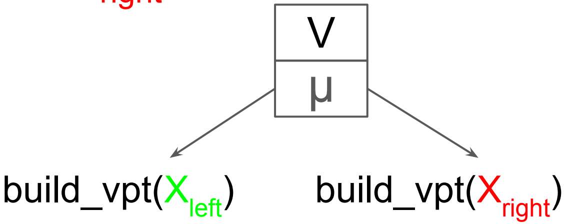

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DEMO
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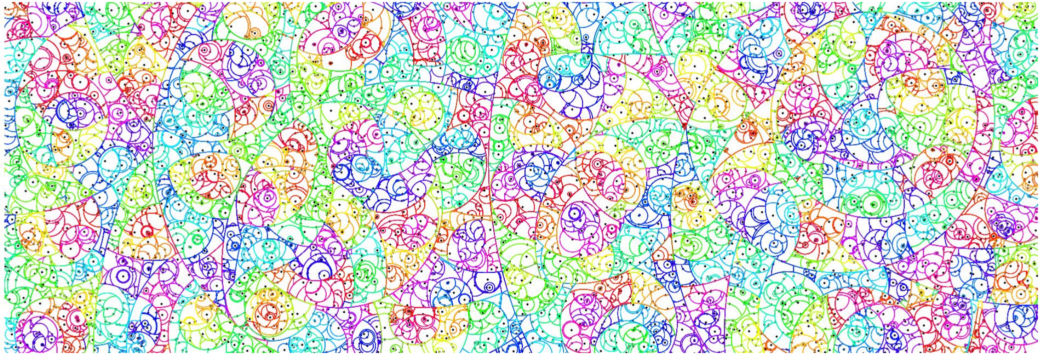
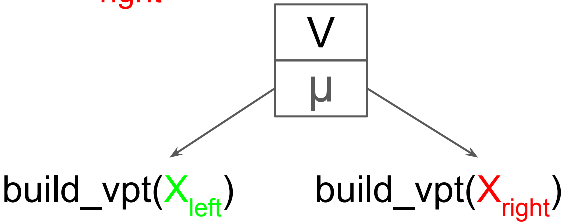
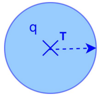




Vantage Point Tree: NN Search

- Given a query point q, we define **τ** the radius of the sphere representing the search space in which we search the NN of q 

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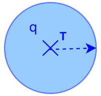
- Given a query point q, we define τ the radius of the sphere representing the search space in which we search the NN of q 

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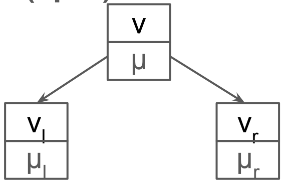

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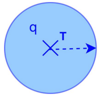
- Given a query point q, we define τ the radius of the sphere representing the search space in which we search the NN of q 

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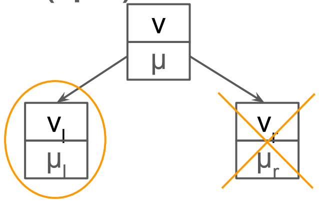

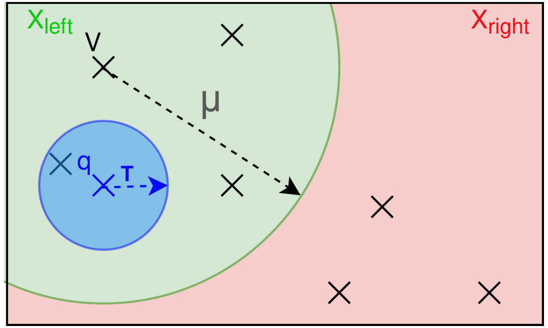

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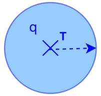
- Given a query point q, we define τ the radius of the sphere representing the search space in which we search the NN of q 
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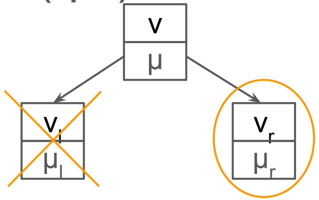

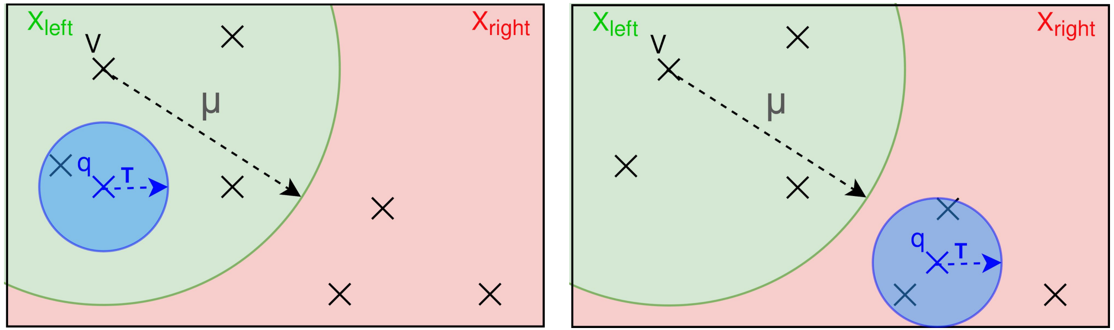

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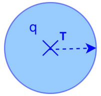
- Given a query point q, we define τ the radius of the sphere representing the search space in which we search the NN of q 
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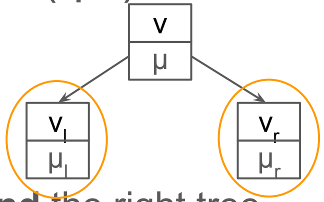
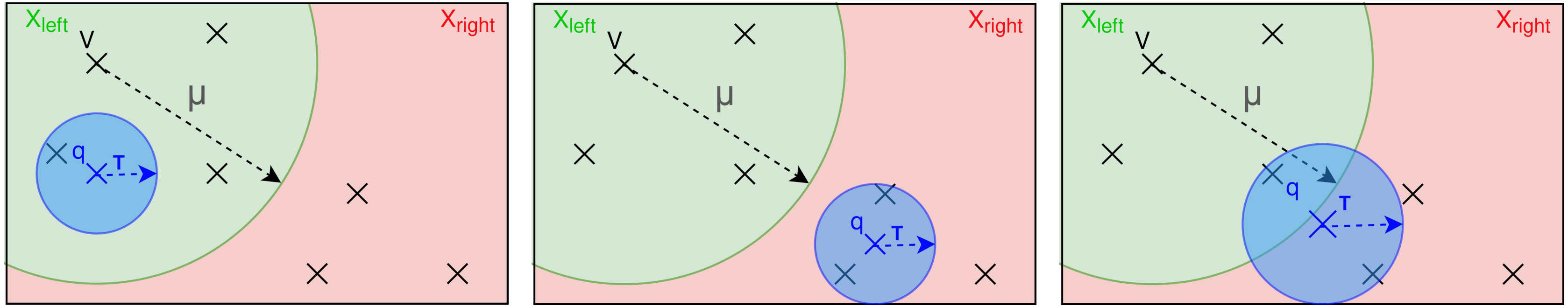
        c. If the q-sphere is inside and outside the v-sphere: explore the left tree **and** the right tree





Vantage Point Tree: Extensions

- VP^S-Tree
  - Instead of retaining only the median, a node retains 2 values per ancestor corresponding to the boundaries of the corresponding subspace

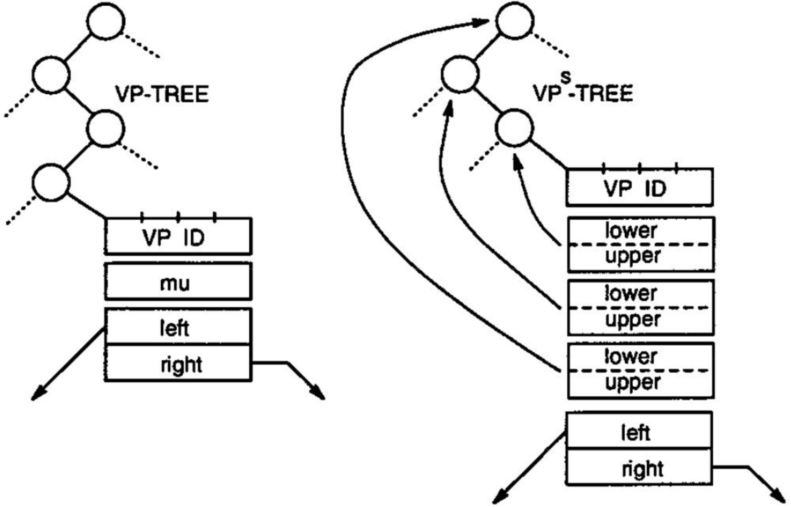


Figure 5: Sample 32-bit machine data structure implementations for the most basic vp-tree, the vp^{s}-tree, and the vp^{sb}-tree.

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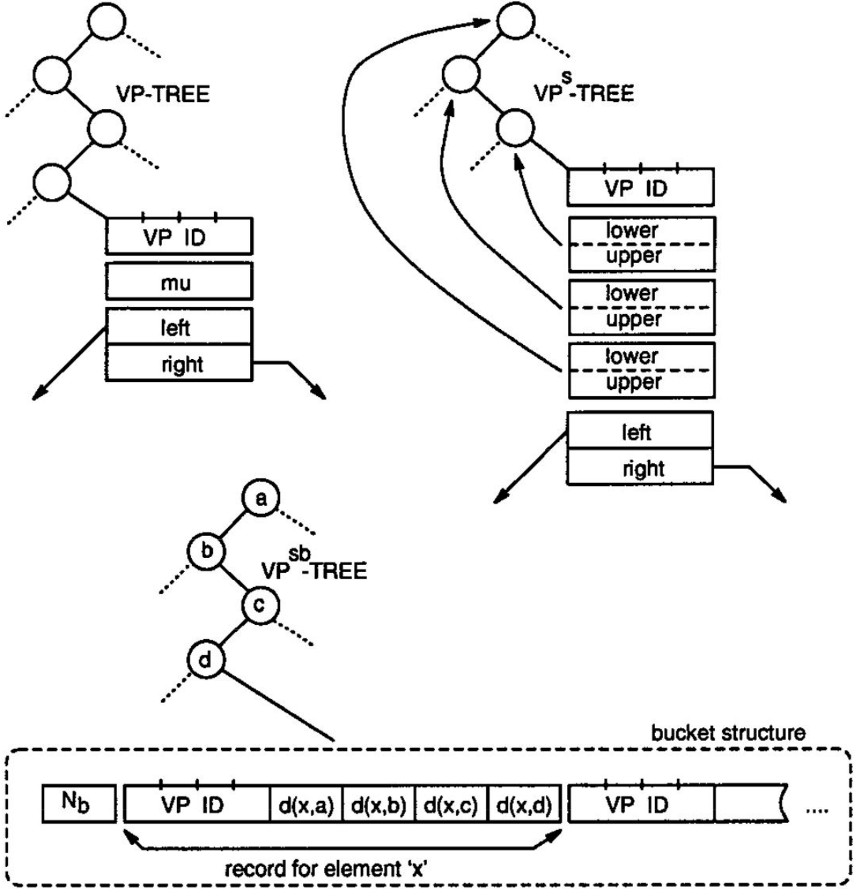
  - A VP^S-Tree in which each leaf contains B points resulting in a bucket structure
  - Reduced memory consumption



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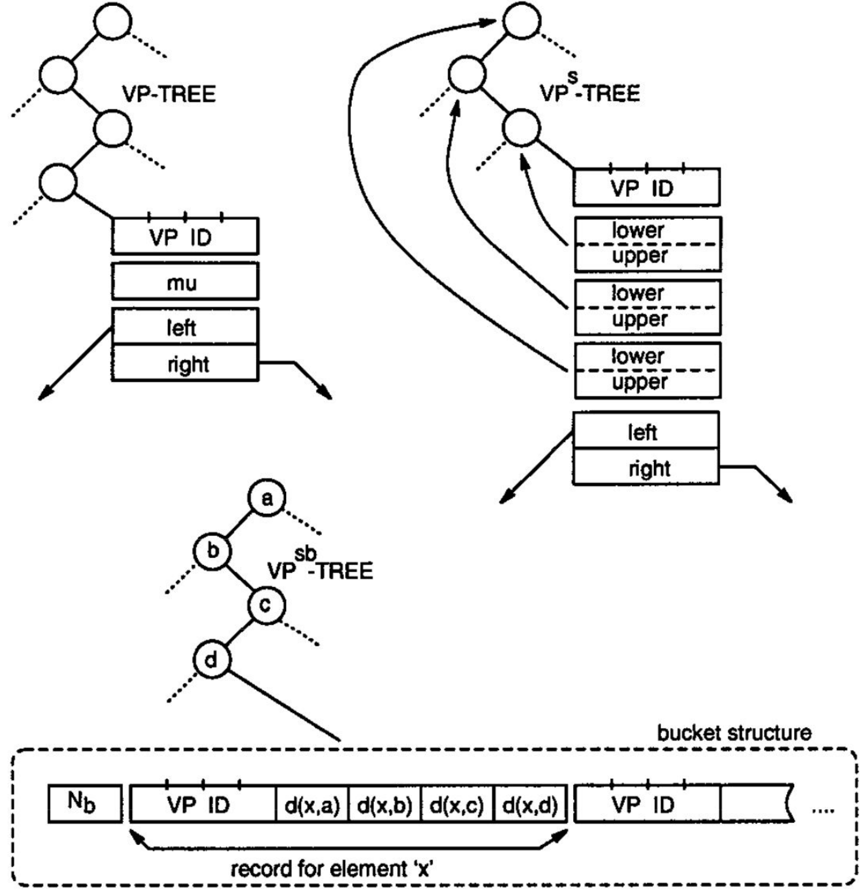
- VP Tree in non-metric space
  - With Bregman divergences: "Bregman Vantage Point Trees for Efficient Nearest Neighbor Queries" (Nielsen et al., 2009)



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Performance comparison with classical metric trees

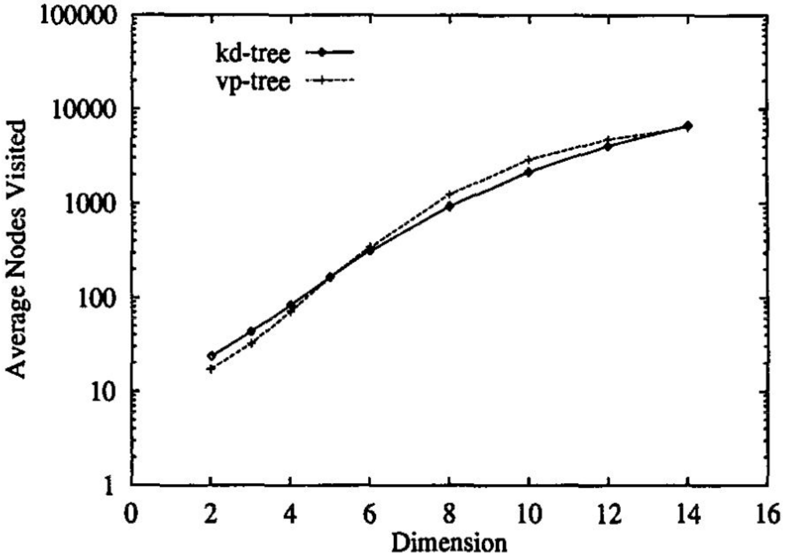

Figure 6: Search Cost vs. Dimension Comparison For L_2 Metric and Based on Nodes Visited

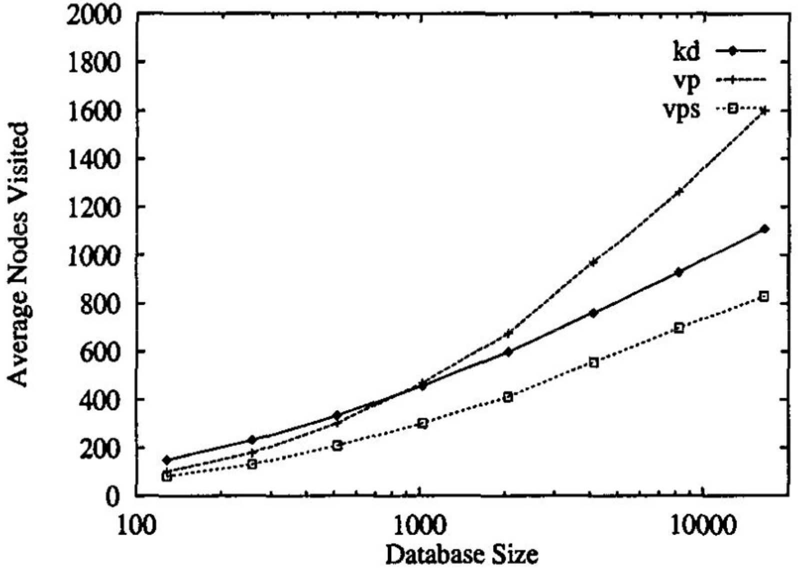

Figure 7: Search Cost vs. Database Size - Dimension 8

Performance comparison with classical metric trees

- "What Is a Good Nearest Neighbors Algorithm for Finding Similar Patches in Images?" (Neeraj et al., 2008)

Table 2. Summary of results. The *vp*-tree performs well in all respects.

Method	Construction Performance	ε-NN Search Performance	k-NN Search Performance
kd-Tree	Excellent	Poor	Poor
PCA Tree	Poor	Fair	Fair
Ball Tree	Fair	Excellent	Excellent
k-Means	Poor	Good	Good
vp-Tree	Excellent	Excellent	Excellent

Merci pour votre attention !