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Fast projector-camera calibration for interactive projection mapping

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***Abstract*—We propose a fast calibration method for projector-camera pairs which does not require any special calibration objects or initial estimates of the calibration parameters. Our method is based on a structured light approach to establish correspondences between the camera and the projector view. Using the vanishing points in the camera and the projector view the internal as well as the external calibration parameters are estimated. In addition, we propose an interactive projection mapping scheme which allows the user to directly place two-dimensional media elements in the tangent planes of the target surface without any manual perspective corrections.**

I. Introduction

With the common availability of digital projectors, the augmentation of real world objects with projected content has immersed as an own form of digital art, often called *projection mapping* or *shape adaptive projection*. The goal is in general the augmentation of three-dimensional target objects with perspectively correct projections which respect the underlying geometry of the object. Applications of this technique not only include marketing campaigns and commercials but are also used in the industry, e.g. in design processes [1].

In its simplest form the problem of a perspectively correct projection to the scene geometry has occurred to almost any projector user in terms of the keystone correction, i.e. correcting the projector images for projection planes which are not parallel to the projector image plane. While (automatic) keystone correction only accounts for plane geometry and therefore only requires the knowledge of the homography between the projector image plane and the scene plane (cmp. [2], [3]), projections to more complicated geometry usually require a complete 3D reconstruction of the target scene or object. In most situations a 3D model of the target object is not available in advance and therefore requires a live 3D reconstruction. Using a camera in addition to the projector, structured light techniques allow a 3D reconstruction of the scene [4]. A pattern which encodes the source pixel coordinates within the projector image is projected to the target object. The camera acquires the pattern images and decodes them to obtain correspondences between the camera and the projector. Nonetheless, a (up to scale) 3D reconstruction of the scene not only requires the correspondences between the projector and the camera but also their calibration parameters. Assuming that the camera and the projector are calibrated (the internal and external parameters of the camera and the projector are known) the 3D reconstruction is obtained by the triangulation of corresponding points.

Especially the requirement of a calibrated projector-camera pair is often a hurdle. In [5] a checkerboard in conjunction with the described structured light technique is used to warp the camera views of the checkerboard to the projector view. The camera view and the warped projector view are treated as a standard stereo image pair such that existing stereo calibration approaches can be used to estimate the internal and external parameters of the camera. While this method leads to great calibration results, checkerboards are often cumbersome to carry around, especially for live events. In [6] the projector calibration is achieved using a three dimensional calibration object. Manually identifying projector pixels such that they project to known three-dimensional points on the calibration object allow a standard calibration from 3D-2D correspondences [7]. The disadvantage of this method, besides the calibration object, is the manual pixel identification process.In [8] an calibration approach for a camera and a stripe laser projector is presented which requires known position of an LED marker. In [9] the authors use a more common calibration object: A white planar surface with known size and orthogonal edges is used to determine the calibration parameters from projected checkerboard images and vanishing points of the scene. The mentioned methods all have in common that they require a calibration object which has to be placed in the scene or even require a prior calibration of a subset of the parameters. While this is perfectly feasible and accurate for static setups where the camera and the projector are calibrated once, it is often in-feasible for live shows where installers or artists bring their own equipment and frequently change the projector and camera zoom, therefore prohibiting any prior calibrations of the projector-camera pair.

Within this article we propose a novel and fast calibration scheme especially suited for artistic purposes in live events which enables an interactive projection mapping of the scene. The contributions of this scheme are: (1) It requires no initial calibration parameters of the projector or the camera. (2) No special calibration object is required, standard indoor geometry like room corners can be used for the calibration. (3) The calibration is fast such that the user is able to quickly re-calibrate in the case of changing projector-camera parameters such zoom, position or rotation. In addition to the calibration scheme we propose an interactive projection mapping method which allows the users to place two-dimensional objects such as images or videos in the tangent planes of the target surfaces in a perspectively correct way (in the case of planar surfaces the tangent planes coincide to the planes themselves). We focus on the speed and the convenience of our method to produce visually pleasing artistic results rather than focusing on overall accuracy. Besides a projector, our method only requires a camera where the resolution should be higher than the projector resolution. Our method can be summarized as follows: A gray-code pattern projection is used to establish 2D correspondences between the camera and projector view which will be explained in III. Afterwards, minimal user interaction is required: The user has to draw three parallel line pairs lying in the three mutually orthogonal planes in the scene. The drawn lines are warped to the projector view. Using the lines in the camera and the projector view we estimate three mutually orthogonal vanishing points in the projector and the camera view (see IV). The internal and external parameters of the camera and the projector are obtained from the corresponding vanishing points (see sections V, VI). Afterwards the user is able to interactively apply our projection mapping method described in section VIII.

II. Camera model

Within this article we model the projector as well as the camera according to the pinhole camera model. We do not model any lens distortions, since the distortion does not influence the overall results too much due to the focal lengths and operating distances of the system. The internal parameters of a pinhole camera are described by the internal matrix

$$\mathbf{K} = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

We assume that the pixels of the camera and the projector are square pixels such that $f_u = f_v$ and $s = 0$. The internal matrices for the camera and the projector will the denoted by $\widetilde{\mathbf{K}}$ and $\mathbf{K}$. Without loss of generality we assume that the camera and the projector look along the positive z-axis in their local left-handed coordinate systems. A point $\mathbf{X} = (X_1, X_2, X_3)^T$ is then mapped to $\mathbf{x} = (x_1, x_2, 1)^T$ on projector image plane as

$$\mathbf{x} = \mathbf{K}\mathbf{R}^T(\mathbf{X} - \mathbf{t}). \tag{2}$$

where the external parameters $\mathbf{R}$ and $\mathbf{t}$ denote the projector orientation and position with respect to the world coordinate frame (cmp. [7] for the notation). The same holds for the camera image formation using $\widetilde{\mathbf{K}}, \widetilde{\mathbf{R}}$ and $\widetilde{\mathbf{t}}$.

III. Gray-code projection

In order to acquire the geometry of the scene we follow the approach in [4] and interpret the projector-camera pair as a stereo camera setup. A fundamental requirement for the 3D reconstruction of a scene from the stereo camera pair is the establishment of correspondences between the two views. Once the correspondences (and the calibration parameters) are known, the 3D reconstruction is obtained by a triangulation of the 2D correspondences. Since the projector is an active device, we are able to uniquely encode each projector pixel using a binary gray-code pattern which is afterwards decoded by the camera. Let $\mathcal{I} : \{0, ..., N-1\} \times \{0, \ldots, M-1\} \to [0, 255]$ denote a projector image of dimension $N \times M$ and $\widetilde{\mathcal{I}} : \{0, \ldots, \widetilde{N}-1\} \times \{0, \ldots, \widetilde{M}-1\}$ denote a camera image of dimension $\widetilde{N} \times \widetilde{M}$. We encode each projector pixel coordinate $(i, j)^T \in \{0, \ldots, N\} \times \{0, \ldots, M\}$ with the bit sequence of the single coordinates resulting in $b_{vmax} = \lceil \log_2 N \rceil$ vertical and $b_{hmax} = \lceil \log_2 M \rceil$ horizontal code images denoted by $\mathcal{I}_{v,b}, b \in \{1, \ldots, b_{vmax}\}$ and $\mathcal{I}_{h,b}, b \in \{1, \ldots, b_{hmax}\}$. In addition we also use the inverse code patterns $\bar{\mathcal{I}}_{v,b}, \bar{\mathcal{I}}_{h,b}$ to stabilize the decoding process [4].

The camera acquires an image for each pattern projection resulting in $2(b_{vmax} + b_{hmax})$ camera images in total and decodes the corresponding projector pixel coordinates in terms of two warp images $\mathcal{W}_v, \mathcal{W}_h$. Examples of the acquired binary

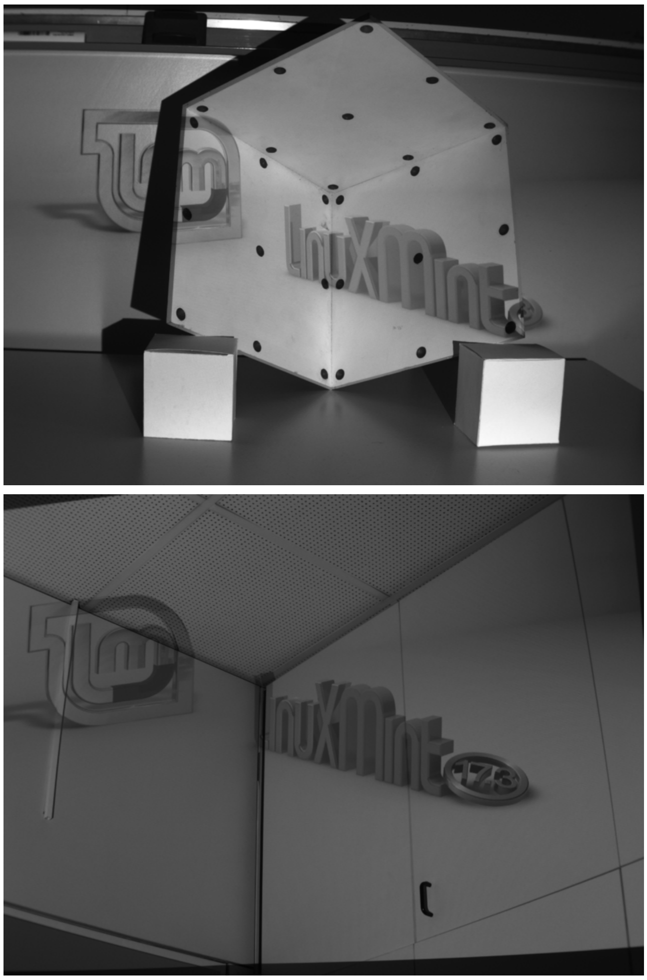

Fig. 1. Two example scenes seen from the camera view. Top: Cube scene, bottom: corner scene.

images and warp images for the cube and corner scene from figure 1 are shown in figure 2. The camera resolution should in general be higher than the projector resolution in order to avoid aliasing artifacts. We favor the gray-code projection due to its simplicity but more advanced structured light schemes exist, which allow a decoding with sub-pixel accuracy [10].

IV. Vanishing Point Estimation

While the correspondences alone already allow a projective reconstruction [7], we are interested in a (up to scale) metric reconstruction of the scene. A necessary requirement is the calibration of the internal and external parameters of the projector-camera pair. Existing approaches like [11] propose a method for complete self-calibration of a projector-camera system from structured light correspondences. They propose to concurrently solve a non-linear optimization problem for the fundamental matrix and the internal parameters of the projector and the camera. In the case of a correct solution, the essential matrix and therefore the external parameters can obtained. The method requires a reasonable initial solution which especially in the case of the internal projector parameters is not always available. We were not able to apply the method in a stable way and often got stuck in local minima. Another complete self-calibration approach has recently been introduced in [12]. An initial calibration is obtained using structured light correspondences similar to [11] and further refined using known points on a calibration object. Instead of a complete self-calibration we propose a fast method which requires minimal user interaction and no special calibration object. At the beginning of the calibration process, the user has to ensure that the projector-camera pair is able to see three mutually orthogonal planes (e.g. a room corner, cmp. figure 1). In each plane the user has to be able to draw a parallel line pair. Overall, this is a reasonable assumption since most rooms are for example equipped with windows and doors which allow to draw the required lines. The camera image of the scene is shown to the user on a screen (e.g. a tablet device) and the user draws the required line pairs by dragging the start- and endpoints of a line segment, resulting in three parallel line pairs $(\widetilde{l}_{1,1}, \widetilde{l}_{1,2}), \ldots, (\widetilde{l}_{3,1}, \widetilde{l}_{3,2})$ where each line $\widetilde{l}_{i,j}$ is described by the two drawn points $(\widetilde{\mathbf{p}}_{i,j}, \widetilde{\mathbf{q}}_{i,j})$. Each annotated line in the camera view is warped to the projector view resulting in $(l_{1,1}, l_{1,2}), \ldots, (l_{3,1}, l_{3,2})$. Due to possible errors and noise in the correspondence estimates we robustify the warping of the lines using a RANSAC scheme. Not only the endpoints of the lines are warped but the line segments in the camera view are sampled and all the sampled points are warped to the projector view. The projector line pairs are then fitted to the warped sampled points using RANSAC. For each parallel line pair we compute its intersection in the camera and projector view respectively which constitute the vanishing points of the scene:

$$\widetilde{\mathbf{v}}_i = \widetilde{l}_{i,1} \cap \widetilde{l}_{i,2} \quad \mathbf{v}_i = l_{i,1} \cap l_{i,2}. \tag{3}$$

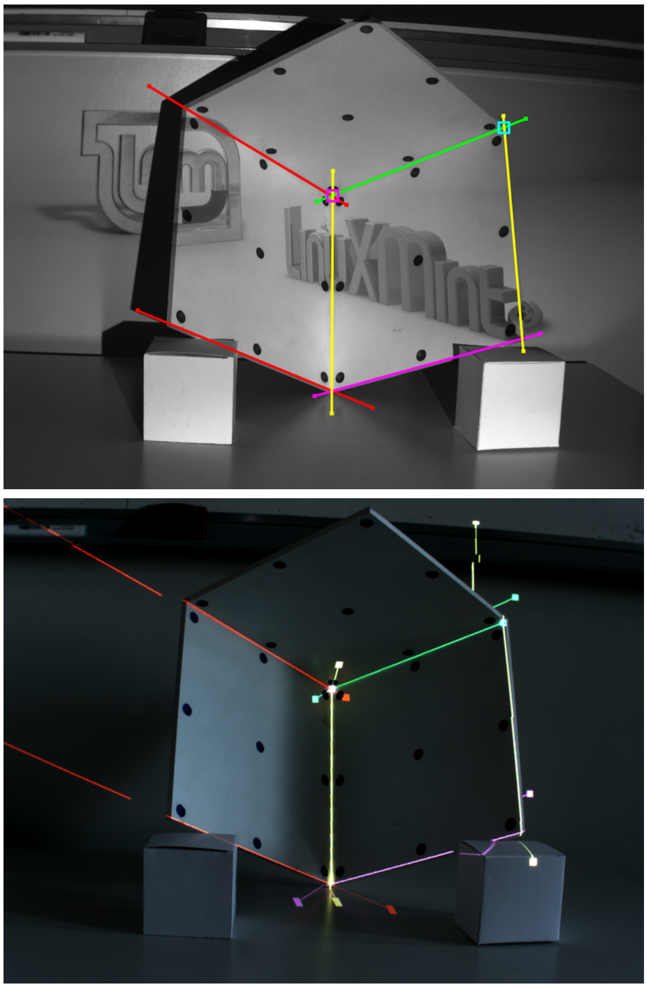

Fig. 3. Top: Annotated mutually orthogonal parallel line pairs on the computer screen (lines have been redrawn for better visualization). Bottom: Warped and projected lines as seen by the camera.

Figure 3 shows the line annotation in the camera view and the warped projected lines seen from the real camera view.

V. Internal Calibration

The vanishing points in the camera and projector views are the key to the internal calibration. Using vanishing points for camera calibration is not a new concept. A comprehensive overview for stereo camera calibration using vanishing points can be found in [13]. The monocular setup is studied in detail in [14] and [7]. Vanishing points have also already been used in [9] for projector-camera calibration. Nonetheless the method requires a white planar calibration object of known size and orthogonal edges. Further the authors assume, that the principal point of the camera and the projector is in the center of the images. In the case of projectors this in general a false assumption. We neither require a calibration object, nor do we impose any constraints on the principal points of the camera or the projector. Without loss of generality we consider only the projector view, since the same principle applies to the camera view as well. It has been shown in [7] that the internal

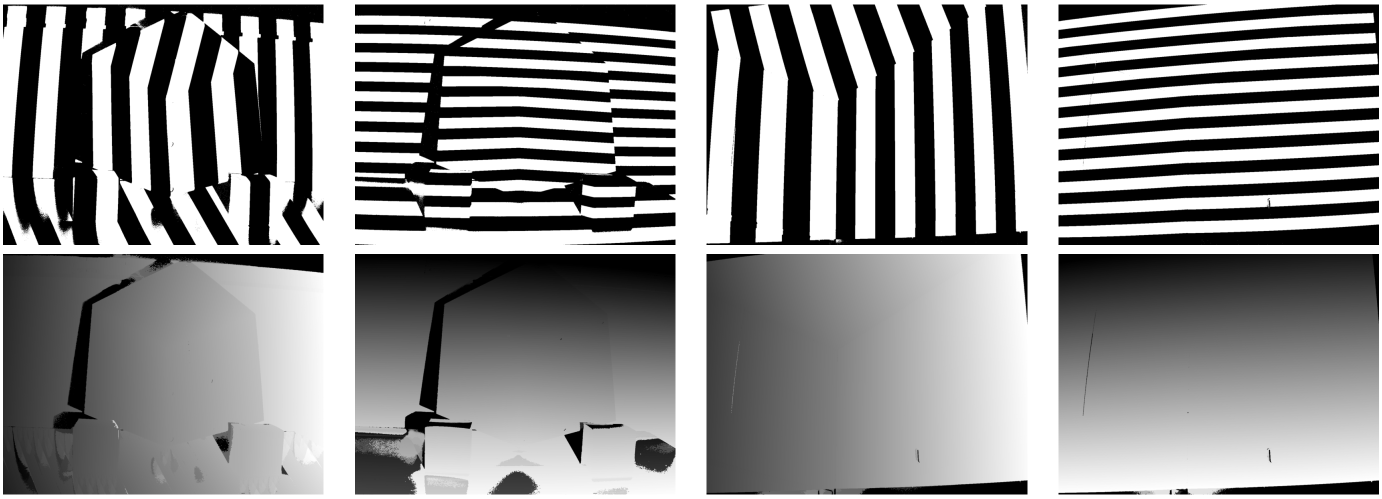

Fig. 2. Top row: Examples of extracted binary patterns from the camera view for the cube and the corner scene. Bottom row: Warp images containing the x and y coordinates in the projector view for the cube and the corner scene.

camera calibration matrix $\mathbf{K}$ is determined by image of the absolute conic ω as

$$\omega = (\mathbf{K}\mathbf{K}^T)^{-1}. \tag{4}$$

Due to the restriction that the pixels of the projector are square pixels, the image of the absolute conic is given by

$$\omega = \begin{bmatrix} \omega_1 & 0 & \omega_2 \\ 0 & \omega_1 & \omega_3 \\ \omega_2 & \omega_3 & \omega_4 \end{bmatrix}. \tag{5}$$

Further, since the vanishing points are mutually orthogonal they impose the three linear constraints

$$\mathbf{v}_1^T \omega \mathbf{v}_2 = 0 \tag{6}$$
$$\mathbf{v}_1^T \omega \mathbf{v}_3 = 0 \tag{7}$$
$$\mathbf{v}_2^T \omega \mathbf{v}_3 = 0 \tag{8}$$

which lead to the linear system

$$\begin{bmatrix} v_{1,1}v_{2,1} & v_{1,1} & 0 & 0 \\ v_{1,2}v_{2,2} & 0 & v_{1,2} & 0 \\ 0 & v_{2,1} & v_{2,2} & 1 \end{bmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix} = \mathbf{0}. \tag{9}$$

The system is solved for ω using the singular value decomposition. Once the image of the absolute conic is known, $\mathbf{K}$ can be obtained using a Cholesky factorization and a matrix inversion since ω is symmetric and positive-definite.

VI. External calibration

The vanishing points $\mathbf{v}_i$ and $\widetilde{\mathbf{v}}_i$ not only describe mutually orthogonal vanishing points but also *corresponding* vanishing points. Using the internal parameters from the previous section, corresponding *directions* in the projector and the camera view can be obtained as

$$\mathbf{d}_i = \mathbf{K}^{-1}\mathbf{v}_i / ||\mathbf{K}^{-1}\mathbf{v}_i|| \quad \text{and} \quad \widetilde{\mathbf{d}}_i = \widetilde{\mathbf{K}}^{-1}\widetilde{\mathbf{v}}_i / ||\widetilde{\mathbf{K}}^{-1}\widetilde{\mathbf{v}}_i||. \tag{10}$$

Assuming that the three directions correspond to the canonical Cartesian directions in the world coordinate system, the rotations for the projector and the camera are given by

$$\mathbf{R} = \begin{bmatrix} \mathbf{d}_1^T \\ \mathbf{d}_2^T \\ \mathbf{d}_3^T \end{bmatrix} \quad \widetilde{\mathbf{R}} = \begin{bmatrix} \widetilde{\mathbf{d}}_1^T \\ \widetilde{\mathbf{d}}_2^T \\ \widetilde{\mathbf{d}}_3^T \end{bmatrix}. \tag{11}$$

The translations $\mathbf{t}, \widetilde{\mathbf{t}}$ of the projector and the camera can be obtained from two known 3D points in the world coordinate system and their corresponding 2D projections in the projector and camera view. Without loss of generality we again only consider the projector view. The translation of the camera is obtained analogously. Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^3$ denote these two known points in world coordinates and let $\mathbf{p} = (p_1, p_2, 1)^T, \mathbf{q} = (q_1, q_2, 1)^T$ denote the corresponding projections in the projector view. Then

$$z_1 \, \mathbf{R}\mathbf{K}^{-1}\mathbf{p} + \mathbf{t} = z_1\mathbf{a} + \mathbf{t} = \mathbf{X} \tag{12}$$
$$z_2 \, \mathbf{R}\mathbf{K}^{-1}\mathbf{q} + \mathbf{t} = z_2\mathbf{b} + \mathbf{t} = \mathbf{Y} \tag{13}$$

where z_1, z_2 denote the distances of $\mathbf{X}, \mathbf{Y}$ from the projector along the z-axis in the projector coordinate frame. Solving

$$\begin{bmatrix} \mathbf{a} & -\mathbf{b} \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \mathbf{x} - \mathbf{y} \tag{14}$$

yields the desired translation

$$\mathbf{t} = \mathbf{x} - z_1 \, \mathbf{R}\mathbf{K}^{-1}\mathbf{p} = \mathbf{y} - z_2 \, \mathbf{R}\mathbf{K}^{-1}\mathbf{q}. \tag{15}$$

Due to the construction of the vanishing points two known 3D points are $\mathbf{X} = (0, 1, 0)^T$ and $\mathbf{Y} = (0, 0, 0)^T$ (the scale of $\mathbf{X}$ is arbitrary). To obtain the corresponding projections, we always declared one line during the vanishing point annotation as the y-axis (e.g. the green line in figure 3) and automatically calculated $\mathbf{p}, \mathbf{q}$ and $\widetilde{\mathbf{p}}, \widetilde{\mathbf{q}}$ as the intersections of the line pairs corresponding to the x-direction in the projector and camera views such that the user had no additional annotation cost (cmp. magenta and cyan squares in figure 3).

VII. Calibration Results

For our experimental setup we used a PointGrey Grashopper GRAS-20S4C-C camera with a resolution of 1600×1200 pixels and a pixel size of $4.4\mu m$ (square pixels) with a Cosmicar/Pentax 12mm lens such that the optimal focal length in the pinhole setting is 2727.27 pixels For the projector we used a BenQ W1070 with a built in zoom lens whose focal length is within $f \in [16.88mm, 21.88mm]$. The maximum resolution of the projector is 1920×1080 pixels. We used a resolution of 1280×720 pixels. The image diagonal of the projector image at a distance of 1m is 1,02m resulting in a pixel size of $11.2\mu m$ at a resolution of 1280×720 such that the expected focal lengths in pixels is $f \in [1498.79, 1942.74]$. We compare the vanishing point based calibration (VP) with a standard calibration method from 3D-2D correspondences (see [7], [6]) using the known reference points on the cube in the scene. The results are shown in table I. The deviations in the v_0 component did not impact the overall quality of the final projections. A 3D reconstruction from the triangulation of the correspondences using the estimated parameters of the vanishing point based method is shown in figure 4.

TABLE I
Internal calibration results

	Camera		Projector	
Parameters	VP	3D-2D	VP	3D-2D
f	2857.75	2609.866	1892.56	1787.12
u_0	911.59	804.422	674.25	579.68
v_0	357.16	669.48	755.56	817.86

VIII. Interactive Projection Mapping

If projectors are used for artistic projection mapping purposes, e.g. installations at fair stands, the content usually consists of several two-dimensional media elements (images, videos) which are supposed to be mapped to the target surfaces. Using the 3D reconstruction of the scene in conjunction with the image data from the camera view, we enable the user to interactively paint those two-dimensional primitives (e.g. rectangles, triangles, circles) in the tangent plane of a target surface without any manual perspective corrections. In the case of planar structures such as the mutually orthogonal planes used in the calibration process, the user is able to directly place the objects within the planes such that

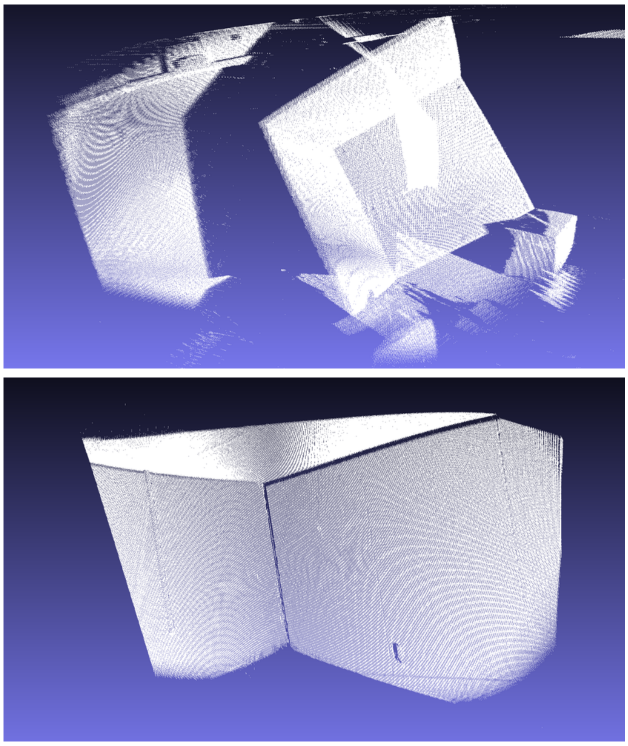

Fig. 4. Three-dimensional reconstruction (pointcloud) of the cube scene (top) and the corner scene (bottom) using the calibration parameters of the vanishing point based method.

he only needs to adjust the desired parameters for the in-plane position (translation), rotation and scale, thus completely avoiding any manual perspective corrections. In order to place a two-dimensional primitive in the tangent plane of the target surface the user selects a target point $\widetilde{\mathbf{p}} = (\widetilde{p_1}, \widetilde{p_2}, 1)^T$ in the camera view. The 2D points $\widetilde{\mathbf{p}}_{i,j} = \widetilde{\mathbf{p}} + (i, j, 0)^T, (i, j) \in [-n, \ldots, n]^2$ from a local $(2n + 1) \times (2n + 1)$ neighborhood and the corresponding 2D points in the projector view $\mathbf{p}_{i,j} = (\mathcal{W}_v(\widetilde{\mathbf{p}}_{i,j}), \mathcal{W}_h(\widetilde{\mathbf{p}}_{i,j}), 1)$ are triangulated resulting in the 3D points $\mathbf{X}_{i,j} = (X_{i,j,1}, X_{i,j,2}, X_{i,j,3})^T$. The surface normal vector and the local tangent plane at $\mathbf{x}$ is obtained from the singular value decomposition of the 3×3 co-variance matrix $\mathbf{C}$ with

$$C_{i,j} = \frac{1}{(2n+1)^2 - 1} \sum_{k=-n}^{n} \sum_{l=-n}^{n} (X_{k,l,i} - \mu_i)(X_{k,l,j} - \mu_j) \tag{16}$$

and

$$\mu_i = \frac{1}{(2n+1)^2} \sum_{k=-n}^{n} \sum_{l=-n}^{n} X_{k,l,i}, i \in \{1, 2, 3\}. \tag{17}$$

The singular vector corresponding to the smallest singular value corresponds to the normal vector of the surface (ensuring that it points towards the camera and the projector view), whereas the two remaining singular vectors describe the local tangent plane. This local basis is used as the local coordinate

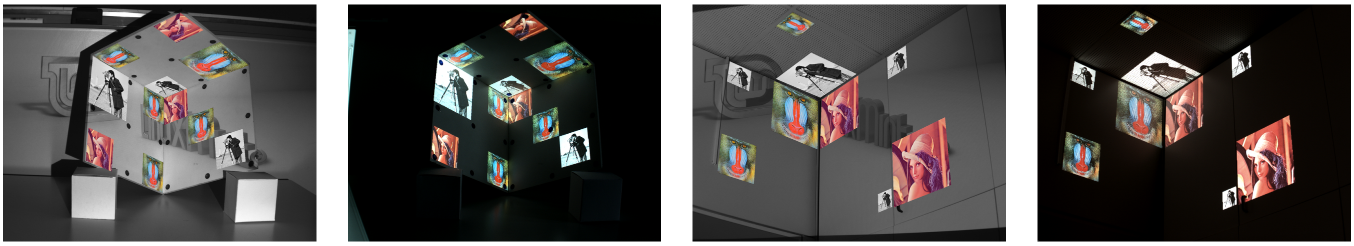

Fig. 5. Virtual camera view the user interacts with and the augmented scenes captured by the camera for the cube and the corner scene.

system for the two-dimensional primitives which are supposed to be rendered in the tangent planes of the surface. Further the user is able to directly manipulate the primitives in the tangent plane (translate, rotate, scale) without any manual perspective correction. Using OpenGL, the camera and the projector view are simulated and the two-dimensional primitives are rendered in a virtual 3D scene from the camera and the projector view. Figure 5 shows the projection results for several two-dimensional quads which have been interactively placed on the surface of a cube and a room corner.

IX. Conclusion and Future Work

We have proposed a fast and flexible calibration approach for projector-camera pairs which requires neither prior calibration parameters nor any calibration objects. It is especially suited for projection mapping installations where parameters of the projector-camera pair might change often and require a flexible re-calibration. In addition we have proposed an interactive projection mapping scheme for perspectively correct projections of two-dimensional media elements in the tangent planes of the scene surfaces without any manual perspective corrections.

Future work will investigate the interactive mapping of non-planar primitives on curved surfaces such as spheres and cylinders using local curvature information estimated from the surface reconstruction.

In addition we will investigate interactive projection mapping methods in the case of dynamic scenes and projection surfaces using real-time depth sensors.

Acknowledgements

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