Extraction of Retinal Blood Vessel using Curvelet Transform and Fuzzy C-Means

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***Abstract*—This paper addresses the automatic blood vessel detection problem in retinal images using matched filtering in an integrated system design platform that involves curvelet transform and fuzzy c-means. Although noise is kept constant in medical CCD cameras, due to a number of factors, the contrast between the background and the blood vessels in retinal images and consequently the visual quality of the images looks very poor. Some form of pre-processing operation is therefore essential for the accurate extraction of these blood vessels. Since curvelet transform can represent lines, edges and curvatures very well as compared to other multi-resolution techniques, this paper uses curvelet transform to enhance the retinal vasculature. Matched filtering is then used to intensify the blood vessels which is further employed by fuzzy c-means algorithm to extract the vessel silhouette from the background. Performance is evaluated on publicly available DRIVE database and is compared with the existing blood vessel extraction methodology that uses curvelet transform. Simulation results demonstrate that the proposed method is very much efficient in detecting long and thick as well as short and thin vessels, wherein the existing methods fail to extract tiny and thin vessels.**

***Index Terms*—Retinal image segmentation, Vessel detection, Curvelet transform, Matched filter, Fuzzy C-Means algorithm**

I. Introduction

Diabetic Retinopathy (DR) is a complication of diabetes and a leading cause of blindness. It occurs when the blood vessels that nourish the retina are blocked and hence deprive several areas of retina with blood and oxygen supply [1]. In proliferative diabetic retinopathy (PDR), new blood vessels start growing along the retina. These blood vessels are fragile and abnormal in nature and cause frequent minor bleeding, sometimes cause permanent vision loss. In most of the retinal screening programs, blood vessels are extracted manually. This is very much time consuming as well as tedious job and requires high degree of skill [2] for the ophthalmologists to detect or identify such a retinal vessel structure. Moreover the results are subject to variability and error and at the same time the skilled ophthalmologists are also limited in number. These altogether, create a pressing demand to design an automated system that enables accurate detection of blood vessels for the diagnosis and treatment of PDR.

Generally, retinal images are rich with thick and thin, small and large vessels of fairly good number of each type. As the vessels travel radially outward from the optic disc at different orientations, the vessels become more and more thinner and the contrast between the background and the thin vessels becomes poorer. Hence, the narrow vessels become almost indistinguishable from the background. Again the retinal images are usually degraded during acquisition or transmission due to introduction of noise, blurring etc. Therefore, accurate extraction of thick and thin as well as tiny and large vessels from the retinal background is a very challenging task.

The existing literature is quite rich with the problem of retinal vessel extraction. The methods are mainly based on pixel based classification techniques using artificial neural network [3], mathematical morphology [4], vessel tracking/ tracing [5], matched filtering [6] etc. The main disadvantage of neural network based methods lies in training process by supervised learning with standard training data [7]. This needs large number of images and for each image in the training set, vascular structure is to be precisely marked by an ophthalmologist. However as observed by Hoover et al. [8], there exist significant differences between the retinal vasculature marked by different experts. Morphological method, on the other hand, does not exploit the known vessel cross-sectional profile [7] and if the length of the structuring element is high, it may cause difficulty in fitting to highly tortous vessels. Vessel tracking methods are not well suited to detect vessels or vessel segments that have no seed point. Furthermore, if any bifurcation point is missed, some of the sub-trees may be undetected. In matched filtering method [6], a 2D linear kernel having a Gaussian profile is rotated and convolved with the retinal image. For each pixel, the highest response is selected and the image is then thresholded to extract vessel from the background. The main advantage of matched filtering is that it can extract retinal vasculature even from a noisy image due to its optimal nature. All these different methods individually are very efficient for detecting blood vessel in retinal images but except matched filtering, the other methods may not be able to extract very thin vascular nets, specially when the image is noisy and unevenly illuminated.

Apart from the above mentioned techniques, multi-resolution methods using curvelet and wavelet transforms are also used for blood vessel detection [9]-[11]. It is reported in several image processing works including retinal image analysis that wavelet and its various variants are very efficient in identifying edges, curves, contours and other boundary information due to their multi-resolution, space-frequency

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localization property. Wavelet transform decomposes an image into approximation, horizontal, vertical and diagonal details. It can represent point singularities and texture information efficiently but fails to describe objects with highly anisotropic elements such as lines, curves and edges. It also fails to extract missing edge information, particularly in low contrast images. On the other hand, curvelet transform is localized in scale (frequency domain), position (spatial domain) and orientation (direction). In the frequency domain, it can capture the structural activity along the radial 'wedges', so it has very high directional sensitivity and anisotropy [9]. It can identify contours, edge directional information, missing and imprecise edges, curvature and boundary information very well. Curvelet edges are much smoother than wavelet edges. Since the retinal image contains a large number of narrow vessels, curvelet transform may be well suited to extract the finest details along the edges of thin vessels even from a low contrast and non-uniformly illuminated background. On the other hand, in most of the cases, wavelet transform fails to extract such tiny and narrow vessels. The ability of curvelet transform to represent lines, curves, edges and object boundaries in different directions efficiently has motivated us to use curvelet based edge enhancement approach in the present work.

Ceylan et al. [10] used 4^{th} level complex wavelet transform and complex-valued artificial neural networks for feature extraction and blood vessel segmentation, respectively. Detection of retinal blood vessels using curvelet transform and morphology operators by multistructure elements is proposed in [11]. This algorithm utilizes a simple thresholding method, as a result, some of the small vessels may be undetected. In [12], vessel enhancement and detection are carried out using Gabor wavelet and multilayered thresholding technique, respectively. Next, a multivariate m-mediods based classifier is proposed for abnormal blood vessel detection and grading of PDR. It is noted from the result that although the method extracts thick vessels accurately but some of the tiny and narrow vessels remain undetected, this may lead to improper grading of PDR. Esmaeili et al. [9] implemented curvelet based contrast enhancement followed by matched filtering, curvelet based edge extraction and length filtering for blood vessel extraction. However, this method requires manual intervention to specify a number of parameters. Hence the method is tedious, time consuming and human specific.

The work in [12] and several others consider vessel detection and PDR analysis as classifier design problem. In PDR, the abnormal vessels that grow along the retina appear to be denser group of neovascular nets. They are normally in the form of very thin vessel segments. Therefore, for proper grading of DR and further medical diagnosis, in addition to accurate extraction of blood vessels, an automated system must be capable of differentiating vessels of different widths. It is difficult to find optimal threshold value that partitions the thick and thin vessels. This is basically a classification problem where the thickness of vessel is subjective. Fuzzy c-means (FCM), which is an unsupervised method of classification may be used to accurately classify the retinal vasculature, extracted by matched filtering into thick and thin vessels.

This paper, as an integrating framework uses curvelet transform, matched filtering and FCM for retinal blood vessel detection. First the curvelet transform is applied on the image. It decomposes the image into a number of subbands wherein the approximated subband is completely suppressed (i.e, the curvelet co-efficients related to coarse approximation are equated to zero) and all other subbands are retained. The inverse curvelet transform is computed which is then superimposed on the original image. A Gaussian kernel based matched filtering is then done on the enhanced image, the output of which undergoes FCM to detect the vessel silhouette from background.

The rest of the paper is organized as follows. Section II discusses on the mathematical preliminaries namely curvelet transform, matched filtering, FCM algorithm. Proposed method of vessel detection is described in Section III. Section IV reports the simulation results and discussion while conclusion and future works are mentioned in Section V.

II. Mathematical Preliminaries

This section briefly presents various tools and techniques used to make this article self understood.

A. Curvelet Transform

1) Continuous time Curvelet Transform: The basic concept of curvelet transform, as proposed by Candes and Donoho [13], is to decompose the entire image into a number of subbands, i.e, to separate the object into a series of disjoint scales. It can handle curves within a structure using a small number of co-efficients. In each scale, it is translated and rotated as copies of the "mother curvelet".

2) Fast Discrete Curvelet Transform: The Fast Discrete Curvelet Transform (FDCT) is the newly constructed and improved version of curvelet transform. This method has attracted the attention of the researchers due to its simplicity, fastness and less redundancy as compared to the original curvelet transform. There are two versions of FDCT:
(i) unequally spaced Fast Fourier transforms (USFFT),
(ii) wrapping function.

The wrapping based FDCT algorithm has been used in this work since it is faster and more efficient than USFFT. In this method, the Fourier plane is divided into a number of concentric circles that represent multi-scale decomposition of the original image. Each of these concentric circles is again divided into a number of angular divisions, referred to as the orientation. This combination of scale and angular division is known as parabolic wedges. Next to find out the curvelet coefficients, inverse FFT is taken on each scale and angle.

B. Matched Filtering using 2D kernel

The application of classical matched filtering (MF) in spatial domain for retinal blood vessel detection was proposed by Chaudhuri et al. [6]. As mentioned in [6], the grey-level profiles of the cross-section of blood vessels in retinal image are Gaussian in nature and the intensity profile is symmetric about the straight line that passes through the center of the vessel. Since the vessel intensity profile is assumed to be Gaussian, the impulse response of the optimum filter should also be Gaussian and can be expressed as $h_{opt}(d) = -exp(d^2/2\sigma^2)$, where σ denotes the scale of the Gaussian function. Here the significance of $-$ve sign implies the fact that the background is brighter than the vessels.

One of the most important properties of the blood vessel that has been assumed in the majority of works is that the two edges of a vessel are parallel to each other and may be approximated as piece-wise linear segments of finite width. Therefore instead of matching a single intensity profile of the cross section of a vessel, performance can be improved significantly by matching a number of cross-sections (of identical profile) along the entire length simultaneously. A prototype matched filter kernel may be mathematically expressed as $K(x,y) = -exp(x^2/2\sigma^2)$ for $|x| \leq (3\sigma)$ and $|y| \leq (L/2)$, where L is the length of the blood vessel segment considered to have fixed orientation. Here the direction of the blood vessel is considered to be aligned along y-axis. In an image, a particular vessel may be oriented at any angle between 0 and π and if the vessel is aligned at $\pi/2$, then only the matched filter will have its peak response. The filter is rotated for all possible angles and convolved with the image under experiment. The corresponding responses are compared and for each pixel only the maximum response is retained. Since the exact width of the vessels are not known apriori, the scale of the Gaussian filter may not perfectly match with all the vessels in retina. Therefore, in this paper, for extracting thick and thin vessels, multi-value matched filtering, i.e, larger σ for thick vessels and smaller σ for thin vessels are used. Matched filtering in multi-scale in the curvelet domain is expected to produce much better result but in that case the computational complexity would increase exponentially. Not only that the filtering operation in curvelet domain is also expensive that leads to high computational cost. To take into account this problem as well as to detect/identify vessels of different types, matched filter in spatial domain and with less number of σ values for Gaussian kernels may be used. However convolution using Gaussian kernel in spatial domain respond not only to the vessels but also to non-vessel edges that have the same Gaussian profile and scale. This in turn leads to false detection of non vessel edges as vessels. To make a better trade-off in detecting the vessel and the non-vessel as well as thick and thin vessels, choice of σ values and computational cost complexity, spatial domain matched filter response of enhanced image may be treated as an unsupervised classification problem that may be better resolved using FCM algorithm.

C. Fuzzy C-Means Algorithm

FCM [14] is a classical unsupervised technique for clustering. It is used to classify a given set of n number of z dimensional data points $X = \{\overrightarrow{X_1}, \overrightarrow{X_2}, \cdots, \overrightarrow{X_n}\}$ into a set of c fuzzy classes or partitions A_i. The algorithm must follow the two criteria as mentioned below.
(1) The summation of the membership values of any particular data point $\overrightarrow{X_p}$ in all the c classes is 1, i.e. $\sum_{i=1}^{c} \mu_{A_i}(\overrightarrow{X_p}) = 1$ for $p = 1, 2, \cdots, n$
(2) All the data points must not belong to the same class with membership 1. Mathematically it can be expressed as $0 < \sum_{p=1}^{n} \mu_{A_i}(\overrightarrow{X_p}) < n$ for $i = 1, 2, \cdots, c$.

The expression for cluster centers $(\overrightarrow{V_i})$ and the corresponding membership values (μ_{A_i}) in each of the classes is given by the following equations.

$$\overrightarrow{V_i} = \frac{\sum_{p=1}^{n}(\mu_{A_i}(\overrightarrow{X_p}))^m \overrightarrow{X_p}}{\sum_{p=1}^{n}(\mu_{A_i}(\overrightarrow{X_p}))^m} \quad (1)$$

Here $m(m > 1)$ is any real number that influences the membership grade.

$$\mu_{A_i}(\overrightarrow{X_p}) = [\sum_{j=1}^{c}(\|\overrightarrow{X_p} - \overrightarrow{V_i}\|^2/\|\overrightarrow{X_p} - \overrightarrow{V_j}\|^2)^{1/m-1}]^{-1} \quad (2)$$

for $1 < i \leq c$ and $1 \leq p \leq n$

D. Performance Metrices

To measure the performance of the proposed algorithm, the following three matrices are used:
(1) Detection Accuracy (ACC) is defined as the ratio of the total number of correctly classified pixels (i.e, the vessel pixels classified as vessel pixels and non-vessel pixels classified as non-vessel pixels) to the number of total vessel pixels in the ground-truth image.
(2) True Positive rate (TPR) is the ratio of the total number of correctly classified pixels to the number of total vessel pixels in the ground-truth.
(3) False positive rate (FPR) which is the fraction of pixels erroneously detected as vessel pixels.

III. Proposed Method

The proposed method for retinal blood vessel extraction is diagrammatically represented in Fig 1. The entire methodology can be divided into the following four parts.

A. Green Channel Extraction

An RGB image is composed of three channels: red, green and blue. If a retinal image is split into these three channels, it is observed that the red channel is saturated and the blue channel tends to be empty, whereas for the green channel image, the contrast between the blood vessels and the background is the maximum. Therefore in this paper, the green channel image is used for vessel extraction.

B. Curvelet based Edge Enhancement

In the present work, the edge enhanced image is obtained in three steps.

1) Step 1: First the retinal image is decomposed into a number of subbands using curvelet transform with different scales and orientations. Curvelet co-efficients are very useful since they contain the most important information like missing and broken boundary information, horizontal, vertical, diagonal and other edge details as well as the course approximation of the image.

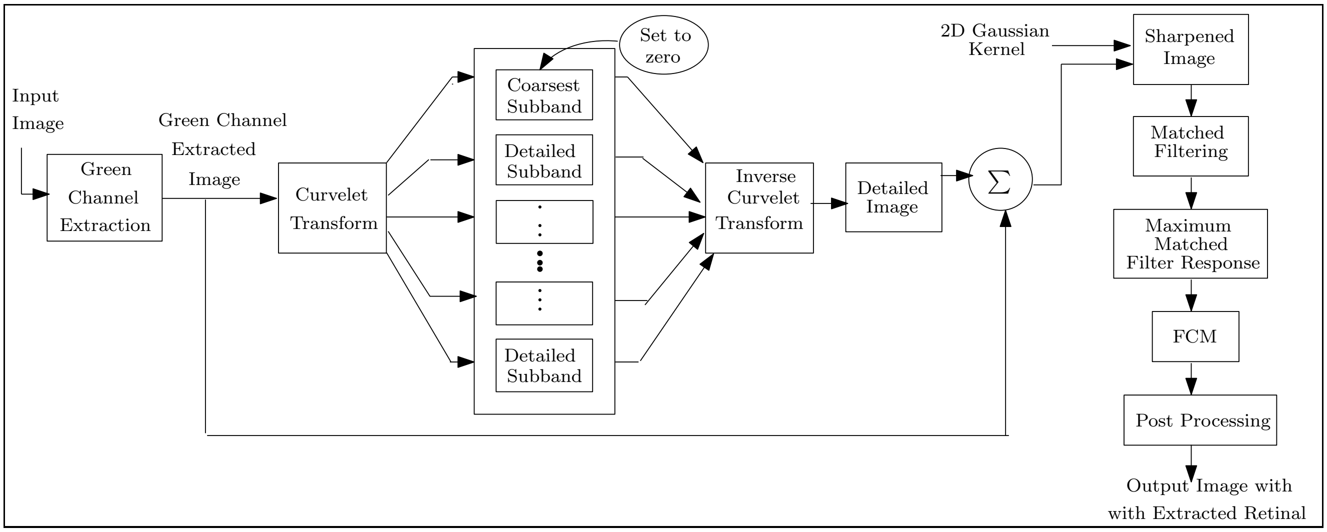

Fig. 1. Schematic Diagram of Proposed Method for Retinal Vessel Detection

2) Step 2: Next the approximate subband, i.e, the coefficients corresponding to coarse approximation of the image are set to zero, keeping the other subbands intact. As a result, the background gets suppressed, while detail edges are highlighted.

3) Step 3: After that the inverse curvelet transform of the background suppressed image is computed and superimposed on the original image. This in turn increases the contrast, specially between thin, tiny, faint vessels and the background. The edges of the narrow vessels which were hardly distinguishable from the background are also sharpened.
Thus, using curvelet transform both the strongest and faintest edges in retinal image can be enhanced.

C. Application of Fuzzy C-means algorithm

To differentiate vessel and non vessel pixels, the FCM algorithm is implemented. The significance of using FCM is that the width of thick and thin vessels may not be fixed in retina, they may vary between a maximum and minimum level. Therefore, different categories may exist within thick and thin vessels depending on the vessel width and can be distinguished using FCM. This paper uses FCM to classify the image into four classes namely background, thin, medium and thick vessels. The thick vessels are further categorized as thick-thin, thick-medium and thick-thick vessels and the thin vessels as thin-thin, thin-medium and thin-thick vessels. According to the FCM algorithm, each and every pixel of the image must have a finite membership in all the four classes. For a pixel that belongs to a vessel in the edge enhanced image, the maximum matched filtered response is very high. On the other hand, for a background pixel, the corresponding response is very weak. A pixel is classified in one of the four classes in which the membership value of its maximum matched filter response is highest.

D. Post Processing

The MFR of the non-vessels and the thin vessels are very low. Therefore, non-vessels like noise and other spurious components may also be detected as thin vessels that in turn increase the FPR. This needs removal of these spurious components through some kind of post processing operation. This work uses connected component analysis to remove the isolated small pixel blocks. After detection, the connected components above a specific area are labeled using 8 connected neighborhood and considered as thin vessel pixels. Thus the false edges are separated from the original vessel edges efficiently.

IV. Simulation Results

This section presents performance of the proposed method tested on DRIVE database [17]. DRIVE database consists of 40 images divided into a training set and a test set, each of them contains 20 images. Both the test and the training set provide manually segmented images. For edge enhancement using curvelet transform, the source code for curvelet transform, available in http://www.curvelet.org is used. For the implementation of the proposed method, first the green channel image is extracted from the original RGB image. Next 5 scales and 16 directions are used in the coarse scale to apply FDCT via wrapping on the green channel extracted image. It causes 32 directions in scales 2^{nd} and 3^{rd} and 64 directions in scales 4^{th} and 5^{th}. The parameter values selected for matched filtering are $\sigma = 1.5$ and $L = 9$ for thick vessels and $\sigma = 1$ and $L = 5$ for thin vessels. Simulation done on the large number of images show that the above values for the number of scales, directions and the values of σ and L are good enough for identifying the different types of vessels. The matched filter kernel is rotated at an increment of 12^0 which in turn generates 15 directions for matched filtering. Initially, the maximum matched filtered response is obtained by setting

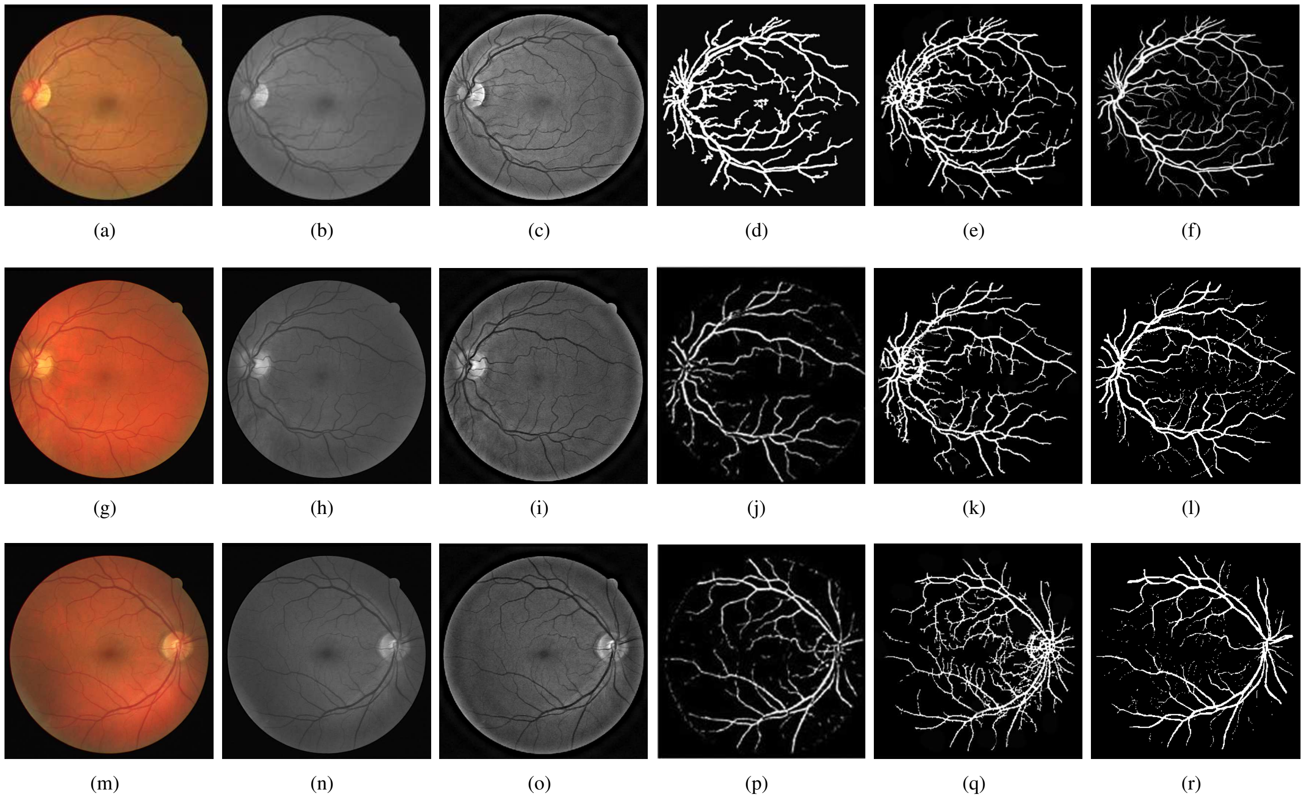

Fig. 2. Simulation Results (a), (g), (m) Original Retinal Image. (b), (h), (n) Green Channel Image. (c), (i), (o) Edge enhanced image using Curvelet Transform. (d) Extracted vessel map by [15] (j), (p) Extracted vessel map by scheme [11]. (e), (k), (q), Extracted blood vessel by proposed method. (f), (l), (r) Manually segmented image.

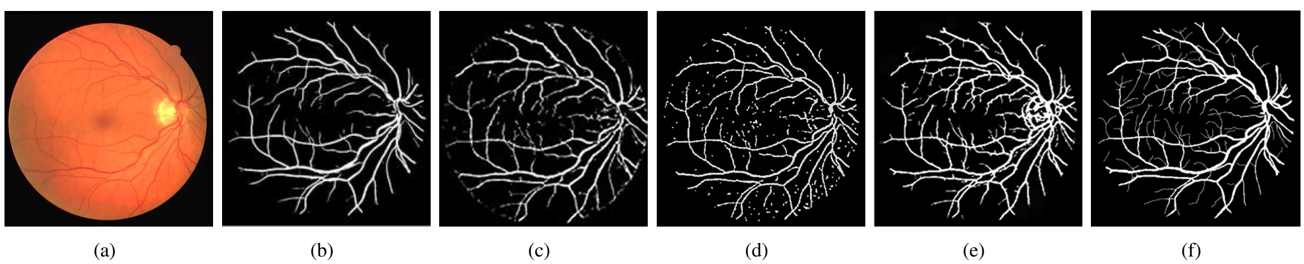

Fig. 3. Simulation Results (a) Original Retinal Image. (b), (c), (d) Vessel Extraction results of [12], [11] and [16], respectively. (e) Extracted vessel map by proposed method. (f) Manually segmented image.

the parameter values σ to 1.5 and L to 9. This extracts the thick vessels from the image. Next with $\sigma = 1$ and $L = 5$ the thin vessels are detected. The extracted blood vessels with the two values of σ are then logically OR-ed to get the complete vasculature.

The proposed method has been applied to all the 40 images of DRIVE database. However, due to space limitation the extracted vasculature are shown only for few images. The results of vessel detection are presented in Fig. 2 and Fig. 3. The original retinal image, the green channel image and the curvelet based edge enhanced image are shown in the 1^{st}, 2^{nd}, 3^{rd} column of Fig 2, respectively. Fig. 2(d) presents the extracted vascular pattern as proposed in [15] and the extracted vessel structure in [11] is shown in Fig. 2(j) and Fig. 2(p). The results of the proposed method are depicted in the 5^{th} column and the hand labeled ground-truth is presented in the last column of Fig 2. Comparing 4^{th} and 5^{th} column, it is observed that some of the tiny vessels which are not detected in Fig. 2(d), Fig. 2(j), Fig. 2(p) are detected by the proposed method shown in Fig. 2(e), Fig. 2(k) and Fig. 2(q), respectively. Therefore, the proposed method outperforms the method in [11] in finding thick and long as well as short and narrow vessels efficiently. Fig. 3 shows the results of vessel extraction by the methods described in [12], [11] and [16], respectively and our proposed method in Fig. 3(e). Here also the results show that the proposed method finds thick and thin

TABLE I
COMPARISON OF VESSEL EXTRACTION RESULTS ON RECENT STUDIES

Method	TPR	FPR	Average ACC
Chaudhuri et al [6]	0.6168	0.0259	0.9284
Mendonca et al [4]	0.7344	0.0236	0.9452
Miri et al [11]	0.7352	0.0205	0.9458
Proposed Method	0.7645	0.0191	0.9439

as well as large and tiny vessels very efficiently, while the existing methods fail to extract tiny and narrow vessels.

The ROC curve which is the graph between FPR and TPR has been depicted in Fig. 4(a). The area under the curve is 0.9415. The values of TPR, FPR, average ACC are shown in Table 1. Numerical values show that TPR of the proposed method is the highest than the methods in [6], [4] and [11], where the average accuracy of our method is almost similar. The main advantage of this algorithm is its simplicity and ease of implementation and can extract the finest details of the vessels. Although TPR values are high, it is also noted that along with the edges of thick and thin vessels, strong edges of lesions and optic disc boundary that have the same Gaussian profile and scale as the matched filter may also be detected. This in turn increases the value of FPR as observed from the saturated region of the ROC curve.

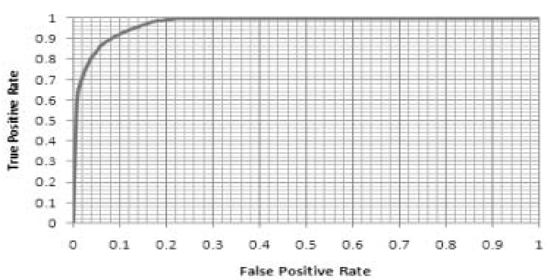

Fig. 4. ROC curve corresponding to the proposed method

V. CONCLUSIONS AND SCOPE OF FUTURE WORK

This paper proposes a method for retinal vessel extraction based on edge enhancement in the curvelet domain, matched filtering and FCM algorithm. Curvelet transform extracts the blood vessels of different orientations with clear boundaries and curvatures. Matched filtering is used to intensify the blood vessels of the edge enhanced image and FCM algorithm classifies the image into background, thick, medium and thin vessels. Experimental results show that this method is very much efficient in extracting thick and long vessels as well as tiny and thin vascular nets. Although the proposed method is efficient in accurately identifying different types of vessels, the optic disc boundary elimination need attention as possible extensions of the work.

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