Differential Area Profiles

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Abstract

In this paper a new feature descriptor, the differential area profile (DAP), is presented. DAPs, like the regular differential morphological profiles, are computed from some size distribution. The proposed method is based on the area metric given by regular connected area filters. Area compared to local width, i.e. the diameter of the structuring element in the corresponding set of openings by reconstruction in classical DMPs, leads to a rather different multiscale decomposition. This is investigated here and an example on a very high resolution satellite image tile is given.

I. Introduction

Urban scenery registered in very high resolution (VHR) satellite images or aerial photographs is often characterized by same class structures of varying shape, size, contrast, intensity and texture. Methods for identifying class labels reliably are therefore focused on intrinsic features of the targeted structures, which are usually obtained at multiple scales. A typical multiscale decomposition of an image is its size distribution computed using granulometries [1], [2]. A granulometry is an ordered set of filters each of which transforms its input into a new image in which features smaller than a particular attribute threshold are absent [3]. Image differentials that find usage in classification problems, can be computed directly from granulometries. Examples are the connected pattern spectra [3], [4] which are histograms containing the number of pixels or the amount of image detail over a range of size/shape classes, and the differential morphological profiles or derivatives of the morphological profiles (DMP) [5] that are computed at each pixel.

The DMP is a set of two vectors in opposite directions, associated to each image element. Each vector entry corresponds to the pixel intensity difference after the application of an attribute filter on the input image for any two consecutive scales. The first vector registers differentials on dark structures and the second on bright structures. This essentially requires that any extensive operator that is to be used, should have an anti-extensive dual. DMPs are defined based on openings and closings by reconstruction [6], [7]. Both operators retain an object (bright or dark respectively) if the radius of the associated structuring element (SE) is smaller than its maximum width. The object is discarded otherwise.

Local width however provides a limited number of scales since for HR and in some cases even for VHR images, most urban structures of interest like buildings, roads, shadows, etc, are only a few pixels wide. This lack of sufficient separability can be countered by introducing the area attribute (Section II). This leads to the notion of differential area profiles or DAPs which are studied in Section III. An example on a VHR satellite image is given in Section IV followed by a comparison between DMPs and DAPs. Conclusions are given in Section V.

II. Area Filters

Morphological operators that retain or reject connected components (binary objects) based on some measurable feature are called connected attribute filters. In the case of area filters, operators are increasing, idempotent and either anti-extensive or extensive, i.e. openings or closings respectively. For any two sets \( X, Y \in E \), with \( E \) being some arbitrary superset and \( \psi \) an operator \( \psi : \mathcal{P}(E) \rightarrow \mathcal{P}(E) \), the three
properties mean that (a) \( \psi(\psi(X)) = \psi(X) \), (b) if \( X \subseteq Y \Rightarrow \psi(X) \subseteq \psi(Y) \) and (c) \( \psi(X) \subseteq X \). The extensive property is the reverse of (c).

Connected operators [8] can either retain components intact or remove them in their entirety, but they cannot introduce new edges. Let \( X \subseteq E \) denote a binary image. The binary connected opening [8] \( \Gamma_x(X) \) of \( X \) at point \( x \in E \) yields the connected component of \( X \) containing \( x \) and \( 0 \) otherwise. Thus \( \Gamma_x \) extracts the connected component to which \( x \) belongs, discarding all others. The binary area opening, which is based on connected openings, can now be defined as:

**Definition 1:** Let \( X \subseteq E \) and \( \lambda \geq 0 \). The binary area opening of \( X \) with scale parameter (area) \( \lambda \) is given by:

\[
\Gamma^\lambda_X = \{ x \in X \mid A(\Gamma_x(X)) \geq \lambda \}. 
\tag{1}
\]

The binary area closing can be defined by duality

\[
\Phi^\lambda_X = [\Gamma^\lambda_X(X^c)]^c. 
\tag{2}
\]

Increasing attribute filters can be computed on a gray-scale image \( f \) by considering its threshold decomposition to \( H \) binary sets \( T_h(f) \), with \( h \in H \), each of which is given by:

\[
T_h(f) = \{ x \in E \mid f(x) \geq h \}. 
\tag{3}
\]

The result is obtained by superimposing the filtered outputs, i.e.

**Definition 2:** The area opening for a mapping \( f : E \to \mathbb{R}^n \) at point \( x \in E \) is given by

\[
\gamma^\lambda_X(f)(x) = \vee\{ h \mid x \in \Gamma^\lambda_X(T_h(f)) \}. 
\tag{4}
\]

The gray-scale area closing \( \varphi^\lambda_X \) is defined by duality:

\[
\varphi^\lambda_X(f) = -\gamma^\lambda_X(-f). 
\tag{5}
\]

In brief, (4) states that an area opening of an image \( f \) assigns each point the highest threshold at which it still belongs to a connected foreground component of area \( \lambda \) or larger, and in the case of (5), the area closing assigns each point the highest threshold at which it still belongs to a connected background component of area \( \lambda \) or larger.

### III. Differential Area Profiles

Attribute filters operated at multiple scales can be used for describing the structural composition of the image contents. Examples are the connected size/shape pattern spectra [3] and the DMP fields [5]. In the case of DMPs, each image point associates to a vector called a morphological profile and each vector entry associates to a specific scale, storing the pixel intensity after the application of the corresponding operator at that scale.

Let \( \Pi(\gamma^\lambda_X(f))(x) \) be the area opening profile of an image \( f \) at a point \( x \) given by:

\[
\Pi(\gamma^\lambda_X(f))(x) = \{ (\gamma^\lambda_X(f))(x) : x \in E \text{ and } \forall \lambda \in \bar{A} \}. 
\tag{6}
\]

\( \bar{A} \) is an \( I \)-long vector of \( \lambda \) values indexed by \( 0 \leq i < I \), i.e. \( \bar{A} = [\lambda_0, \lambda_1, ..., \lambda_{I-1}] \), with \( \lambda_0 = 0 \). Moreover, let the area closing profile be defined by duality as:

\[
\Pi(\varphi^\lambda_X(f))(x) = \{ (\varphi^\lambda_X(f))(x) : x \in E \text{ and } \forall \lambda \in \bar{A} \}. 
\tag{7}
\]

The derivative of the area opening profile at point \( x \) is defined as an \( I - 1 \) long vector \( \Delta^\Pi_X(\gamma^\lambda_X(f))(x) \), with each entry given by:

\[
\Delta_i^\Pi_X(\gamma^\lambda_X(f))(x) = - \frac{d(\gamma^\lambda_X(f))(x)}{di} \bigg|_{i>0}. 
\tag{8}
\]

The term \( di = i - (i - 1) = 1 \) constrains the computation of the differential to any two consecutive area openings or closings. The derivative of the closing profile \( \Delta^\Pi_X(\varphi^\lambda_X(f))(x) \) is defined analogously.

The Differential Area Profile (DAP) of a point \( x \) is the \( 2(I - 1) \) long vector given by the concatenation...
(denoted with □) of $\Delta^\Pi(\gamma_A^\Delta(f))$ and $\Delta^\Pi(\varphi_A^\Delta(f))$ at $x$, i.e.:

$$\text{DAP}(x) = \Delta^\Pi(\gamma_A^\Delta(f))(x) \sqcup \Delta^\Pi(\varphi_A^\Delta(f))(x).$$ (9)

An example is given in Fig. 1 where the DAP of a VHR image tile of Sanaa (Yemen) is computed. Images (b) and (c) show the fields of the derivatives of the area opening and closing profiles respectively. Each vector field consists of 14 slices. Each slice is binarized by setting non-zero pixels to 255 and the 3D connected components are color labeled [9]. Both volumes are shown in color table projection with no transparency. The threshold $\lambda_1$ is set to 2 and the rest are given by $\lambda_{i+1} = 2 \times \lambda_i$.

IV. Comparison Between DAP and DMP

Both, differential area and morphological profiles employ some size metric to differentiate between objects belonging to different classes. DMPs, that rely on openings by reconstruction, employ the size of the associated structuring element as an attribute. This is known to provide only rough categorization based on size, however it is very efficient for sorting objects based on local width. The DAPs instead, employ the area metric as an attribute. The practical difference between them is demonstrated in Fig. 2 where a simple 2-scale DMP and DAP field is computed with $\Lambda_{DMP} = \{0, 5, 10\}$ and $\Lambda_{DAP} = \{0, 100, 200\}$ respectively. Image (b) shows the first scale (field layer) of the DMP field containing a single object of max width equal to 5, and image (c) shows the second scale with the remaining two objects of max width equal to 10. Images (d) and (e) show the two DAP field scales for the respective area thresholds.

The choice between DAP and DMP is application dependent, though computationally the DAP has a number advantages over the DMP. These stem from the fact that the area attribute can be computed incrementally [10]. Moreover, area openings and closings can be computed efficiently using the Max-Tree algorithm [10] or Tarjan’s Union-Find structure [11]. The DAP’s speed advantage compared to algorithms for area openings by reconstruction relies also on the fact that no iterative procedures are required. Moreover, DAPs support a far greater scale resolution, which at most can be one pixel difference between consecutive scales. With DMPs instead, as the radius of the structuring element increases, the structures of interest in VHR imaging reduce sharply.

Fig. 3 shows some indicative layers of the DAP and DMP fields computed on the image of Sanaa (Fig. 1). Note that the DMP is computed with a square SE and each $\lambda_i$ in $\Lambda_{DMP}$ corresponds to the side length of the SE. It can be seen that similar sets of objects can be found at both fields but at rather different scales. In fact if the size of the SE is compared to the corresponding area threshold there is no correlation at all. This is because the DMP will consider any object size for a given layer provided that the width constraint is met. This can be used to track elongated objects and combined with the DAP can produce a richer description of the image contents; see example at the bottom row of Fig. 3.

V. Conclusions

A new feature descriptor was presented based on differentials that are computed from granulometries of connected area filters. The new method does not aim to replace the existing DMP framework but instead to explore a new type of image decompositions. DAPs can in fact complement classical DMPs and this is currently under investigation. The DAP’s computational advantage can be further boosted by introducing an efficient algorithm for its computation. In future work we aim at delivering such an algorithm, that will also support non-increasing attributes.

References

Figure 3. DAP field layers of Sanaa (surface plots): for $\lambda_8 = 256$ (a), $\lambda_{10} = 1024$ (b), and $\lambda_{11} = 2048$. The respective DMP field layers for $\lambda_7 = 7$ (d), $\lambda_9 = 9$ (e), and $\lambda_{11} = 11$ (f). Bottom row: DAP (g) and DMP (h) field layers for $\lambda_{10} = 1024$ and $\lambda_9 = 9$ respectively, after contrast enhancement, and the addition of the two in (i).


