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A Variational Formulation for Fingerprint Orientation Modeling

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Abstract

Fingerprint orientation plays important roles in fingerprint recognition. This paper proposes a framework for modeling the fingerprint orientation field based on the variational principle. The proposed method does not require any prior information about the structure of acquired fingerprints. Comparison has been made with respect to state-of-the-arts in fingerprint orientation modeling.

1. Introduction

In the recent a few years, with the increasing concern on security, the pace of developing and deploying biometrics technology, in particular the fingerprint based technology, has been accelerated tremendously in a wide range of areas from government, defense, air port security, to commercial services. In spite of the wide application of fingerprint technology to our daily life, there remain several issues which have not been adequately addressed. Among them, how to recognize fingerprints acquired with poor quality is still a challenging problem. Key to this problem is to enhance the fingerprint image before the process of recognition. A number of research efforts have been put on this topic and a wealth of techniques has been proposed [1] [2]. Instead of directly employing generic methods in image enhancement for improving the fingerprint image quality, most fingerprint enhancement methods are based on the characteristic structure within the fingerprint, which have been proven to be more effective in practice. One of the most important features in the fingerprint is the highly parallel oriented pattern, and for this reason there are intensive research interests on fingerprint orientation modeling.

In general, fingerprint orientation modeling starts from a local estimation of the orientation, followed by a refining process where prior knowledge or information from a larger scale will be employed. The local estimation can simply be based on the image gradient, or be derived from more sophisticated methods such as statistical techniques [3] [4], filter-bank [5], ridge projection [6], structure tensor [7], integration operator [8] or local voting [9]. After that, a global model can be built and applied in turn to local predictions. A pioneered work in this direction was presented by Sherlock and Monro in 1993 [10], where the orientation field is described by a zero-pole model. The model is formulated in the complex plane with the core point as zero and the delta point as pole. In general, the zero-pole model is almost perfect in regions near singular points, but often unsatisfactory in other regions. An improvement was made by Vizcaya and Gerhardt [11] using a piecewise linear approximation model around singular points to adjust the zero and pole's influence. Similar ideas have been presented in [12] [13]. Very recently, a unified framework was presented in [14] and most of aforementioned models can be regarded as special cases.

For the above global estimation methods, they have one common feature; that is the dependency on the knowledge of singular points. However, the detection of singular points is never a trivial issue and the success of the detection strongly relies on the quality of the derived fingerprint orientation field. Thus, the problem will be as complicated as the chicken-egg paradox. To circumvent this problem, Wang et al [15] presented an orientation estimation method in another way where the modeling problem is formulated as a data fitting problem and trigonometric polynomial is utilized to fit the orientation data as estimated by local methods. A remarkable feature of this model is that it does not require any prior knowledge of singular points. The method has been demonstrated to outperform the singular points dependent method like the combination model in fingerprint image enhancement. The method has recently been extended in [16], where Legendre polynomial is employed as the functional basis and moreover the issue of singularity preservation is addressed through minimizing a cost function.

In this study we will present a new framework to model the fingerprint orientation field. The proposed method is based on the variational principle and need not to have the knowledge of explicit functional form. In addition, singularities can be modeled seamlessly in the framework without any prior knowledge.

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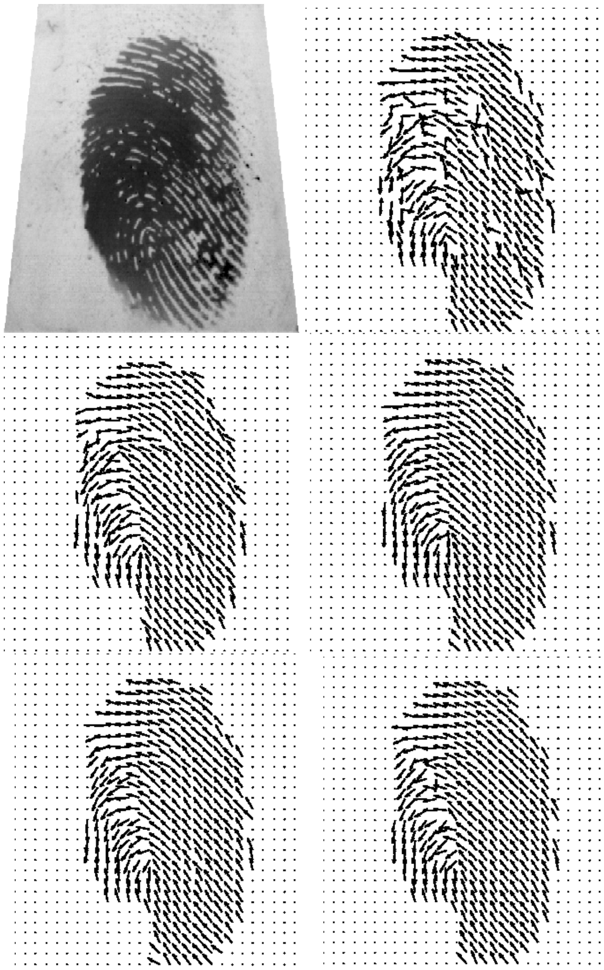

Fig. 1 Row 1: a low-quality fingerprint image (left) and orientation estimation by gradient (t=0, right), where dots indicate background; Row 2: orientation refined by the first term (kinetic energy) at t=1 (left) and 4 (right); Row 3: orientation refined by two energy terms at t=2 (left) and 10 (right).

2. Methods

2.1 Variational Principle

Originated by Leibniz and founded by Euler and Langrange, variational principle is an important method in physics for determining the state or dynamics of a physical system. As stated by Euler [17], "Since the fabric of the Universe is most perfect and is the work of a most wise Creator, nothing whatsoever takes place in the Universe in which some relation of maximum and minimum does not appear". Variational principle seeks the solution through finding the extremum (minimum, maximum or saddle point) of a functional. The method can be expressed using the calculus of variations, which is a branch of mathematics dealing with integral minimization. The functional to be minimized can be formed as an integral involving unknown function f or its derivatives as follows

$$J[f] = \int_{x_x}^{x_2} L(x, f, f')dx. \tag{1}$$

Then the problem is to find the extremal function f^* where the rate of change of the functional $J[f]$ is zero. For more details on the variational principle, interested readers can refer to [18].

Besides enormous applications of the variational principle to physics and chemistry, the method has also been employed frequently to investigate problems in computer vision, such as edge detection, image denoising, super-resolution image reconstruction, optical flow, surface reconstruction, shape from shading, stereo, image inpainting [19] [20]. In the following, we present a formulation of fingerprint orientation modeling using the variational principle.

2.2 Variational approach to orientation modeling

For the convenience of description, let θ denote an orientation field in image domain Ω . Then, the problem in fingerprint orientation modeling is to reconstruct an orientation field φ such that

(C1) φ is smooth and

(C2) true singularities in θ are preserved in φ.

In fingerprint image processing, directions of gradients with difference of π have the same effect on inference of ridge orientation or the design of filters. However, direct operation (integration/summation) will cancel out these directions. To avoid this problem, a common practice in fingerprint orientation modeling is to project θ to the complex domain and take the square as follows:

$$\tilde{\theta} = \exp(i2\theta) \tag{2}$$

which has been termed the double angle approach in literature [15]. Similarly, we have

$$\tilde{\varphi} = \exp(i2\varphi). \tag{3}$$

As will be shown in the following, the estimation of the orientation field can be formulated in the framework of the variational principle. To model the constraint (C1), it is usually accomplished through minimizing the derivatives of the function. In this study, it is modeled as

$$L_1 = \left\|\nabla \tilde{\varphi}\right\|^2 \tag{4}$$

where $\|\bullet\|$ is the magnitude of the complex number. As for the model of the second constraint, it is given by

$$L_2 = \psi\left(\tilde{\theta}\right)\left\|\tilde{\theta} - \tilde{\varphi}\right\|^2. \tag{5}$$

Here $\psi\left(\tilde{\theta}\right)$ stands for a function to capture the singularity in the original orientation field and its value increases with respect to the increase of the saliency of $\tilde{\theta}$, thus singularities in $\tilde{\theta}$ have higher weights in determining the function $\widetilde{\varphi}$. Combined these constraints together, the functional $J(\widetilde{\varphi})$ is defined as:

$$J(\widetilde{\varphi}) = \iint_{\Omega} (\lambda L_1 + L_2) dxdy \tag{6}$$

where $\lambda > 0$ is a regularization parameter. In the language of variational principle, the first term is called the kinetic energy and the second is the potential energy. It should be noted that the design of these two energy terms here is merely for the purpose to illustrate the fingerprint orientation modeling using the variational principle.

2.3 Numerical implementation

For notational convenience, let us write

$$u = \cos 2\varphi \tag{7a}$$

$$v = \sin 2\varphi \tag{7b}$$

Then we have

$$L_1 = \lambda\left(u_x^2 + u_y^2 + v_x^2 + v_y^2\right) \tag{8a}$$

$$L_2 = \psi(\widetilde{\theta})\left((u - \cos 2\theta)^2 + (v - \sin 2\theta)^2\right), \tag{8b}$$

which is very similar to the gradient vector flow formulation for object boundary detection [21]. Substituting (7a) to (8b) to (6), we can derive the solution of the variational problem by solving the associated Euler-Lagrange equations:

$$\lambda \Delta u - \psi(\widetilde{\theta})(u - \cos 2\theta) = 0 \tag{9a}$$

$$\lambda \Delta v - \psi(\widetilde{\theta})(v - \sin 2\theta) = 0 \tag{9b}$$

where Δ stands for the Laplacian operator. Furthermore, regarding the left hand side of the Euler-Lagrange equation as an infinite dimensional gradient, the equations (9a) and (9b) can be solved using the gradient descent method, which leads to the following equations:

$$u_t = \lambda \Delta u - \psi(\widetilde{\theta})(u - \cos 2\theta) \tag{10a}$$

$$v_t = \lambda \Delta v - \psi(\widetilde{\theta})(v - \sin 2\theta). \tag{10b}$$

If negating the second term, Eq. (10) is the heat equation, a special case of the more general diffusion equation, and λ is called thermal diffusivity. The derived partial differentiation equation can be solved using numerical methods. In this study, it is solved by the method of finite difference.

3. Experiments

To validate the performance of the proposed method, experiments have been carried out using publicly available fingerprint database, FVC 2000 Db1 [22]. For illustration, the function ψ is simply taken as the saliency of the original orientation field

$$\psi(x) = \frac{1}{|N_x|} \sum_{x' \in N_x} \left|\sin(\theta(x) - \theta(x'))\right|. \tag{11}$$

Parameter λ is set as 0.25.

Fig. 1 presents an example of low-quality fingerprint image (Row 1 left). The fingerprint image is firstly segmented using the block-based method. If the variance of a block is large, the block will be considered as belonging to foreground. After that, morphological processing is applied to yield a contiguous foreground region. On the right of Row 1 shows the orientation field estimated by the gradient method, where the dots indicate the background. It can be seen that there exist quite a lot of places with evident incoherence among the local orientations. Row 2 shows the results by the heat equation, i.e., only the kinetic energy (KE) is taken into account. It can be seen that great improvement is clearly visible even after one step of iteration (Row 2 left). After four steps (Row 2 right), the random fluctuation is almost perfectly removed. Row 3 depicts the results when both the kinetic and the potential energy is incorporated, where on the left is at time 2 and on the right is at time 10. Compared with the KE results (Row 2), the potential energy term is able to preserve the structure around the fingerprint singular point along with the diffusion process. Nevertheless, when t is small (4 in this study), the difference is very minor.

Table 1. Comparison of equal error rates and CPU time between the Fourier series and the proposed variation method

	Fourier Series	Variation Method
EER (%)	5.69	5.57
CPU time (ms)	140	16

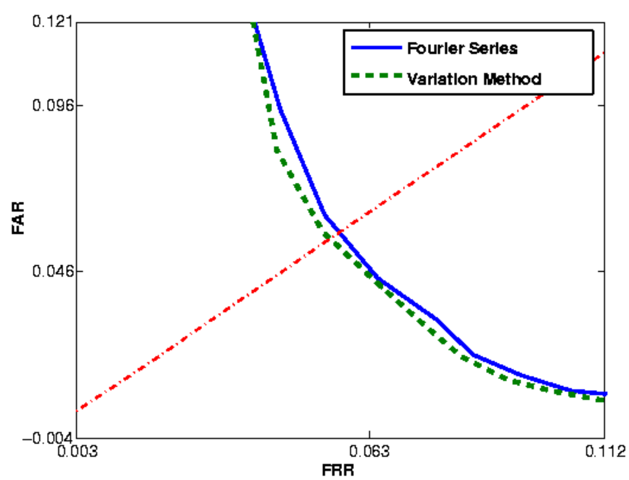

Fig. 2 ROC curve on FVC2000 Db1, where the dotted line represents the proposed method and solid line the Fourier series modeling method.

As aforementioned, the orientation field plays an important role in fingerprint image enhancement. As a way to indirectly measure the performance of orientation modeling, we have carried out matching experiment using the NIST fingerprint software [23] for minutia detection and matching. Fig. 2 plots the ROC curve based on the FVC 2000 Db1 dataset, where the variation method only takes into account the kinetic energy and the performance when both energy terms are considered is very similar. From Fig. 2, it can be seen that the performance between the proposed (dotted line) and the Fourier series modeling method (solid line) [15] is very close. The proposed method is slightly better. The equal error rates are summarized in Table 1, where the CPU time is also given. Both methods are implemented in Matlab and run in a PC with Windows XP operating system, 2.66GHz CPU and 2GB RAM. The proposed method is almost 8 times faster.

4. Conclusion

In this study we introduced a framework for fingerprint orientation modeling which is based on the variational principle. Different from existing methods to approximate the orientation using some function, the variational method needs no explicit form of the approximated function and the solution is derived implicitly from a functional space, where the desired features for the solution are modeled. The proposed framework is advantageous in terms of having less parameters and more freedom to preserve singularities in original fingerprints.

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