OPTIMAL SOURCE RATE ALLOCATION IN BODY SENSOR NETWORKS WITH ENERGY HARVESTING

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ABSTRACT

Body Sensor Networks (BSNs) are promising in pervasive health monitoring applications. One of the major challenges in BSNs is the sustainable power supply since each body sensor has limited battery capacity. In this paper, we optimize the source rate of the sensor to provide an uninterrupted service for BSNs with energy harvesting. First, we employ a discrete-time Markov chain to model the energy harvesting process at each sensor, and then theoretically analyze the relationship between the source rate and the uninterrupted lifetime of the sensor. Second, we formulate a steady-rate optimization problem, which minimizes the rate fluctuation with respect to the average sustainable rate via optimal source rate allocation at each sensor, under the requirement of the uninterrupted service. We propose an analytical solution to solve the optimization problem. In the simulations, we demonstrate that the proposed optimal solution enables the BSN to maintain an uninterrupted service with a steady output rate at each sensor.

Index Terms— Body sensor networks, eHealth, optimal source rate allocation, sustainable power supply, energy harvesting, discrete-time Markov chain, uninterrupted service

1. INTRODUCTION

Body Sensor Networks (BSNs) [1] are promising in pervasive health monitoring applications. A BSN is a body-area wireless sensor network, which consists of multiple wireless *body sensors* and an *aggregator*. A body sensor can be worn on the body or implanted inside the body. The body sensors continuously monitor the patient's vital signs, and then transmit them to the aggregator via wireless channels.

One of the major challenges in BSNs is the sustainable power supply. The body sensors are powered by battery. Battery replacement is impossible for the sensors implanted inside the human body. The most promising approach to deal with the energy supply problem for BSNs is energy harvesting [2][3]. In this approach, the sensor has an energy harvesting device that collects energy from ambient sources such as vibration and motion, light, and heat. However, the energy recharging rate is typically slow and time-varying. Therefore, a sensor may run out of the energy before its battery is recharged, causing an interruption in health monitoring service. Such service interruption may be fatal if the critical data for a severe patient is not captured in time.

Existing power management algorithms for energy-harvesting wireless sensor networks allow a sensor to be inactive for energy saving [4], which is not suitable for mission-fatal body sensor networks. A transmission scheme for BSNs with energy harvesting capabilities was proposed in [3], in which each sensor chooses one of the two transmission modes at each time slot to maximize the successful probability of event reports. However, the relationship between the transmission rate and the energy consumption has not been studied in [3].

Our contribution of this paper is twofold. First, we theoretically analyze the relationship between the source rate and the uninterrupted lifetime of the sensor. Second, we formulate and solve the steady-rate optimization problem, which minimizes the rate fluctuation with respect to the average sustainable rate, subject to the requirement of the uninterrupted service.

2. SYSTEM MODELS

IEEE 802.15.4, which is specifically designed to support low power and low data rate networks, is considered as the promising standard for body sensor sensor networks [5]. In this paper, we adopt IEEE 802.15.4 Time Division Multiple Access (TDMA) as the MAC protocol. Each sensor transmits the data only during its time slots. There is no interference among sensors.

2.1. Energy Harvesting Model

The set of the sensors in the body sensor network is denoted by $\mathbf{N}$. We model the energy harvesting process at sensor i ($\forall i \in \mathbf{N}$) as a discrete-time Markov chain [6], represented by $\{\mathbf{A}_i, \mathbf{Q}_i\}$, where $\mathbf{A}_i$ is the set of the states in the Markov chain, and $\mathbf{Q}_i$ is the transition probability matrix of the Markov chain. The recharging rate at state m ($m \in \mathbf{A}_i$) is denoted by $g_i^{(m)}$. The states in $\mathbf{A}_i$ are organized in such an

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ascending order that $g_i^{(1)} \leq g_i^{(2)} \leq \cdots \leq g_i^{(|\mathbf{A}_i|)}$ where $|\mathbf{A}_i|$ is the number of the states in $\mathbf{A}_i$. In the transition probability matrix $\mathbf{Q}_i$, the element q_{mn} denotes the transition probability from state m to state n. Let $\mathbf{\Pi}_i$ denote the steady-state probability vector at sensor i, which can be calculated from the following relationships [6]: $\mathbf{\Pi}_i{}^T \mathbf{Q}_i = \mathbf{\Pi}_i{}^T$, and $\mathbf{\Pi}_i{}^T \mathbf{I} = 1$ where $\mathbf{I}$ is the *identity vector* with all elements equal to 1. The long-term average recharging rate of sensor i is then given by $g_i^{avg} = \mathbf{\Pi}_i{}^T \mathbf{g}_i$ where $\mathbf{g}_i$ is the vector of the recharging rates at sensor i.

2.2. Power Consumption Model

In a body sensor network, the power consumption at a sensor mainly consists of two parts: *sensing power consumption* and *transmission power consumption*. The sensing power consumption at sensor i, denoted by $P_{s,i}$, is proportional to the source rate r_i at sensor i, and it is given by $P_{s,i} = \psi_i r_i$ where ψ_i is the energy cost for sensing at sensor i. The transmission power at sensor i is given by $P_{t,i} = \beta_i r_i$ [7], where β_i is the transmission energy consumption cost of sensor i, and it is given by $\beta_i = \theta_i + \zeta_i d_i^{m_p}$, where θ_i is the energy cost of transmit electronics of sensor i, ζ_i is a coefficient term corresponding to the energy cost of transmit amplifier at sensor i, and m_p is the path loss exponent.

The total power consumption at sensor i is the sum of the sensing power and the transmission power, and it is given by

$$P_i = P_{s,i} + P_{t,i} = \psi_i r_i + \beta_i r_i = \psi_i r_i + (\theta_i + \zeta_i d_i^{m_p}) r_i, \quad \forall i \in \mathbf{N}. \tag{1}$$

3. ANALYSIS OF THE RELATIONSHIP BETWEEN SOURCE RATE AND UNINTERRUPTED LIFETIME OF THE SENSOR

A sensor consumes the energy through sensing and transmitting the data, and replenishes the energy through energy harvesting from ambient environment. Due to the time-varying energy level at each sensor, we study it in a discrete-time manner. The time is evenly divided into time slots with a fixed length τ. We assume that the recharging rate and the source rate at a sensor remain unchanged during a time slot.

We define the *uninterrupted lifetime* of a sensor as the duration from the time when the sensor starts to work until the time when the energy level of the sensor reaches 0 at the first time. Based on the energy harvesting model and the power consumption model described in Section 2, we have the following theorem on the relationship between the source rate and the uninterrupted lifetime of a sensor.

Theorem 1: Given that, at sensor i, the initial energy is E_i^{ini}, the source rate is r_i, the energy harvesting process is modeled by a Markov chain $\{\mathbf{A}_i, \mathbf{Q}_i\}$, and the power consumption is given by Equation (1), then we have three statements as follows. 1) If $0 < r_i \leq \frac{g_i^{(1)}}{\psi_i + \beta_i}$, the energy level of sensor i will *never* reach 0. 2) If $r_i > \frac{g_i^{(|\mathbf{A}_i|)}}{\psi_i + \beta_i}$, the energy level of sensor i will *definitely* reach 0 at some time, and the uninterrupted lifetime of sensor i is bounded by $T_i^{min} \leq T_i \leq T_i^{max}$ where $T_i^{min} = \frac{E_i^{ini}}{\psi_i r_i + \beta_i r_i - g_i^{(1)}}$ and $T_i^{max} = \frac{E_i^{ini}}{\psi_i r_i + \beta_i r_i - g_i^{(|\mathbf{A}_i|)}}$. The occurrence probability of minimum uninterrupted lifetime of sensor i is given by $P_r(T_i = T_i^{min}) = \pi_1 q_{11}^{\lfloor T_i^{min}/\tau \rfloor}$, and the occurrence probability of maximum uninterrupted lifetime of sensor i is given by $P_r(T_i = T_i^{max}) = \pi_{|\mathbf{A}_i|} q_{|\mathbf{A}_i||\mathbf{A}_i|}^{\lfloor T_i^{max}/\tau \rfloor}$. 3) If $\frac{g_i^{(1)}}{\psi_i + \beta_i} < r_i \leq \frac{g_i^{(|\mathbf{A}_i|)}}{\psi_i + \beta_i}$, the energy level of sensor i will potentially reach 0 at some time, and the uninterrupted lifetime of sensor i is bounded by $T_i \geq T_i^{min}$ where $T_i^{min} = \frac{E_i^{ini}}{\psi_i r_i + \beta_i r_i - g_i^{(1)}}$. The occurrence probability of minimum uninterrupted lifetime of sensor i is given by $P_r(T_i = T_i^{min}) = \pi_1 q_{11}^{\lfloor T_i^{min}/\tau \rfloor}$.

Proof:

During time slot t, the increment of the energy ϵ_i at sensor i is given by $\epsilon_i = \tau(g_i^{(m)} - P_i^{(t)}) = \tau(g_i^{(m)} - (\psi_i r_i + \beta_i r_i))$ where m is the current Markov state. Recall that $g_i^{(1)} \leq g_i^{(m)} \leq g_i^{(|\mathbf{A}_i|)}$ from the definition of $\mathbf{A}_i$. Therefore we have $\tau(g_i^{(1)} - (\psi_i r_i + \beta_i r_i)) \leq \epsilon_i \leq \tau(g_i^{(|\mathbf{A}_i|)} - (\psi_i r_i + \beta_i r_i))$.

1) If $0 < r_i \leq \frac{g_i^{(1)}}{\psi_i + \beta_i}$, then $g_i^{(1)} - (\psi_i r_i + \beta_i r_i) \geq 0$, which means $\epsilon_i \geq \tau(g_i^{(1)} - (\psi_i r_i + \beta_i r_i)) \geq 0$ at any time slot. In other words, the energy level is increased at any time slot. Therefore the energy level of sensor i will *never* reach 0.

2) If $r_i > \frac{g_i^{(|\mathbf{A}_i|)}}{\psi_i + \beta_i}$, then $g_i^{(|\mathbf{A}_i|)} - (\psi_i r_i + \beta_i r_i) < 0$, which means $\epsilon_i \leq \tau(g_i^{(|\mathbf{A}_i|)} - (\psi_i r_i + \beta_i r_i)) < 0$ at any time slot. In other words, the energy level is decreased at any time slot. Therefore the energy level of sensor i will definitely reach 0 when the initial energy is run out.

Given the initial energy E_i^{ini}, the uninterrupted lifetime of sensor i depends on the Markov state at each of the time slots from the beginning to the time when the energy is run out. Sensor i has a minimum uninterrupted lifetime when the sequence of the Markov states is $\{1, 1, 1, \cdots, 1\}$ until the energy level reaches 0 at the first time. In this case, the energy is decreased linearly at a reduction slope $((\psi_i r_i + \beta_i r_i) - g_i^{(1)})$. Hence the minimum uninterrupted lifetime can be found by $T_i^{min} = \frac{E_i^{ini}}{\psi_i r_i + \beta_i r_i - g_i^{(1)}}$. Since the length of time slot is τ, the number of the time slots within the minimum uninterrupted lifetime is given by $n_{min} = \lfloor T_i^{min}/\tau \rfloor$ where $\lfloor x \rfloor$ is a floor function which returns the largest integer not greater than x. The sequence of the Markov states in this case is $\lfloor T_i^{min}/\tau \rfloor$ repeated 1s. Therefore the occurrence probability of minimum uninterrupted lifetime of sensor i is given by $P_r(T_i = T_i^{min}) = \pi_1 q_{11}^{\lfloor T_i^{min}/\tau \rfloor}$ where π_1 is the probability that the sensor is initially at state 1, and q_{11} is the transition probability from state 1 to state 1.

On the other hand, sensor i has a maximum uninterrupted lifetime when the sequence of the Markov states is $\{|\mathbf{A}_i|, |\mathbf{A}_i|, |\mathbf{A}_i|, \cdots, |\mathbf{A}_i|\}$ until the energy level reaches 0 at the first time. In this case, the energy is decreased linearly at a reduction slope $((\psi_i r_i + \beta_i r_i) - g_i^{(|\mathbf{A}_i|)})$. Hence the maximum uninterrupted lifetime can be found by $T_i^{max} = \frac{E_i^{ini}}{\psi_i r_i + \beta_i r_i - g_i^{(|\mathbf{A}_i|)}}$. The number of the time slots within the maximum uninterrupted lifetime is given by $n_{max} = \lfloor T_i^{max}/\tau \rfloor$. Therefore the occurrence probability of maximum uninterrupted lifetime of sensor i is given by $P_r(T_i = T_i^{max}) = \pi_{|\mathbf{A}_i|} q_{|\mathbf{A}_i||\mathbf{A}_i|}^{\lfloor T_i^{max}/\tau \rfloor}$ where $\pi_{|\mathbf{A}_i|}$ is the probability that the sensor is initially at state $|\mathbf{A}_i|$, and $q_{|\mathbf{A}_i||\mathbf{A}_i|}$ is the transition probability from state $|\mathbf{A}_i|$ to state $|\mathbf{A}_i|$.

3) If $\frac{g_i^{(1)}}{\psi_i + \beta_i} < r_i \le \frac{g_i^{(|\mathbf{A}_i|)}}{\psi_i + \beta_i}$, then ϵ_i may be positive or negative or 0 at a time slot. The energy level of sensor i will potentially reach 0 at some time, or will potentially never reach 0.

Sensor i has a minimum uninterrupted lifetime when the sequence of the Markov states is $\{1, 1, 1, \cdots, 1\}$ until the energy level reaches 0 at the first time. In this case, the energy is decreased linearly at a reduction slope $((\psi_i r_i + \beta_i r_i) - g_i^{(1)})$. Hence the minimum uninterrupted lifetime can be found by $T_i^{min} = \frac{E_i^{ini}}{\psi_i r_i + \beta_i r_i - g_i^{(1)}}$. The number of the time slots within the minimum uninterrupted lifetime is given by $n_{min} = \lfloor T_i^{min}/\tau \rfloor$. The sequence of the Markov states in this case is $\lfloor T_i^{min}/\tau \rfloor$ repeated 1s. Therefore the occurrence probability of minimum uninterrupted lifetime of sensor i is given by $P_r(T_i = T_i^{min}) = \pi_1 q_{11}^{\lfloor T_i^{min}/\tau \rfloor}$.

$\diamondsuit$

4. STEADY-RATE OPTIMIZATION PROBLEM

The dynamic energy harvesting process leads to a dynamic energy replenishment at each sensor. In a health monitoring system, a steady source rate is desired. Therefore, we formulate the *steady-rate optimization problem*, which minimizes the rate fluctuation under the constraint of the uninterrupted service.

4.1. Problem Formulation

We define the *average sustainable rate* b_i of sensor i as the source rate, at which sensor i will consume the same energy as the harvested energy in a long run. Based on the Markov model of energy harvesting, we can find the average recharging rate g_i^{avg} at sensor i. From the definition of the average sustainable rate b_i of sensor i, we have $g_i^{avg} = \psi_i b_i + \beta_i b_i$, from which we can get $b_i = \frac{g_i^{avg}}{\psi_i + \beta_i}$.

The energy level may be different at the beginning of each time slot, and the recharging rate is also time-varying. A sensor needs to adaptively adjust its source rate based on the remaining energy. The adjustment of the source rate at sensor i can be achieved by varying the frequency of data acquisition. A steady rate from each sensor is desirable in a health monitoring system. Therefore, we set an objective to minimize the sum of the squares of the rate fluctuations of all the sensors in a BSN. The objective function at time slot t is mathematically expressed by $f_{obj}^{(t)} = \sum_{i \in \mathbf{N}} (r_i^{(t)} - b_i)^2$. The *steady-rate optimization problem* in a body sensor network at time slot t can be stated as: to minimize the sum of the squares of the rate fluctuations of all the sensors, subject to the requirement of the uninterrupted service. Mathematically, the problem is formulated as follows.

$$
\begin{array}{ll}
\text{minimize}_{(\mathbf{r}^{(t)})} & \sum_{i \in \mathbf{N}} (r_i^{(t)} - b_i)^2 \\
\text{subject to} & P_i^{(t)} = \psi_i r_i + \beta_i r_i, \forall i \in \mathbf{N}, \\
& E_i^{(t+1)} = E_i^{(t)} + \tau \phi_i^{(t)} - \tau P_i^{(t)} - F_i^{(t)}, \forall i \in \mathbf{N}, \\
& E_i^{min} \le E_i^{(t+1)} \le E_i^{max}, \forall i \in \mathbf{N}, \\
& r_i^{(t)} \ge 0, \forall i \in \mathbf{N},
\end{array}
\tag{2}
$$

where the optimization variable $\mathbf{r}^{(t)}$ is the vector of the source rates at time slot t, $E_i^{(t)}$ is the energy of sensor i at the beginning of time slot t, $\phi_i^{(t)}$ is the energy recharging rate of sensor i at time slot t, $F_i^{(t)}$ is the amount of the energy not being collected during time slot t due to battery overflow, E_i^{min} is the minimum energy level required to be maintained at sensor i, and E_i^{max} is the battery capacity of sensor i.

4.2. Optimal Analytical Solution

The optimization problem (2) can be decomposed into $|N|$ independent sub-problems, where $|N|$ is the number of the sensors in the set $\mathbf{N}$. Sub-problem i associated with sensor i is given by

$$
\begin{array}{ll}
\text{minimize}_{(r_i^{(t)})} & (r_i^{(t)} - b_i)^2 \\
\text{subject to} & P_i^{(t)} = \psi_i r_i + \beta_i r_i, \\
& E_i^{(t+1)} = E_i^{(t)} + \tau \phi_i^{(t)} - \tau P_i^{(t)} - F_i^{(t)}, \\
& E_i^{min} \le E_i^{(t+1)} \le E_i^{max}, \\
& r_i^{(t)} \ge 0.
\end{array}
\tag{3}
$$

By solving the sub-problem (3) for any $i \in \mathbf{N}$, respectively, we can get the optimal solution to the problem (2). The sub-problem (3) actually aims to find the source rate $r_i^{(t)}$ closest to the average sustainable rate b_i based on the energy $E_i^{(t)}$ at the beginning of time slot t and the recharging rate $\phi_i^{(t)}$ during time slot t. The optimal source rate $r_i^{(t)^*}$ for the sub-problem (3) can be found by an *analytical solution* given as follows.

- **Case 1**: if $\phi_i^{(t)} < g_i^{avg}$,
  - *Case 1.1*: if $E_i^{(t)} - E_i^{min} \ge \tau(g_i^{avg} - \phi_i^{(t)})$, the optimal source rate $r_i^{(t)^*} = b_i$, the objective function $f_{sub}^{(i)}$ for the sub-problem (3) is $f_{sub}^{(i)} = (b_i -$

$b_i)^2 = 0$, and the energy at the beginning of time slot $(t+1)$ will be $E_i^{(t+1)} = E_i^{(t)} + \tau(\phi_i^{(t)} - g_i^{avg})$.

  - *Case 1.2*: if $E_i^{(t)} - E_i^{min} < \tau(g_i^{avg} - \phi_i^{(t)})$, the optimal source rate ${r_i^{(t)}}^* = \frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i + \beta_i)}$, the objective function $f_{sub}^{(i)}$ for the sub-problem (3) is $f_{sub}^{(i)} = (b_i - \frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i + \beta_i)})^2$, and the energy at the beginning of time slot $(t+1)$ will be $E_i^{(t+1)} = E_i^{min}$.

- **Case 2**: if $\phi_i^{(t)} \geq g_i^{avg}$,
  - *Case 2.1*: if $E_i^{max} - E_i^{(t)} \geq \tau(\phi_i^{(t)} - g_i^{avg})$, the optimal source rate ${r_i^{(t)}}^* = b_i$, the objective function $f_{sub}^{(i)}$ for the sub-problem (3) is $f_{sub}^{(i)} = 0$, and the energy at the beginning of time slot $(t+1)$ will be $E_i^{(t+1)} = E_i^{(t)} + \tau(\phi_i^{(t)} - g_i^{avg})$.
  - *Case 2.2*: if $E_i^{max} - E_i^{(t)} < \tau(\phi_i^{(t)} - g_i^{avg})$, the optimal source rate ${r_i^{(t)}}^* = b_i$, the objective function $f_{sub}^{(i)}$ for the sub-problem (3) is $f_{sub}^{(i)} = 0$, and the energy at the beginning of time slot $(t+1)$ will be $E_i^{(t+1)} = E_i^{max}$. In this case, the amount of wasted energy due to battery overflow is $F_i^{(t)} = \tau(\phi_i^{(t)} - g_i^{avg}) - (E_i^{max} - E_i^{(t)})$.

Theorem 2: The proposed *analytical solution* is the optimal solution to the optimization problem (3).

Proof:

The objective function in the optimization problem (3) is $f_{sub}^{(i)} = (b_i - r_i^{(t)})^2 \geq 0$. In cases 1.1, 2.1, and 2.2 of the proposed analytical solution, the obtained objective values are all 0. If the source rates obtained from cases 1.1, 2.1, and 2.2 of the analytical solution satisfy the constraints of the optimization problem (3), they will be the optimal solution to the optimization problem (3).

In *Case 1.1* where $\phi_i^{(t)} < g_i^{avg}$ and $E_i^{(t)} - E_i^{min} \geq \tau(g_i^{avg} - \phi_i^{(t)})$, the source rate obtained from the analytical solution is ${r_i^{(t)}}^* = b_i$. The energy consumption during time slot t at sensor i is $E_{con,i}^{(t)} = \tau(\psi_i + \beta_i)b_i$. The energy replenishment during time slot t at sensor i is $E_{rep,i}^{(t)} = \tau\phi_i^{(t)}$.Then the energy increment during time slot t at sensor i is given by $E_{inc,i}^{(t)} = E_{rep,i}^{(t)} - E_{con,i}^{(t)}$. We have $b_i = g_i^{avg}/(\psi_i + \beta_i)$. Therefore $E_{inc,i}^{(t)} = E_{rep,i}^{(t)} - E_{con,i}^{(t)} = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)b_i = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)(g_i^{avg}/(\psi_i + \beta_i)) = \tau(\phi_i^{(t)} - g_i^{avg}) < 0$. Therefore the amount of wasted energy due to battery overflow is zero, which means $F_i^{(t)} = 0$. The energy at the beginning of time slot $(t+1)$ is given by $E_i^{(t+1)} = E_i^{(t)} + \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)b_i - F_i^{(t)} = E_i^{(t)} + \tau(\phi_i^{(t)} - g_i^{avg}) < E_i^{(t)} \leq E_i^{max}$. Since $E_i^{(t)} - E_i^{min} \geq \tau(g_i^{avg} - \phi_i^{(t)})$, we have $E_i^{(t+1)} = E_i^{(t)} + \tau(\phi_i^{(t)} - g_i^{avg}) \geq E_i^{min}$. Therefore, $E_i^{min} \leq E_i^{(t+1)} \leq E_i^{max}$. The source rate b_i obtained from the analytical solution satisfies the constraints of the optimization problem (3), and it achieves the minimum objective value of 0. Therefore it is the optimal solution to the optimization problem (3).

In *Case 2.1* where $\phi_i^{(t)} \geq g_i^{avg}$ and $E_i^{max} - E_i^{(t)} \geq \tau(\phi_i^{(t)} - g_i^{avg})$, the source rate obtained from the analytical solution is ${r_i^{(t)}}^* = b_i$. The energy increment during time slot t at sensor i is given by $E_{inc,i}^{(t)} = E_{rep,i}^{(t)} - E_{con,i}^{(t)} = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)b_i = \tau(\phi_i^{(t)} - g_i^{avg}) \geq 0$. Since $E_i^{max} - E_i^{(t)} \geq \tau(\phi_i^{(t)} - g_i^{avg})$, the energy level won't reach the battery capacity at the end of time slot t. Therefore $F_i^{(t)} = 0$. The energy at the beginning of time slot $(t+1)$ is given by $E_i^{(t+1)} = E_i^{(t)} + \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)b_i - F_i^{(t)} = E_i^{(t)} + \tau(\phi_i^{(t)} - g_i^{avg}) \geq E_i^{(t)} \geq E_i^{min}$. From $E_i^{max} - E_i^{(t)} \geq \tau(\phi_i^{(t)} - g_i^{avg})$, we have $E_i^{(t+1)} = E_i^{(t)} + \tau(\phi_i^{(t)} - g_i^{avg}) \leq E_i^{max}$. Therefore, $E_i^{min} \leq E_i^{(t+1)} \leq E_i^{max}$. The source rate b_i obtained from the analytical solution satisfies the constraints of the optimization problem (3), and it achieves the minimum objective value of 0. Therefore it is the optimal solution to the optimization problem (3).

In *Case 2.2* where $\phi_i^{(t)} \geq g_i^{avg}$ and $E_i^{max} - E_i^{(t)} < \tau(\phi_i^{(t)} - g_i^{avg})$, the source rate obtained from the analytical solution is ${r_i^{(t)}}^* = b_i$. The energy increment during time slot t at sensor i is given by $E_{inc,i}^{(t)} = E_{rep,i}^{(t)} - E_{con,i}^{(t)} = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)b_i = \tau(\phi_i^{(t)} - g_i^{avg}) \geq 0$. Since $E_i^{max} - E_i^{(t)} < \tau(\phi_i^{(t)} - g_i^{avg})$, the energy level will exceed the battery capacity at the end of time slot t, and the wasted energy due to battery overflow is given by $F_i^{(t)} = E_i^{(t)} + \tau(\phi_i^{(t)} - g_i^{avg}) - E_i^{max}$. The energy at the beginning of time slot $(t+1)$ will be $E_i^{(t+1)} = E_i^{max} > E_i^{min}$. The source rate b_i obtained from the analytical solution satisfies the constraints of the optimization problem (3), and it achieves the minimum objective value of 0. Therefore it is the optimal solution to the optimization problem (3).

In *Case 1.2* where $\phi_i^{(t)} < g_i^{avg}$ and $E_i^{(t)} - E_i^{min} < \tau(g_i^{avg} - \phi_i^{(t)})$, the source rate obtained from the analytical solution is ${r_i^{(t)}}^* = \frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i + \beta_i)}$. We first verify if ${r_i^{(t)}}^*$ is a feasible point of the optimization problem (3). The energy increment during time slot t at sensor i is given by $E_{inc,i}^{(t)} = E_{rep,i}^{(t)} - E_{con,i}^{(t)} = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i){r_i^{(t)}}^* = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)\frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i + \beta_i)} = -(E_i^{(t)} - E_i^{min}) \leq 0$. Hence $F_i^{(t)} = 0$. The energy at the beginning of time slot $(t+1)$ is given by $E_i^{(t+1)} = E_i^{(t)} + E_{inc,i}^{(t)} = E_i^{(t)} - (E_i^{(t)} - E_i^{min}) = E_i^{min} < E_i^{max}$. The source rate ${r_i^{(t)}}^* = \frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i + \beta_i)}$ obtained from the analytical solution satisfies the constraints of the optimization problem (3), therefore it is a feasible point of the optimization problem (3). We next prove that the objective function achieves the minimum value at the feasible point ${r_i^{(t)}}^* = \frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i+\beta_i)}$. Since $0 \le E_i^{(t)} - E_i^{min} < \tau(g_i^{avg} - \phi_i^{(t)})$, we have $b_i - {r_i^{(t)}}^* = \frac{g_i^{avg}}{\psi_i+\beta_i} - \frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i+\beta_i)} = \frac{\tau(g_i^{avg} - \phi_i^{(t)}) - (E_i^{(t)} - E_i^{min})}{\tau(\psi_i+\beta_i)} > \frac{\tau(g_i^{avg} - \phi_i^{(t)}) - \tau(g_i^{avg} - \phi_i^{(t)})}{\tau(\psi_i+\beta_i)} = 0$. We assume that there exists another feasible point ${r_i^{(t)}}' \neq {r_i^{(t)}}^*$ such that the objective value at ${r_i^{(t)}}'$ is smaller than that at ${r_i^{(t)}}^*$. In other words, $({r_i^{(t)}}' - b_i)^2 < ({r_i^{(t)}}^* - b_i)^2$, from which we get ${r_i^{(t)}}^* < {r_i^{(t)}}' < b_i$ or $b_i < {r_i^{(t)}}' < 2b_i - {r_i^{(t)}}^*$. We examine the two cases as follows, respectively.

1. When ${r_i^{(t)}}^* < {r_i^{(t)}}' < b_i$, the energy increment during time slot t at sensor i is given by $E_{inc,i}^{(t)} = E_{rep,i}^{(t)} - E_{con,i}^{(t)} = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i){r_i^{(t)}}' < \tau\phi_i^{(t)} - \tau(\psi_i+\beta_i){r_i^{(t)}}^* = \tau\phi_i^{(t)} - \tau(\psi_i+\beta_i)\frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i+\beta_i)} = -(E_i^{(t)} - E_i^{min})$. The energy at the beginning of time slot $(t+1)$ is given by $E_i^{(t+1)} = E_i^{(t)} + E_{inc,i}^{(t)} < E_i^{(t)} - (E_i^{(t)} - E_i^{min}) = E_i^{min}$.

2. When $b_i < {r_i^{(t)}}' < 2b_i - {r_i^{(t)}}^*$, the energy increment during time slot t at sensor i is given by $E_{inc,i}^{(t)} = E_{rep,i}^{(t)} - E_{con,i}^{(t)} = \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i){r_i^{(t)}}' < \tau\phi_i^{(t)} - \tau(\psi_i + \beta_i)b_i = \tau(\phi_i^{(t)} - g_i^{avg}) < -(E_i^{(t)} - E_i^{min})$. The energy at the beginning of time slot $(t+1)$ is given by $E_i^{(t+1)} = E_i^{(t)} + E_{inc,i}^{(t)} < E_i^{(t)} - (E_i^{(t)} - E_i^{min}) = E_i^{min}$.

We have $E_i^{(t+1)} < E_i^{min}$ when ${r_i^{(t)}}^* < {r_i^{(t)}}' < b_i$ or $b_i < {r_i^{(t)}}' < 2b_i - {r_i^{(t)}}^*$. Therefore ${r_i^{(t)}}'$ is not a feasible point of the optimization problem (3). Therefore, we conclude that ${r_i^{(t)}}^* = \frac{E_i^{(t)} - E_i^{min} + \tau\phi_i^{(t)}}{\tau(\psi_i+\beta_i)}$ is the optimal point in *Case 1.2* of the optimization problem (3).

In summary, the source rate obtained from the proposed analytical solution are optimal for the optimization problem (3).

$\diamondsuit$

Since the *steady-rate optimization problem* (2) can be decomposed into $|N|$ independent sub-problems, each of which can be solved with the proposed analytical solution, the vector of the source rates $\{{r_i^{(t)}}^* | \forall i \in \mathbf{N}\}$ is the optimal solution to the steady-rate optimization problem (2).

5. SIMULATIONS

The typical body sensors in health monitoring applications are 1) body temperature sensor, 2) pulse oxygen sensor, 3) blood pressure sensor, 4) electrocardiography (ECG) sensor, and 5) electroencephalography (EEG) sensor [8]. Therefore we deploy the 5 sensors in a BSN in the simulations. The distance from a sensor to the aggregator is uniformly distributed between 0.3 m and 0.7 m [9]. In the power consumption model for sensor i, we set $\psi_i = 2 \times 10^{-8} J/b$, $\theta_i = 4 \times 10^{-8} J/b$, $\zeta_i = 1.3 \times 10^{-8} J/b/m^{2.4}$. The path loss exponent is set to $m_p = 2.4$ based on the measurement results in [9]. The initial energy of each sensor is set to 0.1 J. Each sensor has a battery capacity 0.11 J. The minimum energy required for each sensor is 0.01 J. The energy harvesting process at a sensor is modeled by a two-state Markov chain with state 1 and state 2 [2]. The transition probability from state 1 to state 2 is uniformly distributed between 0.6 and 0.8, and the transition probability from state 2 to state 1 is uniformly distributed between 0.2 and 0.4. We set the length of the time slot to 5 s.

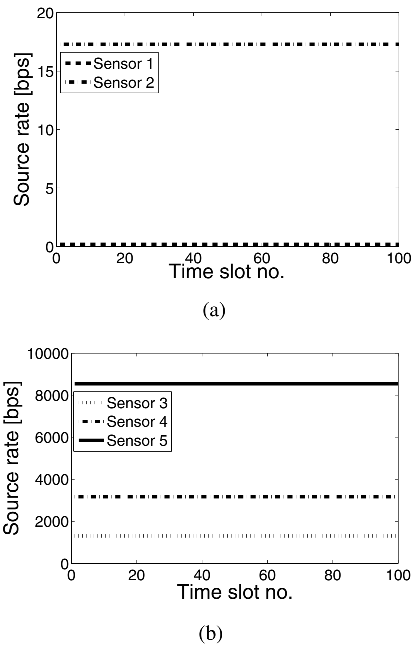

Fig. 1. Variations of the source rates during 100 time slots: (a) at sensors 1-2, and (b) at sensors 3-5

The variations of the source rates of 5 body sensors in a BSN during 100 time slots is shown in Fig. 1. Each sensor minimizes the rate fluctuation with respective to the average sustainable rate, subject to the requirement of uninterrupted service. As shown in Fig. 1, each sensor maintains a constant source rate, equal to the average sustainable rate, during the 100 time slots, which indicates a steady data transmission. The source rates are heterogeneous among sensors. Sensor 1 (body temperature sensor) and sensor 2 (pulse oxygen sensor) have a much smaller source rate, shown in Fig. 1(a), compared to sensor 3 (blood pressure sensor), sensor 4 (ECG sensor), and sensor 5 (EEG sensor), shown in Fig.1(b).

The energy level of each sensor varies over time, as shown

Source rate [bps]	847	969	1090	1211	1332	1453	1574	1695
Minimum uninterrupted lifetime [s]	∞	∞	13228	6614	4409	3307	2646	2205
Maximum uninterrupted lifetime [s]	∞	∞	∞	∞	∞	∞	13228	6614

Table 1. Relationship between the source rate and the lifetime of sensor 3

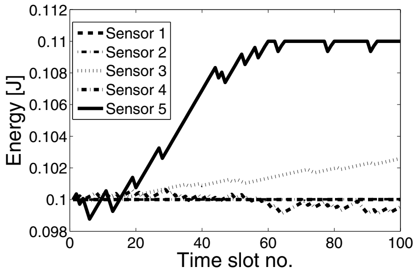

Fig. 2. Energy variation during 100 time slots at each sensor

in Fig. 2, because the energy harvesting is a random process. When the harvested energy is larger than the consumed energy, the energy level of the sensor is increased. When the harvested energy is smaller than the consumed energy, it is decreased. The variation of the energy level is limited to the range between the battery capacity 0.11 J and the minimum energy 0.01 J. During time slots 60-62, 65-77, 80-90, and 93-100, the battery of sensor 5 is overflowed, which causes the wasted energy.

There is a tradeoff between the source rate and the lifetime of a sensor, as shown in Table 1. The uninterrupted lifetime of a sensor is the duration from the starting time of the sensor to the time when the energy level of the sensor reaches 0 at the first time. Table 1 shows that the expected uninterrupted lifetime of a sensor will be reduced as the source rate is increased. As shown in Table 1, when the source rate of sensor 3 is 847 bps, the uninterrupted lifetime of the sensor will be infinity. When the source rate of sensor 3 is increased to 1695 bps, the uninterrupted lifetime of the sensor will be between 2205 s and 6614 s.

6. CONCLUSION

Energy harvesting is an promising approach to provide sustainable power supply for BSNs. In this paper, we model the energy harvesting process at each sensor as a discrete-time Markov chain, and then theoretically analyze the relationship between the source rate and the uninterrupted lifetime of the sensor. A steady source rate is desired for health monitoring applications. Therefore we formulate the steady-rate optimization problem, which minimizes the rate fluctuation with respect to the average sustainable rate, subject to the requirement of the uninterrupted service. The optimization problem is solved analytically. The simulation results demonstrate the steady output rate of the sensor and confirm the theoretical analysis of the rate-lifetime relationship.

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