GRADIENT PROJECTION BASED QOS DRIVEN CROSS-LAYER SCHEDULING FOR VIDEO APPLICATIONS

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ABSTRACT

We proposed a novel cross-layer optimization approach for video streaming applications in orthogonal frequency division multiplexing (OFDM) based networks. Parameters in media access control (MAC) layer and physical layer are optimized jointly in a cross-layer framework. The main objective is to maximize video quality by reducing the end-to-end distortion in application layer for each user. Potential games theory is adopted to solve this cross-layer problem in distributed way. Convergence of game is guaranteed by gradient projection, which reduces the time complexity of optimization considerably. Simulation results have confirmed that the gradient projection based cross-layer optimization can significantly enhance the video quality performance with low time complexity and high scalability.

Index Terms-cross-layer, QoS, gradient projection, potential games, video streaming

1. INTRODUCTION

With the growing demand for high data rate communication, OFDM has become one of the most appealing physical layer techniques for next generation wireless communication systems due to its high spectrum efficiency and robustness to multipath fading. However, wireless channel has two nature characteristics such as time-varying and resource-limited. It is necessary to introduce smart resource allocation strategies in order to achieve better spectral efficiency in an OFDM network. A feasible method is to deal with traffics experiencing a best channel condition at any given time [1-3].

As the development of processing capacity of mobile terminals and increasing requirements of users, wireless video streaming applications such as visual telephone, video games, video conference and video on demand become more and more popular in nowadays. However, as a type of applications with high data rate and strong content relevance, video streaming applications are restricted by wireless channels. As a result of time-varying and resource-limited characteristics of wireless channels differing from wired ones, parameters of different layers, from application layer to physical layer, may be jointly adjusted to enhance the network performance, which is called as cross-layer design. Cross-layer design refers to protocol design done by actively exploiting the dependence between protocol layers to obtain performance gains [4]. For example, when a wireless channel is experiencing poor quality, optimizer can adjust the modulation and coding scheme, schedule the proper user to minimize the packet loss rate and enhance perceived video quality of users over wireless networks. Therefore, cross-layer optimization is an effective solution to improve the OFDM network performance.

Many proposed cross-layer scheduling and resource allocation methods exploit the time-varying nature of the wireless channel to maximize the throughput of the network while maintaining fairness across multiple users [5-7]. However, throughput is just a MAC layer indicator, which can not reflect the video quality for resource allocation. Thus we propose a video content distortion estimation, which can measure the performance in video quality.

A wealth of work exists on video streaming over wireless networks. One area, which has received significant attention, is that the source content and channel model are jointly considered to determine the optimal source encoding modes [8-12]. However, we focus on downlink video streaming where the media server is at a different location from the base station, and the video encoding cannot be adapted to changes in the channel. Therefore, we assume the video is pre-encoded and packetized at the server. Packet scheduling for the streaming of pre-encoded video is also a well-studied topic [13-15]. Paper [13] proposed a gradient based scheduling scheme in which user data rates are dynamically adjusted based on channel quality as well as the gradients of a utility function. Paper [14] presented a cross-layer algorithm, including the application layer, MAC layer and physical layer. A compressed method is proposed to estimate video quality contribution caused by each packet in application layer. However, scheduling for video streaming over wireless networks to multiple clients has conventionally focused on allocating wireless resource in a centralized way. Our main contribution is to propose a distortion aware scheduling scheme for packet-based video transmission over OFDM based long term evolution (LTE) networks. Parameters from physical layer and MAC layer are jointly adjusted by the cross-layer optimizer under an objective function of application layer. Cross-layer optimization problem is formulated as a potential game through element mapping. Therefore, the existence and uniqueness of Nash Equilibrium is guaranteed. We propose the gradient projection decision rule to select the optimal strategy for optimal video quality. Convergence of game can be achieved by this gradient projection. As a result, time complexity of algorithm is reduced considerably, as well as the scalability of cross-layer design is enhanced.

The rest of this paper is organized as follows: we propose the cross-layer architecture in section 2. Section 3 describes

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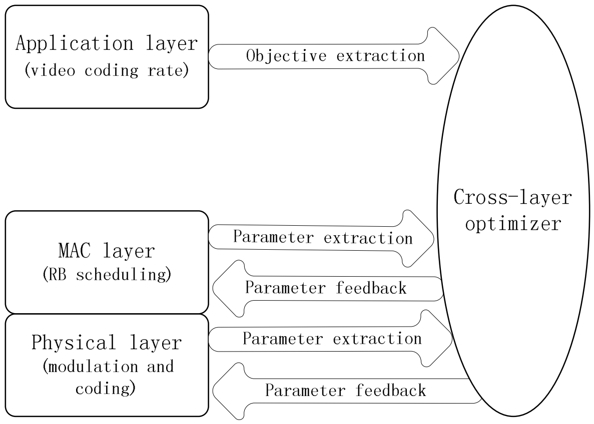

Figure 1. Framework of cross-layer design

the distortion based video quality model and defines the objective function of application layer. In section 4 physical layer parameters are optimized through link adaptation. We propose a dynamic scheduling scheme based on gradient projection in MAC layer in section 5. In section 6 we simulate the cross-layer algorithm in performance and price respectively. Conclusions are made in section 7.

2. CROSS-LAYER ARCHITECTURE

The proposed cross-layer architecture consists of cross-layer optimizer (CLO), parameter abstraction and parameter feedback as demonstrated in Figure 1. The CLO is the center of the model, selecting the proper parameter of each OSI layer to make the utility function optimal, such as the scheduling user as well as the modulation and channel coding parameters.

Our cross-layer architecture is composed of physical layer, MAC layer and application layer, as visualized in Fig. 1. The CLO jointly optimizes multiple networks layers, making predictions on their states and selecting proper values for their parameters. These steps of cross-layer optimization are repeated every TTI, which is 1ms specified in OFDM based LTE networks by 3GPP [20].

The joint cross-layer strategy S can be defined as:

$$S=\{PHY_1,\cdots,PHY_N,MAC_1,\cdots,MAC_K\}$$

Where $PHY_i, i=(1,2,\cdots,N)$ represents the strategies in physical layer, $MAC_i, i=(1,2,\cdots,K)$ represents the strategies in MAC layer. The cross-layer optimizer attempts to find the optimal strategy profile to make the overall video quality optimal. In our proposed cross-layer scheme, fairness can be guaranteed under the constraint of latency.

There are many solution orders for cross-layer adaptation and optimization. For example, top-down approach, bottom-up approach, integrated approach and so on. In this paper, bottom-up approach is adopted in order to reduce the complexity of algorithm.

3 QoS DRIVEN CROSS-LAYER OPTIMIZATION

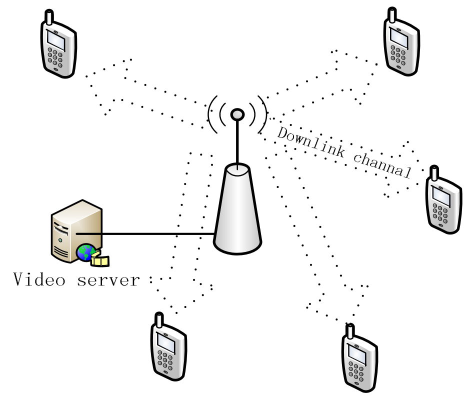

Figure 2. Downlink video streaming application scenario

3.1. System modeling

In this paper, as shown in figure 2, a downlink video streaming application scenario in LTE is considered, video server transmits requested H.264 video streaming to each user. The process of cross-layer optimization is mapped to a potential game. Interplay between users can be considered adequately, as well as the Nash equilibrium (NE) of optimization is guaranteed.

In H.264 video streaming, a slice could either be as small as a few macro blocks (MBs), or as large as an entire video frame. Each slice header acts as a resynchronization marker, which allows the slices to be independently decodable and to be transported out of order. When a video stream is requested by a client, packets of this video will be transmitted through optical fiber to the CLO at an eNB with multiple clients. We assume that the optical fiber is lossless with enough bandwidth. For simplicity, we assume that all users are video users with same requirements of QoS level. The overall objective function of CLO for scheduling is:

$$\min\sum_{k}^{M}\sum_{i=1}^{N}\xi_i^k D_k, \text{ subject to}$$

$$t_w^k+t_c^k<t^k, k=1,\cdots,M$$

Where D_k denotes the frame distortion caused by lost slice of user k. ξ_i^k denotes the scheduling result for resource block (RB) i, if user k is scheduled in RB k, ξ_i^k is equal to 1. M and N is the number of users and resource blocks respectively. The constraint indicates the latency requirements for playback of video streaming. t_w^k, t_c^k and t^k are the waiting time, the transmitting time and the latency bound of the kth user. If the sum of waiting time and transmitting time is bigger than the latency bound, transport block (TB) will be discarded.

Once the expected distortion for slice is determined, the task of the cross-layer optimizer simply becomes choosing the best strategy profile with respect to the desired objective function.

3.2. Distortion based video quality

We assume the whole slice is lost if one TB of this slice is lost. The cross-layer optimizer selects the optimal parameter values that maximize the expected user-perceived video quality. Let D be the overall end-to-end frame distortion, defined as the mean square error (MSE) between the received video frame and the one transmitted by eNB. The video distortion is caused by block error rate (BLER) and dependency of video content under the constraint of transmitting delay, while the BLER is a function of the modulation and coding scheme of RB, as well as SINR of wireless channel.

At the decoder side, we employ error concealment scheme, if a slice is lost, all MBs of this slice are supposed to be lost. In this case, the decoder simply copies the MBs at the same location from the previous decoded frame. With this simple and efficient error concealment method, we develop a statistical analysis of the video distortion. For a MB in a slice, in case of no channel errors, its reconstruction value is $\hat{F}(n,j)$. If the slice is lost, the reconstruction value of MB j is $\tilde{F}(n-1,j)$, which is copied from the previous decoded frame. Therefore, the expected frame distortion is

$$\begin{aligned}
D(n) &= E\left(\left[\hat{F}(n,j)-\tilde{F}(n,j)\right]^2\right) \\
&= bler\cdot E\left(\left[\hat{F}(n,j)-\tilde{F}(n-1,j)\right]^2\right) \\
&= bler\cdot E\left(\left[\hat{F}(n,j)-\hat{F}(n-1,j)+\hat{F}(n-1,j)-\tilde{F}(n-1,j)\right]^2\right) \\
&= bler\cdot E\left(\left[\hat{F}(n,j)-\hat{F}(n-1,j)\right]^2\right) \\
&+ bler\cdot E\left(\left[\hat{F}(n-1,j)-\tilde{F}(n-1,j)\right]^2\right) \\
&= bler\cdot RFD(n,n-1)+bler\cdot D(n-1)
\end{aligned}$$

Where $RFD(n,n-1)$ represents the MSE between the reconstructed frame n and frame $n-1$. $bler$ denotes the BLER of corresponding RB. Note that I slice and P slice lead to different distortion for prediction.

4. LINK ADAPTATION

In wireless networks, fading, interference and noise can greatly impact the link capacity, and in turn decreases the video quality in transmitting. To achieve a robust link performance, the most well-known technique is link adaptation through adaptive modulation and coding (AMC). The objective of AMC is to maximize the link level data rate by adjusting transmission parameters to the available channel condition. AMC can effectively decrease the block error rate while satisfying the constraint of latency. In this paper, we adopt the following approximated bit error rate (BER) expression [12]

$$p_m^e(\gamma)=\frac{a_m}{e^{\gamma\cdot b_m}}$$

Table 1. Parameters of different modes

Mode index	m=1	m=2	m=3	m=4	m=5	m=6
Modulation	BPSK	QPSK	QPSK	16-QAM	16-QAM	64-QAM
Coding rate	1/2	1/2	3/4	9/16	3/4	3/4
R_m	0.50	1.00	1.50	2.25	3.00	4.50
a_m	1.1369	0.3351	0.2197	0.2081	0.1936	0.1887
b_m	7.5556	3.2543	1.5244	0.6250	0.3484	0.0871

Where m is the mode index of modulation and coding schemes, R_m is the bit number per symbol, and γ is the received SINR. a_m and b_m are obtained from the table 1.

Therefore, through link adaption technology we can select proper modulation and coding scheme in physical layer in terms of SINR of wireless channel. Optimal BLER of RBs can be obtained, which is used as the reference of user scheduling.

5. DYNAMIC SCHEDULING BASED ON GRADIENT PROJECTION

5.1. Definition of exact potential game

Let $\Gamma=\langle N,Y,u\rangle$ be a game in strategic form with a finite number of players, and the number of players is N. The set of strategies of player i is Y_i, and the payoff function of player i is u_i: $Y\rightarrow R$, where $Y=Y_1\times Y_2\times\cdots\times Y_N$ is the set of strategy profiles, and R denotes the set of real numbers. Then we define the exact potential function $P:Y\rightarrow R$, if for $i\in N$ and $y_{-i}\in Y_{-i}$

$$u_i(y_{-i},x)-u_i(y_{-i},z)=P(y_{-i},x)-P(y_{-i},z),\quad \forall x,z\in Y_i$$

Where Y_{-i} denotes the strategy profiles except strategies of player i. Γ is called a exact potential game if it admits an exact potential. Here the potential function can reflect the change in the utility accrued by every unilaterally deviating player. It is easy to show that the necessary and sufficient condition for an exact potential game is

$$\frac{\partial u_i(y)}{\partial y_i}=\frac{\partial P(y)}{\partial y_i},\quad \forall i\in N, y\in Y$$

Next, as the relationship between elements in a game and those in an OFDM based network can be expressed in table 2, we can map a cross-layer optimization problem to a game. As depicted in table 2, RBs are mapped to players in a game, which is a decision maker in the interactive decision process. RBs can jointly schedule the user to improve the overall

Table 2. The relationship between elements in games and optimization

Game	⟺	Cross-layer optimization process
Player	⟺	RBs in cell
Strategy	⟺	User scheduling
Payoff	⟺	Distortion with time delay constraint
Preference	⟺	Minimum of utility function

video quality. The utility function of each RB i can be expressed as

$$u_i = \sum_{k=1}^{M} \xi_i^k D_k$$

Obviously, this cross-layer game is a self-motivated potential game, and the potential function

$$P = \sum_{k}^{M} \sum_{i=1}^{N} \xi_i^k D_k$$

Self-motivated potential game Γ possesses a pure strategy Nash Equilibrium [19]. As the strategy space is convex and potential function P is continuously differentiable on the strategy space, then the NE of Γ is a stationary point of P. As P is convex, then every NE of Γ is a minimum point of potential function P.

5.2. Convergence of distributed dynamic process

After deriving conditions for a potential game to have at least one NE, we need to design the rules that each player must follow to reach equilibrium. To this end we assume that the same game could be myopically played repeatedly. In each iteration, every player has neither memory of past iteration nor speculation of future events, but it chooses its own strategy according to some decision rules that depend on the current state of the game. A new iteration of the same game is then played, until a NE of the game is reached. The main role of these rules is to help players make proper decision that guarantee the asymptotically stability of NE of the game. We call these rules as stable decision rules and denote the set of stable rules for the ith player by $D_i(x)$.

1) Best response:
$$D_i(x) = \{x_i^* \in X_i(x_{-i}) : x_i^* = \arg\max_{x_i} u_i(x_i, x_{-i})\}$$

2) Better response:
$$D_i(x) = \{x_i^* \in X_i(x_{-i}) : u_i(x_i^*, x_{-i}) > u_i(x_i, x_{-i})\}$$

3) Gradient projection response:
$$D_i(x) = [x_i + \alpha_i \nabla_i u_i(x_i, x_{-i})]_{X_i(x_{-i})}$$

Where $\alpha_i \in \left[\varepsilon_i, \frac{2}{K_i} - \varepsilon_i\right], 0 < \varepsilon_i < \frac{2}{K_i}$ is the step size of iteration, and $K_i > 0$ is a Lipschitz constant of marginal payoff. $[c]_{X_i(x_{-i})}$ denotes the Euclidean projection of c on the set $X_i(x_{-i})$.

$$[c]_{X_i(x_{-i})} = \arg\min_{x_i \in X_i} \|c - x_i\|_2^2$$

In this paper, gradient projection response is used as the stable decision rules. Given the stable decision rules, two different approaches can be followed in the choice of the iterative approaches to be performed by the players, namely Gauss-Seidel and Jacobi based schemes. We choose the Jacobi iterative approach in this paper. Players may update their own strategies in a parallel way.

$$x_i^{t+1} = D_i(x_1^t, \cdots, x_{i-1}^t, x_i^t, x_{i+1}^t, \cdots, x_N^t)$$

6. SIMULATIONS AND ANALYSIS

In the simulation, we use the H.264/AVC video streaming with different coding rate. We assume the whole slice is lost if one of the TB constituting the slice is lost. This assumption is reasonable since usually the slice can be independently decodable [18]. When a slice is lost during transmission, we use the temporal replacement (TR) error concealment strategy. In the simulation, simulator randomly generates network topologies consisting of 19 eNBs. Each cell contains five users with the requirements of video streaming services. We adopt the 6 paths Rayleigh model to calculate the frequency selective fading. The expected video quality at the receiver is measured by the average of the peak signal-to-noise ratio (PSNR). We compare the PSNRs, under the same network conditions, of the reconstructed video sequences achieved from the proposed gradient projection based QoS driven cross-layer scheduling (GQCS) with those achieved from PF, MAX C/I and GBS method proposed in [12].

Five video sequences with varied content: “foreman”, “coastguard”, “mother and daughter”, “news” and “mobile” in QCIF (176x144) format are used for the simulations. The wireless network is modeled as an OFDM based LTE system. The system parameters used in the simulations are shown in table 3.

6.1. Performance analysis

Since PSNR is most commonly used as a measure of quality of reconstruction of lossy compression codec, we adopt PSNR to illustrate simulation results.

$$PSNR = 10\log_{10}\left(255^2 / MSE\right)$$

Table 3. Simulation parameters

Parameter		Value
Inter-site distance(m)		1000
Centre Frequency		2 GHz
BS transmit power		46dBm
Lognormal Shadowing		Standard deviation: 10 dB
MS Antenna Gain	9dB	0.5
	0 dBi	1
Path-Loss		128.1+37.6log10(R), R in km
Thermal Noise Density		-174 dBm/Hz
BS Antenna pattern(horizontal) (For 3-sector cell sites with fixed antenna patterns)		$A(\theta) = -\min\left[12\left(\frac{\theta}{\theta_{3dB}}\right)^2, A_m\right]$ θ_{3dB} = 70 degrees, Am = 20 dB
L2S interface		EESM

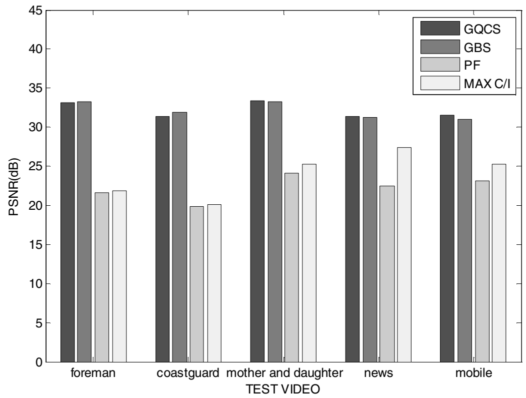

Figure 3. Average PSNR of video streaming

Fig. 3 shows the average quality of the received five video streams requested by different users. Each video sequence is encoded in a bit rate of 300kbps. Fig. 4 shows the average quality at each frame of "foreman". It is obvious that our proposed cross-layer scheme performs analogously with the GBS method.

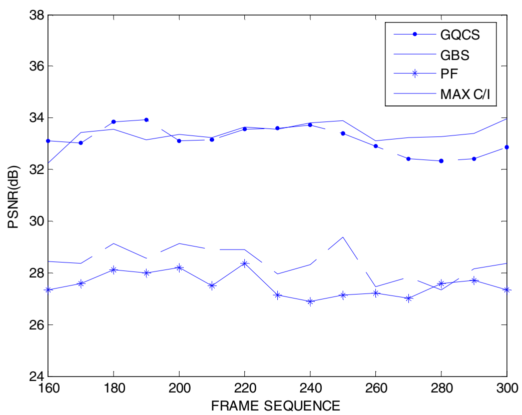

Figure 4. Frame PSNR of coast

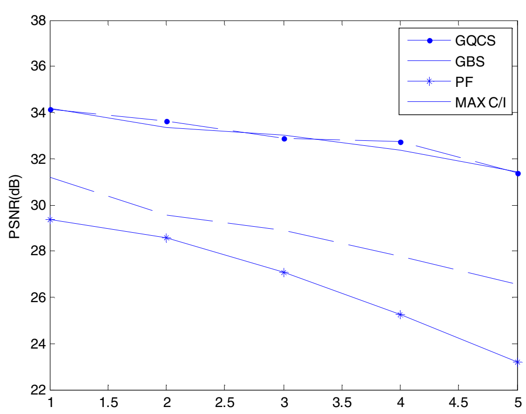

Figure 5. PSNR with different bit rates

Fig. 5 shows the average PSNR of video streams with different bit rates. We can see that the average PSNR of our proposed gradient projection based cross-layer algorithm is similar with the GBS scheme.

6.2. Price analysis

Taking advantage of potential game theory, system overall optimization can be achieved through iterative optimization of players. However, the theoretical optimal solution usually cannot be achieved by NE. Therefore, we need to define an indicator to measure the inefficiency of equilibrium. E. Koutsoupias proposed a framework to systematically study this issue [21]. The framework presupposes a strategic environment, a definition for the outcome of games, and a real-valued, nonnegative objective function defined on the possible outcomes of the game. The price of anarchy is then defined as the ratio between the objective function value of equilibrium and that of an optimal solution.

$$POA = \frac{T_{obj}(S^e)}{T_{obj}(S^*)}$$

Where S^e denotes the strategy profile under equilibrium, and S^* denotes the optimal strategy selection.

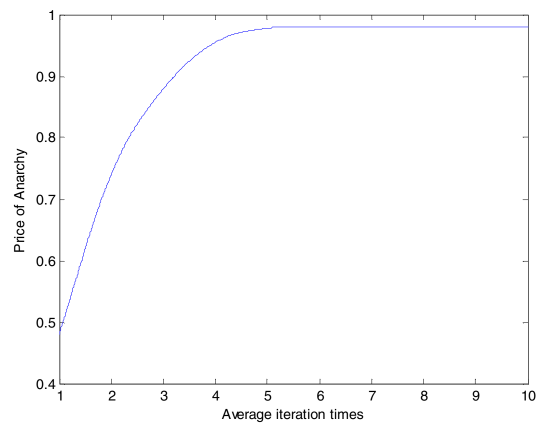

Figure 6. Price of anarchy of gradient projection

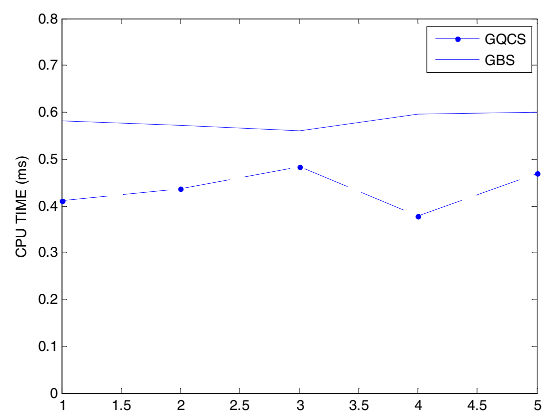

Figure 7. Time complexity comparison

If the price of anarchy of a game is 1, then its equilibrium is fully efficient. Ideally, as game Γ is an exact potential game, NE is the optimal point of the objective function. Therefore, the solution of our gradient projection is an optimal solution. However, considering the calculation accuracy and latency constraint, the price of anarchy of this cross-layer game is

$$POA = \frac{T_{obj}(S^e)}{T_{obj}(S^*)} \approx 1$$

Fig. 7 shows the comparison of the time complexity of the cross-layer method and GBS method for one eNB by average CPU time. The simulation of the two different methods are compiled in visual studio 2008 platform, and is run on a Intel Core 2 2.53 GHz CPU based personal computer running Windows XP professional. In Fig. 7, obviously, the average computation cost of cross-layer scheduling is less than GBS scheme, while the GQCS scheme brings analogous performance with GBS method.

7 CONCLUSIONS

In this paper, we propose a gradient projection based cross-layer optimized framework for multimedia streaming over OFDM based networks. In the proposed system, the end-to-end distortion at the application layer, user scheduling at the MAC layer, and the adaptive modulation and coding at the physical layer have been jointly optimized. The optimal combination of system parameters are obtained by using gradient projection based potential game. Experimental results indicate that the proposed joint optimization framework can effectively achieve a strong video quality performance with less time complexity and enhanced scalability. However, there may be further performance gain if we consider the adaptive video source rate with the scalable video coding. We will focus on this work in future research.

REFERENCES

[1] C. Wong, R. Cheng, K. Letaief, and R. Murch, "Multiuser OFDM with adaptive subcarrier, bit and power allocation," IEEE J. Select. Areas Commun., vol. 17, no. 10, pp. 1747–1758, October 1999.

[2] W. Rhee and J. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in Proc. IEEE VTC 2000 Spring, Tokio, Japan, May 2000, pp. 1085–1089.

[3] G. Song and Y. Li, "Cross-Layer Optimization for OFDM Wireless Networks - Part I: Theoretical Framework", IEEE Trans. on Wireless Comm. vol. 4, no. 2, pp. 625-634, March 2005

[4] Vineet Srivastava, Mehul Motani. Cross-Layer Design: A Survey and the Road Ahead [J]. IEEE Communications Magazine, 2005, vol. 12, pp. 112-19.

[5] S. Shakkottai, R. Srikant, and A. Stolyar, "Pathwise Optimality and State Space Collapse for the Exponential Rule," in Proc. IEEE International Symposium on Information Theory, 2002.

[6] Y. Liu and E. Knightly, "Opportunistic Fair Scheduling over Multiple Wireless Networks," in Proc. of IEEE INFOCOMM, March 2003.

[7] P. Liu, R. Berry, and M. Honig, "Delay Sensitive Packet Scheduling in Wireless Networks," in Proc. IEEE WCNC 2003, March 2003.

[8] Z. He, J. Cai, and C. W. Chen, "Joint source channel rate-distortion analysis for adaptive mode selection and rate control in wireless video coding," IEEE Trans. Circuits Sys. Video Technol., vol. 12, no. 6, pp.511–523, June 2002.

[9] C. E. Luna, Y. Eisenberg, R. Berry, T. N. Pappas, and A. K. Katsaggelos,"Joint source coding and data rate adaptation for energy efficient wirless video streaming," IEEE J. Select. Areas Commun., vol. 21, no. 10, pp.1710–1720, December 2003.

[10] Y. Eisenberg, F. Zhai, T. N. Pappas, R. Berry, and A. K. Katsaggelos, "VAPOR: variance-aware per-pixel optimal resource allocation," IEEE Trans. Image Processing, vol. 15, no. 2, pp. 289–299, February 2006.

[11] Sabir M. F., Heath R. W., Bovik A. C.. "Joint source-channel distorion modeling for MPEG-4 video ". IEEE Trans. Image process., 2009, vol. 18, pp. 90-105.

[12] Haiyan Luo, Dalei Wu, Song Ci, et al."TFRC-based Rate Control for Real-Time Video Streaming over Wireless Multi-hop Mesh Networks".in Proc. IEEE ICC 2009.

[13] Peshala Pahalawatta,Randall Berry and Thrasyvoulos Pappas. "Content-Aware Resource Allocation and Packet Scheduling for Video Transmission over Wireless Networks". IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 25, NO. 4, MAY 2007.

[14] F. Li, G. Lin, L. He. "Cross-layer scheduling for multiuser H.264 video transmission over wireless networks". IET Commun., 2010, Vol.4, Iss. 8, pp.1012-1025.

[15] Qinwen Liu, Shengli Zhou, and Georgios B. Giannakis. "Queuing with Adaptive Modulation and Coding over Wireless Links: Cross-layer Analysis and Design". IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 4, NO. 3, MAY 2005.

[16] Mihaela Van, Davis Sai. "Cross-layer Wireless Multimedia Transmission: Challenges, Principles, and New Paradigms". IEEE Wireless Communications, 2005, VOL. 8, pp.50-58.

[17] S. Khan, Y. Peng, and E. Steinbach. "Application-driven Cross-layer Optimization for Video Streaming over Wireless Networks". IEEE Communications Magazine, 2006, VOL. 1, pp. 122-130.

[18] WIEGAND T., SULLIVAN G., BJONTEGAARD G., LUTHRA A.Overview of the H.264/AVC video coding standard,IEEE Trans. Circuits Syst. Video Technol., 2003, 13, (7), pp. 560–576.

[19] D. Monderer, L. Shapley. Potential games [J]. Games and Economics Behavior, 1996, vol. 14: 124-43.

[20] 3GPP TS 36.300 v8.8.0. Evolved Universal Terrestrial Radio Access (E-UTRA) and Evolved Universal Terrestrial Radio Access Network Technical Specification [S]. 2009.

[21] E. Koutsoupias and C. Papadimitriou. Worst-case equilibria. In 16th Annual Symposium on Theoretical Aspects of Computer Science, pages 404--413, Trier, Germany, March 1999.