A GRADIENT-LIKE VARIATIONAL BAYESIAN APPROACH: APPLICATION TO MICROWAVE IMAGING FOR BREAST TUMOR DETECTION

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ABSTRACT

In this paper a nonlinear inverse scattering problem is solved by means of a variational Bayesian approach. The objective is to detect breast tumor from measurements of the scattered fields at different frequencies and for several illuminations. This inverse problem is known to be non linear and ill-posed. Thus, it needs to be regularized by introducing *a priori* information. Herein, prior information available on the sought object is that it is composed of a finite known number of different materials distributed in compact regions. It is accounted for by tackling the problem in a Bayesian framework. Then, the true joint posterior is approximated by a separable law by mean of a gradient-like variational Bayesian technique. The latter is adapted to complex valued contrast and used to compute the posterior estimators through a joint update of the shape parameters of the approximating marginals. Both permittivity and conductivity maps are reconstructed and the results obtained on synthetic data show a good reconstruction quality and a convergence faster than that of the classical variational Bayesian approach.

Index Terms— Inverse scattering problem, breast tumor detection, Gauss-Markov-Potts prior, Gradient-like Variational Bayesian Approximation.

1. INTRODUCTION

Due to the non negligible dielectric contrast that exists between cancerous and normal healthy breast tissues and to the non-ionizing nature of microwaves, microwave imaging appears to be a promising new technique for the early detection of breast cancer [1, 2], innocuous and cheap as compared to the more popular X-ray mammography [3].

Microwave imaging is handled herein as a non linear inverse scattering problem where the goal is to retrieve a contrast function representative of the dielectric properties (permittivity and conductivity) of an unknown object (the breast), from measurements of the scattered field that results from its interaction with a known interrogating wave. The dielectric contrast that can be quite high between the different breast tissues precludes the consideration of weak scattering linearizing assumptions as this can result in a significant loss of accuracy. As for deterministic nonlinear inversion methods [4, 5, 6], that rely upon the iterative minimization of a cost functional, their reliability highly depends upon the initial estimate of the solution and they do not allow taking easily into account *a priori* information. Yet introducing such an information is mandatory as, in addition to be nonlinear, the inverse problem at hand is also known to be ill-posed, which means that it needs to be regularized and such a regularization is usually done by introducing *a priori* information.

Contrarily to deterministic inversion methods, the Bayesian framework gives a real favorable ground for taking into account the latter. Herein, we would like to account for the fact that the breast is composed of a finite number of different tissues distributed in homogeneous regions. This *a priori* is introduced via a Gauss-Markov-Potts model which consists in Gauss-Markov fields with hidden Potts label fields [7]. Then, two options are usually used: a stochastic sampling, such as in the Markov Chain Monte Carlo method (MCMC) [8], and a variational approximation, such as in the Variational Bayesian Approach (VBA) [9]. Both of them have been applied to optical diffraction tomography [10] and VBA have shown its effectiveness in terms of quality of reconstruction and fastness compared to MCMC [11, 12]. However, while both of them provide good results, MCMC and VBA methods are highly time consuming. This is the reason why we have developed a new method based on the variational Bayesian approach, but with a simultaneous gradient-like updating of the shaping parameters [13] in order to accelerate the convergence of the classical VBA. This new technique has already been studied in X-ray tomography, dictionary decomposition and astrophysical map-making [14, 15]. The originality herein is to apply it to an inverse scattering problem and to test it for breast cancer detection where the unknown contrast is complex valued, which means that both permittivity and conductivity maps have to be reconstructed. Hence the variational technique is adapted to the complex contrast case by assuming that permittivity and conductivity have the same segmentation, i.e. the same hidden field, but are independent conditionally to this hidden variable.

The paper is organized as follows: section 2 describes the measurement configuration and the forward modeling of the problem. In section 3, we present the Bayesian framework used to solve the inverse scattering problem. The new method, from now on denoted as Gradient-like Variational Bayesian Approach (GVBA), is detailed in section 4, whereas section 5 displays some results obtained from synthetic data. Finally some conclusions are given in section 6.

2. THE FORWARD MODEL

Let us consider a 2D-TM configuration where the media are supposed to be cylindrical of infinite extension along the z axis and are illuminated by a line source that generates an electric field polarized along the latter axis. A homogeneous breast (domain $\mathcal{D}_2$), whose relative dielectric permittivity and conductivity are $\epsilon_r = 6.12$ and $\sigma = 0.11$ Sm^{-1}, respectively, and of 9.6-cm-diameter circular cross-section, is placed in the air (domain $\mathcal{D}_1$). The breast is affected by a tumor (domain $\mathcal{D}_3$) of 2-cm-diameter circular cross-section, whose relative dielectric permittivity and conductivity are $\epsilon_r = 55.3$ and $\sigma = 1.57$ Sm^{-1}, respectively, and is illuminated by the source from 16 various angular positions uniformly distributed around a 7.5-cm-radius circle centered at the origin and at 6 different frequencies in the band 0.5 - 3 GHz that yields a good compromise between resolution and penetration of the wave in the breast. For each frequency and illumination angle, 32 measurements of the scattered field are performed at angular positions uniformly distributed around the same circle. It can be noted that, in the inversion process, the data corresponding to the different frequencies and illuminations are processed simultaneously in order to reduce the ill-posedness of the inverse problem.

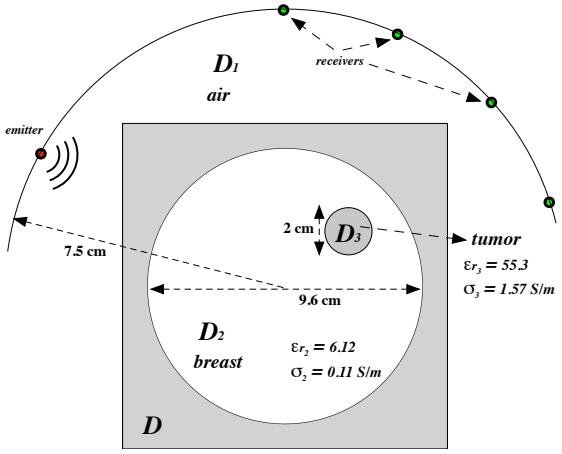

Fig. 1. The measurement configuration

The different media are characterized by their propagation constant $k(\boldsymbol{r})$ such that $k(\boldsymbol{r})^2 = \omega^2\epsilon_0\epsilon_r(\boldsymbol{r})\mu_0 + i\omega\mu_0\sigma(\boldsymbol{r})$, where ω is the angular frequency, ϵ_0 and μ_0 are the permittivity and the permeability of free space, respectively, $\boldsymbol{r} \in \mathcal{D}$ is an observation point in a test domain $\mathcal{D}$ and $\epsilon_r(\boldsymbol{r})$ and $\sigma(\boldsymbol{r})$ are the relative permittivity and conductivity of the medium. We now consider a contrast function χ defined in $\mathcal{D}$ and null outside the object, such that $\chi(\boldsymbol{r}) = (k(\boldsymbol{r})^2 - k_1^2)/k_1^2$, where k_1 is the propagation constant of the embedding medium, and we define $w(\boldsymbol{r})$ as the Huygens type sources induced within the target by the incident wave, i.e. $w(\boldsymbol{r}) = \chi(\boldsymbol{r})E(\boldsymbol{r})$ where $E(\boldsymbol{r})$ is the total field in the target.

Modeling is based upon a domain integral representation of the electric fields which consists in two coupled contrast-source integral equations whose discrete counterparts are obtained by means of a method of moments [16] with pulse-basis and point matching. This amounts to partition the test domain $\mathcal{D}$ into elementary square pixels small enough in order to consider the field and the contrast as constant over each of them. The discrete model then reads:

$$\boldsymbol{y}_\nu = \boldsymbol{G}_f^o\, \boldsymbol{w}_\nu + \boldsymbol{\epsilon}_\nu \quad (1)$$

$$\boldsymbol{w}_\nu = \boldsymbol{X}_f\, \boldsymbol{E}_\nu^{inc} + \boldsymbol{X}_f\, \boldsymbol{G}_f^c\, \boldsymbol{w}_\nu + \boldsymbol{\xi}_\nu, \quad (2)$$

where $\boldsymbol{E}$, $\boldsymbol{\chi}$ and $\boldsymbol{w}$ are vectors that contain the values of $E(\boldsymbol{r}')$, $\chi(\boldsymbol{r}')$ and $w(\boldsymbol{r}')$ at the centers $\boldsymbol{r}'$ of the pixels ($\boldsymbol{r}' \in \mathcal{D}$), $\boldsymbol{y}$ is the vector containing the values of the scattered field $y(\boldsymbol{r})$ at the measurement points $\boldsymbol{r}$, $\boldsymbol{X} = \mathrm{diag}(\boldsymbol{\chi})$, $\boldsymbol{G}^o$ and $\boldsymbol{G}^c$ are huge matrices whose elements result from the integration of the Green's function over the pixels [10] and $\boldsymbol{\epsilon}$ and $\boldsymbol{\xi}$ are variables that account for the model and measurement errors and that are supposed to be centered and white and to satisfy Gaussian laws, i.e. $\boldsymbol{\epsilon} \sim \mathcal{N}(\boldsymbol{\epsilon}|\boldsymbol{0}, v_\epsilon\mathbf{I})$ and $\boldsymbol{\xi} \sim \mathcal{N}(\boldsymbol{\xi}|\boldsymbol{0}, v_\xi\mathbf{I})$. The subscripts f and ν, that will be omitted from now on, account for the different frequencies and for both the different frequencies and source positions, respectively.

3. BAYESIAN INVERSION

Now, the inverse problem consists in estimating the contrast $\boldsymbol{\chi}$, or more precisely the relative permittivity ϵ_r and the conductivity σ, from the scattered fields $\boldsymbol{y}$, given the incident fields $\boldsymbol{E}^{inc}$. It can be noted that, the induced sources $\boldsymbol{w}$ being unknown, they have to be estimated at the same time as $\boldsymbol{\chi}$.

The first step is to account for the *a priori* information available on the sought solution. Herein, we would like to account for the fact that the object is composed of a finite number K of different materials distributed in compact homogeneous regions. This is done by modeling the contrast as a Gaussian mixture. Each pixel $\chi(\boldsymbol{r})$ is assigned to a material (class) $z(\boldsymbol{r})$. For each class, the pixel contrast conditional distribution is a Gaussian law $p(\chi(\boldsymbol{r})|z(\boldsymbol{r}) = k) = \mathcal{N}(m_k, v_k)$, where $k = 1, \ldots, K$, with a mean value m_k and a variance v_k that depend upon the material properties.

The information that the different materials are distributed in compact homogeneous regions is accounted for by means

of a Potts-Markov model on $\boldsymbol{z}$ that expresses the spatial dependence between the neighboring pixels:

$$p(\boldsymbol{z}|\lambda) = \frac{1}{T(\lambda)} \exp\left\{\lambda \sum_{\boldsymbol{r}\in\mathcal{D}} \sum_{\boldsymbol{r}'\in V_{\boldsymbol{r}}} \delta\left(z(\boldsymbol{r}) - z(\boldsymbol{r}')\right)\right\}, \tag{3}$$

where $z = \{z(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{D}\}$ represents the image of the labels (segmentation), λ determines the correlation between neighbors (herein $\lambda = 1$), $T(\lambda)$ is a normalization factor and $V_{\boldsymbol{r}}$ is a neighborhood of $\boldsymbol{r}$, herein made of the four nearest pixels. Conjugate prior laws are assigned to other hyper-parameters, i.e. $p(m_k) \sim \mathcal{N}(m_k|\mu_0, \tau_0)$, $p(v_k) \sim \mathcal{IG}(v_k|\eta_0, \phi_0)$, $p(v_\epsilon) \sim \mathcal{IG}(v_\epsilon|\eta_\epsilon, \phi_\epsilon)$, $p(v_\xi) \sim \mathcal{IG}(v_\xi|\eta_\xi, \phi_\xi)$, where $\mathcal{N}(m|\mu, \tau)$ and $\mathcal{IG}(v|\eta, \phi)$ stand for Gaussian and inverse-gamma distributions, respectively, and μ_0, τ_0, η_0, ϕ_0, η_ϵ, ϕ_ϵ, η_ξ and ϕ_ξ are meta-hyper-parameters appropriately set to obtain almost non-informative prior distributions.

It can be noted that a semi-supervised context is considered herein as K is supposed to be known, whereas the contrast $\boldsymbol{\chi}$, the induced currents $\boldsymbol{w}$, the segmentation $\boldsymbol{z}$ and the hyper-parameters of the model $\boldsymbol{\psi} = \{\boldsymbol{m}, \boldsymbol{v}, v_\epsilon, v_\xi\}$ are estimated simultaneously. Afterwards, we apply the Bayes formula to get the joint posterior distribution of the unknowns:

$$\begin{aligned} p(\boldsymbol{\chi}, \boldsymbol{w}, \boldsymbol{z}, \boldsymbol{\psi}|\boldsymbol{y}) \quad &\propto \quad p(\boldsymbol{y}|\boldsymbol{w}, v_\epsilon)\, p(\boldsymbol{w}|\boldsymbol{\chi}, v_\xi)\, p(\boldsymbol{\chi}|\boldsymbol{z}, \boldsymbol{m}, \boldsymbol{v}) \\ &\times \quad p(\boldsymbol{z}|\lambda)\, p(\boldsymbol{m}|\mu_0, \tau_0)\, p(\boldsymbol{v}|\eta_0, \phi_0) \\ &\times \quad p(v_\epsilon|\eta_\epsilon, \phi_\epsilon)\, p(v_\xi|\eta_\xi, \phi_\xi). \end{aligned} \tag{4}$$

Applying the joint maximum *a posteriori* (JMAP) or the posterior mean (PM) to compute the joint posterior distribution yields intractable forms and an approximation is needed to obtain a practical solution.

4. THE GVBA APPROACH

The gradient-like variational Bayesian approach (GVBA) is derived from the classical variational Bayesian approach (VBA, [9]) that aims at approximating the joint posterior distribution $p(\boldsymbol{x}|\boldsymbol{y})$ ($\boldsymbol{x} = \{\boldsymbol{\chi}, \boldsymbol{w}, \boldsymbol{z}, \boldsymbol{\psi}\}$) by a separable law $q(\boldsymbol{x}) = \prod_i q_i(\boldsymbol{x}_i)$ which is as close to the posterior distribution as possible in terms of the Kullback-Leibler divergence. It can be noted that minimizing the KL divergence is equivalent to maximizing the negative free energy derived from statistical physics $\mathcal{F}(\boldsymbol{q}) = \int q(\boldsymbol{x}) \ln\left(p(\boldsymbol{y}, \boldsymbol{x})/q(\boldsymbol{x})\right) \mathrm{d}\boldsymbol{x}$. The solution of this problem can be obtained by alternate optimization with respect to each $q_i(x_i)$ and is given by:

$$q_i(\boldsymbol{x}_i) \propto \exp\left\{\left\langle \ln(p(\boldsymbol{x}, \boldsymbol{y}))\right\rangle_{\prod_{j\neq i} q_j(\boldsymbol{x}_j)}\right\}. \tag{5}$$

The computation of $\boldsymbol{q}_i$ requires the knowledge of all $\boldsymbol{q}_j$, $j \neq i$. However, recently, other ways than this classical alternate optimization have been investigated [14]. In fact, the optimization involved in VBA is an infinite dimensional concave problem. Hence, approximating densities $q_i(x_i)$ can be obtained by adapting a classical optimization algorithm, such as a gradient method, to VBA. Using the notion of optimal step, the approximating marginals have an iterative functional form. At iteration $n+1$, they read:

$$\begin{aligned} \widetilde{q}_i^{(n+1)}(\boldsymbol{x}_i) \quad &\propto \quad \left(\widetilde{q}_i^{(n)}(\boldsymbol{x}_i)\right)^{(1-\alpha)} \\ &\times \quad \exp\left\{\alpha\left\langle \ln(p(\boldsymbol{x}, \boldsymbol{y}))\right\rangle_{\prod_{i\neq j} \widetilde{q}_j^{(n)}(x_j)}\right\} \end{aligned} \tag{6}$$

where α ($\alpha \geq 0$) is a descent step that minimizes the negative free energy at each iteration. A strong separation is chosen:

$$q(\boldsymbol{x}) = q(v_\epsilon)q(v_\xi) \times \prod_i q(\chi_i)q(w_i)q(z_i) \prod_k q(m_k)q(v_k).$$

Then, using equation (6), the approximating marginal for each unknown variable can be computed by means of functional optimization. Updating the approximate posterior requires 7 different gradient steps. However, since there is no dependence between χ, z, v_ϵ, v_ξ, v_k and m_k, their gradient steps can be set to 1 in order to accelerate the convergence. Hence, only the contrast source updating step α_w is computed. This is done in an optimal way from the negative free energy function by means of a Newton's method. The optimal step then reads:

$$\alpha_w^{opt} = \Delta\mathcal{F}(\alpha_w)/\Delta^2\mathcal{F}(\alpha_w)\Big|_{\alpha_w=0}, \tag{7}$$

with $\Delta\mathcal{F} = \partial\mathcal{F}/\partial\alpha_w$ et $\Delta^2\mathcal{F} = \partial^2\mathcal{F}/\partial^2\alpha_w$. At this point, it can be mentioned that taking the likelihood in the exponential family and using conjugate priors will result in joint posteriors and marginals ranging in the exponential family. Optimization with respect to $q_i(x_i)$ then results in optimizing the parameters of these laws. In the following we summarize the results for the Gaussian - inverse gamma case:

$$\begin{aligned} q(\boldsymbol{w}) &= \mathcal{N}(\widetilde{\boldsymbol{m}}_w, \widetilde{\boldsymbol{V}}_w), & q(\boldsymbol{\chi}) &= \mathcal{N}(\widetilde{\boldsymbol{m}}_\chi, \widetilde{\boldsymbol{V}}_\chi), \\ q(m_k) &= \mathcal{N}(\widetilde{\mu}_k, \widetilde{\tau}_k), & q(v_k) &= \mathcal{IG}(\widetilde{\eta}_k, \widetilde{\phi}_k), \\ q(v_\epsilon) &= \mathcal{IG}(\widetilde{\eta}_\epsilon, \widetilde{\phi}_\epsilon), & q(v_\xi) &= \mathcal{IG}(\widetilde{\eta}_\xi, \widetilde{\phi}_\xi), \end{aligned}$$

$$q(\boldsymbol{z}) = \widetilde{\boldsymbol{\zeta}}_k \propto exp\left(\lambda \sum_{\boldsymbol{r}\in\mathcal{D}} \sum_{\boldsymbol{r}'\in V(\boldsymbol{r})} \widetilde{\zeta}(\boldsymbol{r}')\right), \tag{8}$$

where $\widetilde{\boldsymbol{m}}_w^n = \widetilde{\boldsymbol{m}}_w + \alpha_w \widetilde{\boldsymbol{V}}_w^n \boldsymbol{d}_w$, $\boldsymbol{d}_w = \Big[\overline{v_\epsilon^{-1}} \boldsymbol{G}^{o^H} (\boldsymbol{y} - \boldsymbol{G}^o \widetilde{\boldsymbol{m}}_w) + \overline{v_\xi^{-1}}\Big(\widetilde{\boldsymbol{m}}_\chi \boldsymbol{E}^{inc} - \boldsymbol{G}^{c^H}(\widetilde{\boldsymbol{m}}_\chi^2 + \widetilde{\boldsymbol{V}}_\chi)\boldsymbol{E}^{inc} - \widetilde{\boldsymbol{m}}_w + \widetilde{\boldsymbol{m}}_\chi \boldsymbol{G}^c \widetilde{\boldsymbol{m}}_w + \boldsymbol{G}^{c^H} \widetilde{\boldsymbol{m}}_\chi^* \widetilde{\boldsymbol{m}}_w - \boldsymbol{G}^{c^H}(\widetilde{\boldsymbol{m}}_\chi^2 + \widetilde{\boldsymbol{V}}_\chi) \times \boldsymbol{G}^c \widetilde{\boldsymbol{m}}_w\Big)\Big]$.
$\widetilde{\boldsymbol{V}}_w^n$ is given in [17], whereas other shaping parameters are detailed in [10]. α_w^{opt} then reads:

$$\alpha_w^{opt} = \frac{\widetilde{\boldsymbol{m}}'_{w0}\boldsymbol{d}_w + \frac{1}{2}\sum_i \boldsymbol{S}_{w_i}^2}{\widetilde{\boldsymbol{m}}''_{w0}\boldsymbol{d}_w + \widetilde{\boldsymbol{m}}'_{w0}\boldsymbol{d}'_w + \frac{1}{2}\sum_i \boldsymbol{S}_{w_i}^3}, \tag{9}$$

where $'$ and $''$ stand for the first and second derivatives, respectively, and $\boldsymbol{S}_{w_i} = \boldsymbol{R}_{w_i}\widetilde{\boldsymbol{V}}_{w0_i} - 1$ (see details in [17]).

5. NUMERICAL EXPERIMENTS AND RESULTS

The above method (GVBA) is applied to synthetic data obtained with the configuration of section 2. The initial values of the unknowns $\boldsymbol{\chi}^{(0)}$ and $\boldsymbol{w}^{(0)}$ are obtained by backpropagating the scattered field data from the measurement circle onto $\mathcal{D}$, whereas the initial segmentation $\boldsymbol{z}^{(0)}$ is given by *K-means* clustering [18], with empirical estimators for $\boldsymbol{\psi}^{(0)}$. It can be noted that the same segmentation is used for the real and imaginary parts of the contrast. The test domain $\mathcal{D}$ is a 12.16-cm-side square partitioned into 64×64 1.9-mm-side square pixels. The results obtained after 2000 iterations of GVBA are displayed in Figure 2 and compared to those obtained by means of classical VBA and Contrast Source Inversion (CSI, [6]) methods after 2000 and 1000 iterations, respectively, the former two being initialized by a few CSI iterations. It appears that GVBA outperforms VBA with respect to both the resolution and speed of convergence, as shown in Table 1 that displays the CPU time needed by the different methods and the corresponding peak signal to noise ratio, and that both methods outperform the deterministic CSI method particularly concerning the retrieved conductivity values. This is confirmed by the profiles retrieved along an horizontal line crossing the center of the tumor (Figure 2-bottom).

Table 1. Comparison of the different approaches

Method	CSI	VBA	GVBA
CPU time (min)	74	87	51
PSNR (dB)	77.64	82.78	85.74

6. CONCLUSION

Herein, microwave imaging for breast cancer detection is handled in a Bayesian framework with a Gauss-Markov-Potts prior. A new algorithm (the so called gradient-like variational Bayesian approximation) based on the classical variational Bayesian approximation and a gradient descent method is used to compute the posterior with a free-form distribution with respect to complex quantities as both permittivity and conductivity maps have to be retrieved. As a first step, good results have been obtained on synthetic data corresponding to a simple object and it has been shown that the new approach performs better than the classical VBA, especially in terms of speed of convergence. Application of the above method to more realistic scattering configurations and to experimental data collected in controlled situations is under investigation.

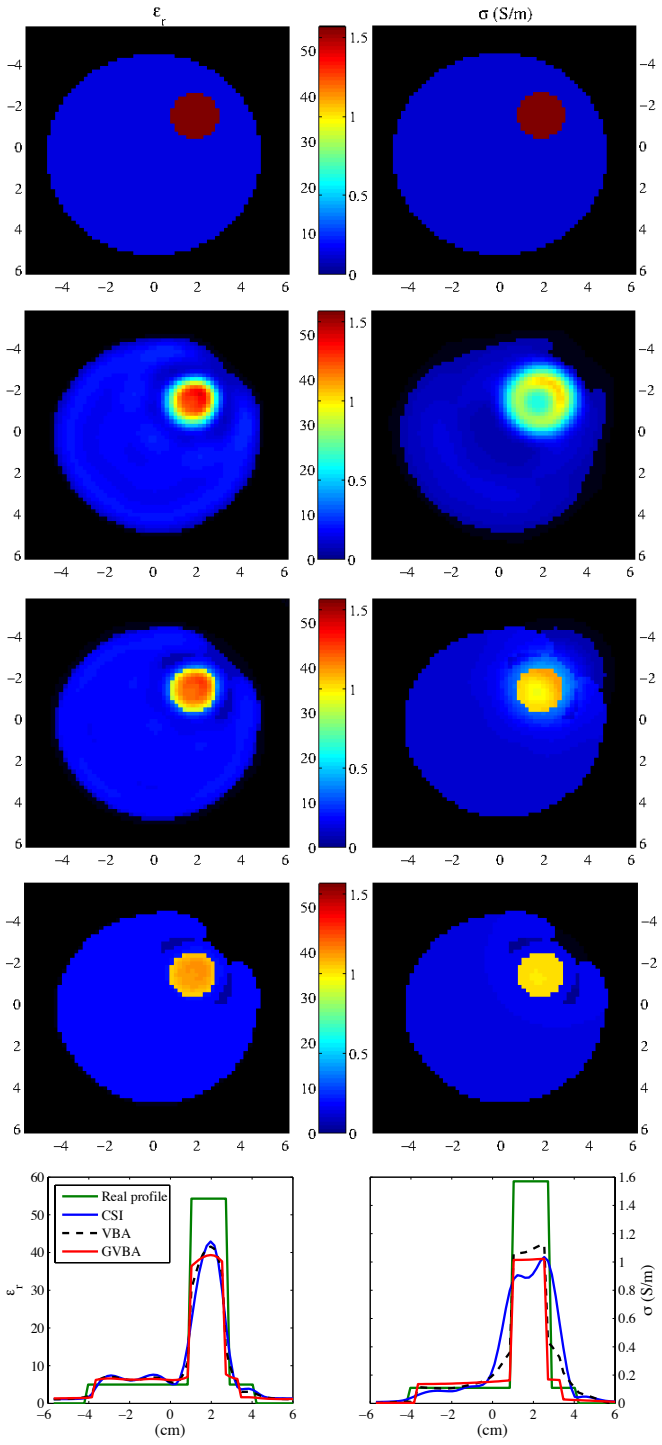

Fig. 2. Permittivity (left) and conductivity (right) reconstructed by means of CSI (second row), VBA (third row) and GVBA (forth row) compared to the real object (top) and the profiles reconstructed with CSI (green line), VBA (dashed line) and GVBA (red line) along an horizontal line passing through the center of the tumor (bottom).

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