IMAGE ENHANCEMENT BY ENTROPY MAXIMIZATION AND QUANTIZATION RESOLUTION UPCONVERSION

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ABSTRACT

This article introduces a new contrast enhancement algorithm of tone-preserving entropy maximization. Its design objective is to present the maximal amount of information content in the enhanced image, or being optimal in an information theoretical sense, while preventing the loss of tone continuity. The resulting optimization problem can be graph-theoretically modeled as the construction of K-edge maximum-weight path, and it can be solved efficiently by dynamic programming. Moreover, the proposed algorithm is made more effective by being combined with a preprocess of image restoration that aims to correct quantization errors caused by the analog-to-digital conversion of image signals. Empirical evidences are provided to demonstrate the superior visual quality obtained by the new image enhancement algorithm.

I. INTRODUCTION

In image enhancement the presentation of visual signals is manipulated or improved to better suit the needs of users engaged in specific tasks. Among all subjects of image processing and computer vision, image enhancement, despite its long history, is technically least rigorous; most of image/video enhancement techniques are more an art than science. This is largely because criteria for good image representations are highly subjective and often vary in application problems. Due to this lack of generality and rigor many image enhancement techniques are not robust and have undesired side effects. In this research, we set out to address these shortcomings and introduce a new, more principled approach for image enhancement.

For most users and in most applications the main purpose of image enhancement is to bring conspicuity to features and details that are otherwise obscured in the original image. In this case a common approach of image enhancement is the point process of grey level transform, as exemplified by the histogram equalization technique[1]. This classic approach was reexamined in depth and cast into a framework of optimal contrast-tone mapping (OCTM) by Wu [2]. In OCTM an integer-to-integer transform $j = T(i)$, which maps input grey level i to output grey level j, is so constructed that the expected difference $E\{T(i+1) - T(i)\}_p$ is maximized, where the expectation is taken over the probability mass function p (histogram) of grey levels. Regardless the choice of the grey level transform $T(i)$, for any given pixel its processed value depends only on its original value, independent of its spatial context. Because of this property image enhancement via grey level transform is classified as context-free in [2].

An important criterion in the design of image enhancement algorithms is entropy, which is meaningful not only information-theoretically, but also psychovisually if the quest of enhancement is the conspicuity of image details. This is because the entropy, as a statistical measure for the information content of the image signal, can predict the richness of details exhibited by the processed image. In this line of reasoning, the design objective of grey level transform $T(i)$ ought to be the maximal entropy $H(p)$ under suitable constraint(s) to prevent pathological cases, p being the probability mass function of grey levels of the processed image. But considering that $T(i)$ is context free, the following two conditions are needed so that $T(i)$ retains the spatial structures of image signal; otherwise the white noise image would be produced in pursue of maximum entropy. 1. $T(i)$ is a mathematical function, i.e., single valued, so that it cannot produce random noises; 2. $T(i)$ preserves the relative ranking of input intensity levels, namely, $T(i) \leq T(j)$ if $i < j$, so that enhanced local details do not alter the two-dimensional spatial patterns of the original image. Now these conditions would seemingly result in a paradox: the entropy is maximized by the identity transform $i = T(i)$, i.e., doing nothing, if the input image and display device have the same number of grey levels, which is by far the most common case in practice. This is because any other single-valued transforms that map two or more input grey levels into one output grey level can only reduce the entropy.

However, the above difficulty turns out to be only operational, rooted in the digitization of the input image signals; it becomes a nonissue if the histogram is replaced by a continuous probability density function. Tracing back to the physical problem the light intensity is a continuous quantity in the first place prior to analog-to-digital conversion. If the quantization resolution of the input discrete pixel values is upconverted significantly beyond that of the display by an image restoration process, then the histogram of the restored image will have far more bins than the output grey levels of the display. As such, the transform function $T(i)$ will necessarily map multiple adjacent grey levels of the restored image to a same output grey level, and consequently the entropy maximization-based image enhancement will no longer have a trivial solution of $i = T(i)$. Our insight of aiding image enhancement by image restoration fills a vital missing link towards the development of the new image enhancement approach of this paper.

II. UPCONVERSION OF QUANTIZATION RESOLUTION

As discussed in the introduction, in the maximum entropy approach for image enhancement, it is necessary that the input histogram has more grey levels than the display. For input images of the same bit depth as the display, the quantization resolution of pixel values needs to be increased. Here the objective is to estimate the underlying continuous light amplitude values and requantize them into higher resolution. This is a problem of image restoration in the domain of pixel amplitude. Without multiple exposures like in acquisition of high dynamic range images, we need to make the restoration possible by exploiting other form of signal correlations. For most natural images the underlying two-dimensional luminance signal is highly smooth on object surfaces, and even on object boundaries it is also smooth along the edge trajectories. Therefore, an edge-directed low-pass filtering technique is suited to estimate the continuous luminance image from the observed grey level image of insufficient amplitude quantization resolution.

The underlying continuous luminance image $\mathbf{I}$ is modeled as an piece-wise autoregressive (PAR) process

$$\mathbf{I}(x, y) = \sum_{(i,j)\in\mathcal{S}(x,y)} \alpha_{i,j}\mathbf{I}(x+i, y+j) + \eta \qquad (1)$$

where $\alpha_{i,j}$ are the model parameters, (x, y) is the pixel location and $S(x, y)$ is a local neighborhood of (x, y),η is the model fitting error. The PAR model is locally adaptive and it can preserve edges well [3]. Assuming that the 2D signal $\mathbf{I}$ is stationary in a local area $N(x, y)$ surrounding (x, y), the adaptive model parameters for $\mathbf{I}(x, y)$ can be estimated using samples drawn from the input image $\mathbf{J}$ in locality $N(x, y)$. The pixel values of J carry errors $e = \mathbf{I} - \mathbf{J}$ that are caused by the analog-to-digital conversion, imaging sensor noises, and possibly compression noises. As these error causes are statistically independent, e is white Gaussian and of zero mean, hence we adopt the following least-squares estimation

$$\underset{\alpha}{\operatorname{argmin}} \sum_{(x,y)\in N(x,y)} \|J(x, y) - \sum_{(i,j)\in\mathcal{S}(x,y)} \alpha_{i,j} J(x+i, y+j)\|_2^2 \qquad (2)$$

to obtain the PAR model parameters α.

Having the above estimated PAR model parameters, the continuous luminance image $\mathbf{I}$ can be restored based on the model. Although the PAR model is piecewise, all luminance values of $\mathbf{I}$ in the entire image domain are restored jointly as the following

$$\min_{\mathbf{I}} \|\mathbf{I} - \mathbf{A}\mathbf{I}\|_2^2 + \lambda\|\mathbf{I} - \mathbf{J}\|_2^2, \qquad (3)$$

where matrix $\mathbf{A}$ consists of PAR model parameters; the i-th row of $\mathbf{A}$ is the estimated parameter vector α_i at pixel location i.

The above PAR-based image restoration method is not the only choice; it can be replaced by other restoration methods, such as those based on popular sparsity signal processing, as long as they generate a real-valued luminance image. The remaining task is to enhance the input image $\mathbf{J}$ by a global transform function $T(\cdot)$ to requantize the restored image $\mathbf{I}$ into K discrete levels, K being the number of grey levels of the display device ($K = 255$ for typical optoelectronic displays). Note that in general the upconversion of quantization resolution maps an original grey level into different finer grey levels. Although $T(\cdot)$ is single-valued with respect to the histogram of restored image of high amplitude quantization resolution, it is no longer so with respect to the histogram of the original input image. This is because an original grey level can be converted and requantized to different finer grey levels at different pixel locations. Furthermore, since the restored image of higher amplitude resolution is generated by a spatial operator that exploits sample correlations, the refined pixel values depend on the local waveforms. In this way, the classic image enhancement approach of grey level transform becomes context-sensitive and hence more adaptive to the two-dimensional image waveform.

III. ENHANCEMENT BY TONE-PRESERVING ENTROPY MAXIMIZATION

The amplitude quantization resolution upconversion described in the proceeding section is only a necessary preprocessing for the image enhancement approach of entropy maximization. The restored image $\mathbf{I}$ is not our final result; image $\mathbf{I}$ will be enhanced by the global grey level transform $T(\cdot)$ that maximizes the entropy of the output image under certain constraint as we reasoned in the introduction.

For a display of K grey levels and given the probability density function p_I of the grey levels of the restored image $\mathbf{I}$, the classic histogram equalization transform

$$k = T(i) = K \int_0^i p_I(t)dt \qquad (4)$$

will maximize the entropy of the resulting image. But the problem is more complex than histogram equalization. As well known to the users histogram equalization is prone to contour artifacts. This is because histogram equalization can map a large dynamic range of luminance into a single grey level. This problem was treated in [2], where the author argued that the tone continuity is another aspect of

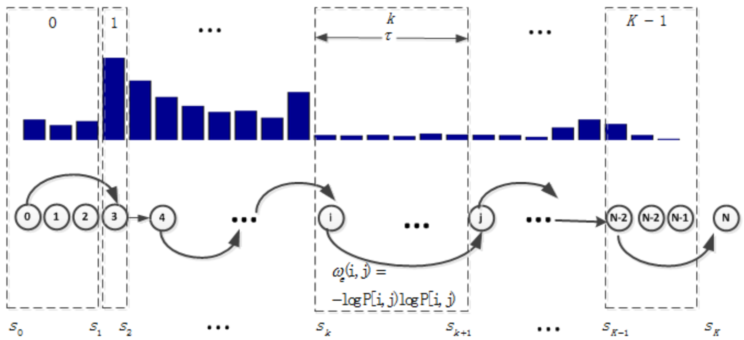


Fig. 1. The graph-theoretical representation of maximum entropy requantization problem.

perceptual image quality that should be balanced against high contrast and proposed to bound tone distortion in grey level transform. Similarly, we impose an upper bound τ on the tone error to constrain the solution of entropy maximization.

The proposed tone-preserving entropy maximization algorithm is one of constrained discrete optimization. The input of the algorithm is the histogram of the restored image $\mathbf{I}$; this histogram is generated by uniform quantization of the luminance range into N bins, $N >> K$. In this setting, a grey level transform T is defined by an ordered integer-valued vector $\mathbf{s} = (s_1, s_2, \cdots, s_{K-1})$ such that all input grey levels $i \in [s_k, s_{k+1})$ are mapped to output level k, written as $T(s_k, s_{k+1}) = k$, $0 \leq k < K$, where $s_0 \equiv 0$, $s_K \equiv N$. Denote by $P[i, j)$ the probability that a grey level in $\mathbf{I}$ falls into the interval $[i, j)$. Then, the image enhancement by tone-preserving entropy maximization (TPEM) can be formulated as the following constrained optimization problem:

$$\min_{\mathbf{s}} \sum_{k=0}^{K-1} -P[s_k, s_{k+1}) log P[s_k, s_{k+1}) \qquad (5)$$
$$\text{subject to } s_{k+1} - s_k \leq \tau \ \forall k$$

where τ is the above mentioned tone distortion bound.

The problem of (5) can be modeled as a K-edge maximum-weight path in the directed acyclic graph (DAG), denoted by $G(V, E)$ and shown in Fig. 1. The nodes of the DAG vertex set V are grey levels of the input histogram, labeled by $0, 1, 2, \cdots, N$. A pair of grey levels i and j, $i < j$, induce an edge $e(i, j) \in E$ if the grey levels in $[i, j)$ are permitted to be mapped to an output grey level k by T, i.e., $T(i, j) = k$. Edge $e(i, j)$ is assigned a weight $-P[i, j) \log P[i, j)$. In the construction of the above DAG $G(V, E)$, we impose the upper bound on tone distortion by not allowing any edge from node i to node j if $j - i > \tau$. The solution of the optimization problem (5) corresponds to a path from 0 to N of K edges such that the sum of the edge weights is maximal, which can be effectively solved by dynamic programming [4].

IV. EXPERIMENTAL RESULTS

(a) original image(6.61) (b) HE(5.76)

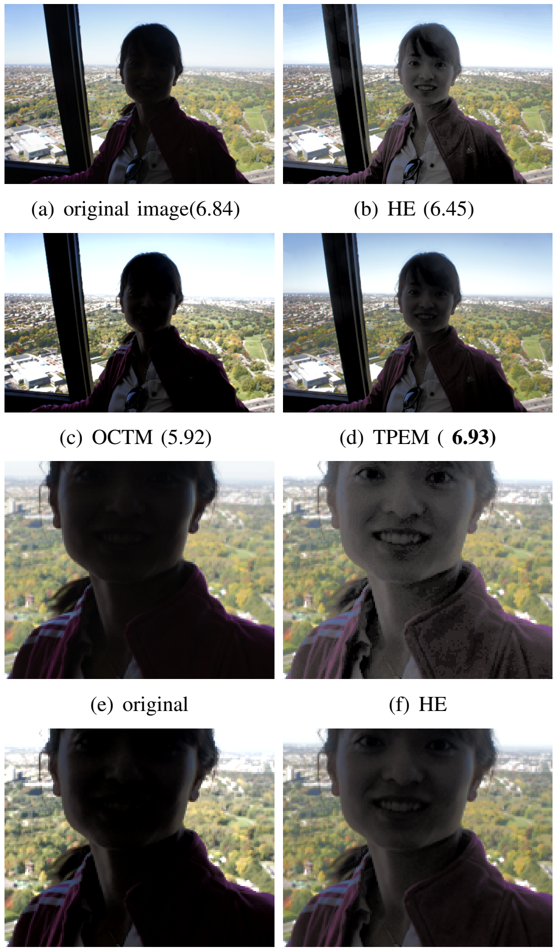
(c) OCTM(6.54) (d) TPEM (**7.56**)

(e) original (f) HE (g) OCTM (h) TPEM

Fig. 2. The visual quality comparison between three techniques on dark indoor image.

Experiments are conducted to evaluate the proposed TPEM image enhancement technique in comparison with the traditional HE technique and the recent OCTM technique [2]. Diverse range of natural images are tested, three of which are presented in Figs.2 through 4 together with the output images of the three techniques, representing dark indoor, over-exposure and back-lighting conditions respectively. The regions enclosed by boxes are zoomed in for closeup visual examination.

The entropies of the tested image enhancement techniques are listed in Table 1. As we discussed in the introduction,



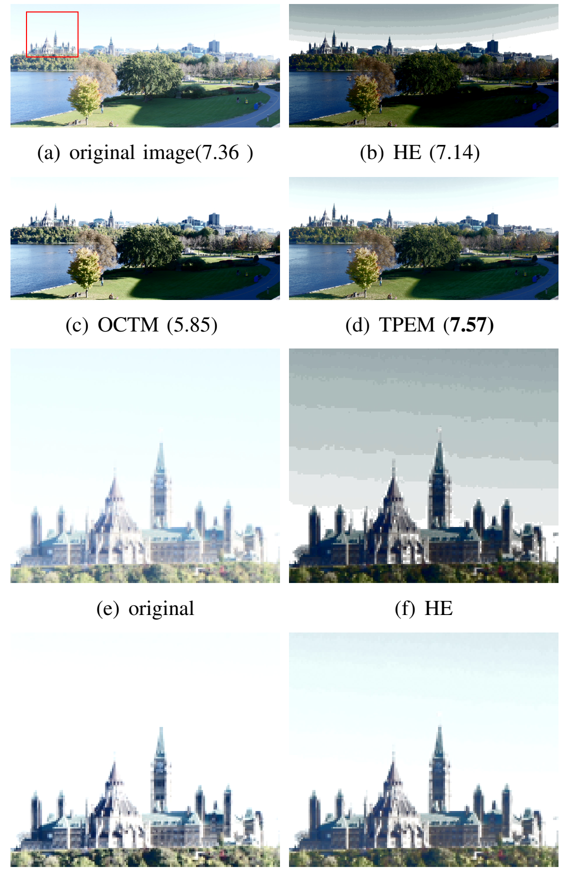
(a) original image(6.84) (b) HE (6.45)
(c) OCTM (5.92) (d) TPEM (**6.93**)
(e) original (f) HE
(g) OCTM (h) TPEM

Fig. 3. The visual quality comparison between three techniques on back-lighting image.

	Original	HE	OCTM	TPEM
Indoor	6.61	5.76	6.54	**7.56**
Back Lighting	6.84	6.45	5.92	**6.93**
Over-exposure	7.36	7.14	5.85	**7.57**

Table I. Entropies of the results of different image enhancement techniques.

both HE and OCTM techniques decrease the entropy of the original image as they cannot split a histogram bin. In contrast, the proposed TPEM algorithm can increase the entropy of the original image thanks to the quantization resolution upconversion. The increased entropy appears to correspond to superior image quality of TPEM. All three images have rather large dynamic range; it is challenging to enhance details in all grey level ranges. HE tends to either overexpose the objects in lighter regions (see the sculpture in Fig. 2 and the face in Fig. 3) or underexpose the objects in darker regions (see the lawn in Fig. 4). Although OCTM offers in general better perceptual quality than HE, but it is



(a) original image(7.36) (b) HE (7.14)
(c) OCTM (5.85) (d) TPEM (**7.57**)
(e) original (f) HE
(g) OCTM (h) TPEM

Fig. 4. The visual quality comparison between three techniques on over-exposure image.

less robust and has inferior quality compared to TPEM (see Fig. 3).

Also, it is evident that HE suffers from serious tone distortions, creating false contours on smooth surfaces (see the face and jacket in Fig.3, and the sky in Fig.4). This is because HE does not restrict the maximum step size of the transform function, allowing two close grey levels to be mapped quite apart from each other. This weakness is overcome by TPEM because it imposes an upper bound on tone distortion in the entropy maximization process. In terms of perceptual quality, TPEM appears to be the best of the three methods by presenting fine detailed foreground objects in natural-looking background.

V. ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China (No. 61301288,61372131, 61227004,11204014)the Ph.D. Programs Foundation of Ministry of Education of ChinaNo.20130203130001)the Fundamental Research Funds for the Central Universities (No. K5051302057, JB140218).

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