ULTRASOUND IMAGE RECONSTRUCTION USING THE FINITE-RATE-OF-INNOVATION PRINCIPLE

*Satish Mulleti, *Sudarshan Nagesh, †Rajesh Langoju, †Abhijit Patil, and *Chandra Sekhar Seelamantula

*Department of Electrical Engineering,
*Indian Institute of Science, Bangalore - 560012, Karnataka, India
†General Electrical Global Research Centre, Bangalore, Karnataka, India
Emails: {satishm, sudarshan}@ee.iisc.ernet.in, {rajesh.langoju,Abhijit.Patil1}@ge.com, chandra.sekhar@ieee.org

ABSTRACT

Recently, a method of finding the spectral samples of non-periodic-finite-rate-of-innovation (NP-FRI) signals using a sum-of-sincs (SoS) sampling kernel was proposed in the literature. In the SoS approach, the kernel is repeated at a rate dependent on the delays of the FRI signal. The number of repetitions depends on both the duration and the delays of pulses constituting the FRI signal. In this paper, we show that the kernel repetition can be avoided and perfect reconstruction can be obtained by working with the SoS kernel directly provided that certain sampling criteria are satisfied. We place a lower bound on the sampling rate to ensure that exact signal reconstruction is achieved using filtered samples. To suppress the effect of noise, we use Cadzow denoising technique. Reconstruction is achieved using the annihilating filter method. We report results on data simulated using Field II software as well as real cardiac ultrasound data. The experimental results show that, with nearly 10 times less data than that required by the standard technique, the proposed method gives a comparable quality of reconstruction. The reconstruction accuracy can be controlled by choosing the model order of the NP-FRI signal appropriately.

***Index Terms*—** Finite rate of innovation, ultrasound imaging, annihilating filter, Cadzow denoising, sum-of-sincs kernel.

1. INTRODUCTION

Recently, Vetterli et al. [1] proposed a sampling and reconstruction methodology for the class of finite-rate-of-innovation (FRI) signals, which may not be bandlimited nor lie in a shift-invariant space, but specified by a finite number of free parameters per unit interval. For example, consider

$$x(t) = \sum_{\ell=1}^{L} a_\ell \, g(t - t_\ell), \tag{1}$$

which is an example of a non-periodic FRI (NP-FRI) signal, where $g(t)$ is a known signal and the amplitudes and delays $\{a_\ell, t_\ell\}_{\ell=1}^{L}$ are unknown free parameters. The FRI signal model is applicable to many real-world signals such as those encountered in RADAR [2], ultrasound [3], frequency-domain optical-coherence tomography [4], etc. In this paper, we are specifically interested in ultrasound image reconstruction using the FRI signal modeling approach. In ultrasound imaging, a high-frequency (greater than 20 kHz) sound wave pulse emitted by an ultrasound transducer is used to probe a specimen (such as subcutaneous body structures). The pulse gets reflected from the specimen due to change in acoustic impedance. The reflected pulses are collected by the receiver. Tur et al. [3] showed that the ultrasound signal can be modeled as an NP-FRI signal as shown in (1), where $g(t)$ is the transmitted pulse and $x(t)$ is the received signal, L represents the number of distinct layers of the specimen assuming a piecewise-constant acoustic impedance, $\{a_\ell\}_{\ell=1}^{L}$ denote the acoustic reflectivity of each layer, and $\{t_\ell\}_{\ell=1}^{L}$ denote the round-path time delay of the transmitted pulse from each layer and is proportional to the geometric distance of the corresponding layer of specimen from a reference plane. We assume that the delays $\{t_\ell\}_{\ell=1}^{L}$ are ordered, that is, $t_1 < t_2 < \cdots < t_{L-1} < t_L$.

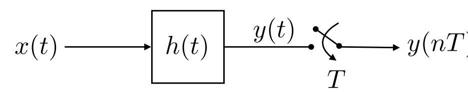

Fig. 1. Generic sampling setup.

In Figure 1, we show a generic sampling setup with $h(t)$ as the sampling kernel. The goal of FRI sampling and reconstruction method is to design the sampling kernel $h(t)$ such that, with a suitable sampling interval T, one should be able to estimate the unknown parameters $\{a_\ell, t_\ell\}_{\ell=1}^{L}$ from the samples $y(nT)$.

1.1. Related work

Vetterli et al. [1] considered sampling and reconstruction of FRI signals such as stream of Dirac impulses, nonuniform splines, differential Dirac impulses and piecewise-polynomial signals. For these classes, they proposed the use of sinc and Gaussian sampling kernels, and showed that the amplitudes and delays can be estimated from a finite number of samples by employing annihilating filter. Dragotti et al. [5] extended the class of sampling kernels to include polynomial-reproducing kernels, exponential-reproducing kernels, and kernels with rational transfer functions. Tur et al. [3] proposed a sum-of-sincs (SoS) sampling kernel and showed applications to ultrasound imaging. The SoS kernel has the advantage that it can be applied to a generic class of pulses $g(t)$ in (1). Sun [6] proposed an average sampling method for FRI signals instead of instantaneously sampling and showed that efficient reconstruction is feasible using average sampling. The performance of Sun's method in presence of noise is reported by Bi et al. [7]. Sampling and reconstruction of piecewise sinusoids and a combination of piecewise sinusoids and polynomials is shown in [8]. Seelamantula and Unser [9] proposed a multichannel sampling method for FRI signals, using a physically realizable sampling kernel based on a resistor-capacitor circuit. The reconstruction method does not require the use of the annihilating filter. Some alternative multichannel sampling approaches have been proposed by Gedalyahu et al. [10], Kusuma and Goyal [11], Olkkonen and Olkkonen [12], and Asl et al. [13]. Uriguen et al. [14] proposed the use of arbitrary sampling kernels for FRI signals, whereas Matusiak and Eldar [15] addressed the sampling and reconstruction of FRI signal with unknown pulse shapes. Multi-dimensional extensions of the FRI signal sampling were reported in [16–20].

1.2. This paper

In FRI literature, sampling and reconstruction of both periodic and non-periodic FRI signals has been reported. With the sinc and Gaussian sampling kernels [1], both periodic and non-periodic FRI signals can be reconstructed. However, these sampling kernels have infinite impulse responses and are practically unrealizable. The SoS kernel [3] proposed for periodic FRI signals has a finite duration. To extend the sampling framework to NP-FRI signals, Tur et al. [3] proposed repetition of the SoS kernel and the number of repetitions depends on the width of the pulse $g(t)$ and the maximum time-delay t_L in (1). Perfect repetition of $h(t)$ may be difficult to realize in practice. In this paper, we show that it is possible to sample and reconstruct NP-FRI signals using the SoS kernel directly without the need for repetition. We also compute the minimum sampling rate to enable accurate reconstruction. We also show applications of the SoS kernel to simulated data obtained using Field II software, which produces realistic ultrasound images, and real cardiac ultrasound image data.

2. SAMPLING AND RECONSTRUCTION USING SUM-OF-SINCS SAMPLING KERNEL

Given the FRI signal $x(t)$ in (1), we assume that $g(t)$ and L are known and that $g(t)$ is time-limited. The continuous-time Fourier transform (CTFT) of $x(t)$ is given by

$$X(\omega) = G(\omega) \sum_{\ell=1}^{L} a_\ell \, e^{-\mathrm{j}\omega t_\ell}, \tag{2}$$

where $G(\omega)$ is the CTFT of $g(t)$. Consider the frequency-domain function $F(\omega) = \frac{X(\omega)}{G(\omega)}$ and samples of $F(\omega)$ at locations $k\omega_0$ (assuming $G(k\omega_0) \neq 0$), for integer $k \in \mathcal{K}$. The set $\mathcal{K}$ is a set of consecutive integers of cardinality greater than or equal to $2L$. The samples are given by,

$$F(k\omega_0) = \sum_{\ell=1}^{L} a_\ell \, e^{-\mathrm{j}k\omega_0 t_\ell}. \tag{3}$$

If $\omega_0 \, t_L < 2\pi$ then from the samples of $F(k\omega_0)$, we can uniquely determine the delays $\{t_\ell\}_{\ell=1}^{L}$ by employing the annihilating filter method [21]. Once the delays are estimated, the amplitudes $\{a_\ell\}_{\ell=1}^{L}$ can be calculated using a least-squares (LS) approach. Hence, the goal is to design a sampling kernel to obtain $F(k\omega_0)$ for $k \in \mathcal{K}$ from $x(t)$.

Since $x(t)$ is not bandlimited, sampling $x(t)$ directly at any sampling rate will give rise to aliasing at all frequencies of $X(\omega)$. Due to aliasing, one cannot measure samples of $X(\omega)$ from discrete-time Fourier transform (DTFT) of sampled signal $y(nT)$. If we employ an ideal lowpass filter (anti-aliasing filter) before sampling and take the samples at Nyquist rate, we can measure the samples of $X(\omega)$ from DTFT of $y(nT)$ if $k\omega_0$ lies in the passband of the lowpass filter [1] for $k \in \mathcal{K}$. However, an ideal lowpass filter is not realizable, and hence, we need a practical alternative. Since the requirement is to estimate $2L$ samples of $X(\omega)$, it is desirable to design a finite-duration sampling kernel and choose the sampling rate such that there is no aliasing at frequencies $k\omega_0$ for $k \in \mathcal{K}$. In this case, we show that $X(k\,\omega_0)$ can be computed directly from the frequency-domain samples of $y(nT)$.

2.1. Estimating frequency samples of NP-FRI signal by SoS filter

Tur et al. [3] showed that by applying a sum-of-sincs (SoS) sampling kernel, one can measure the Fourier series coefficients of periodic FRI signals. The frequency response of the SoS kernel is

$$H(\omega) = \sum_{p=-L}^{L} \operatorname{sinc}\left(\frac{\omega}{\omega_0} - p\right), \tag{4}$$

and the corresponding time-domain representation is

$$h(t) = \operatorname{rect}\left(\frac{t}{T_0}\right) \sum_{p=-L}^{L} e^{\mathrm{j}p\omega_0 t}, \tag{5}$$

where $T_0 = \frac{2\pi}{\omega_0}$ and $\operatorname{rect}\left(\frac{t}{T_0}\right) = 1$ for $\frac{-T_0}{2} \leq t \leq \frac{T_0}{2}$, and zero elsewhere. The SoS kernel in (5) nullifies the aliasing at frequencies $k\omega_0$ for $k \in \mathcal{K}$ (where $\mathcal{K} = \{-L, -L+1, -L+2, \cdots, L-1, L\}$) provided that the sampling rate is chosen appropriately. From the DTFT of samples $y(nT)$, one can measure $X(k\omega_0)$ for $k \in \mathcal{K}$.

For NP-FRI signals in (1), Tur et al. [3] proposed a repetition of SoS kernel in (5) as follows:

$$h_r(t) = \sum_{q=-M}^{M} h(t - qT_r), \tag{6}$$

where M is measure of number of repetitions and depends on duration of the pulse $g(t)$ and the maximum time delay t_L in (1). The period of repetition T_r depends on t_L.

In this paper, we show that, one can estimate the frequency samples $X(k\omega_0)$ using the SoS kernel directly in (5) without requiring to perform any repetitions of $h(t)$.

The CTFT of filtered signal is $Y(\omega) = X(\omega)\,H(\omega)$ and the Fourier transform of the sampled signal with sampling interval T is given by,

$$Y_s(\omega) = \sum_{m\in\mathbb{Z}} \sum_{p=-L}^{L} X(\omega + m\omega_s) \operatorname{sinc}\left(\frac{\omega + m\omega_s}{\omega_0} - p\right), \tag{7}$$

where $\omega_s = \frac{2\pi}{T}$. If we sample $Y_s(\omega)$ at $\omega = k_0\omega_0$ and choose the sampling frequency as $\omega_s = N\,\omega_0$, where $k_0 \in \mathcal{K}$ and $N \in \mathbb{Z}^+$,

$$\begin{aligned} Y_s(k_0\omega_0) &= \sum_{m\in\mathbb{Z}} \sum_{p=-L}^{L} X(k_0\omega_0 + mN\omega_0) \operatorname{sinc}(k_0 + mN - p) \\ &= \sum_{p=-L}^{L} X(k_0\omega_0) \operatorname{sinc}(k_0 - p) \\ &+ \sum_{\substack{m\in\mathbb{Z} \\ m\neq 0}} \sum_{p=-L}^{L} X(k_0\omega_0 + mN\omega_0) \operatorname{sinc}(k_0 + mN - p)\,. \end{aligned} \tag{8}$$

For $k_0 \in \mathcal{K} = \{-L, -L+1, \cdots, L-1, L\}$, the first summation will have only one non-zero term at $p = k_0$. We can write (8) as

$$Y_s(k_0\omega_0) = X(k_0\omega_0) + \sum_{\substack{m\in\mathbb{Z}\\ m\neq 0}} \sum_{p=-L}^{L} X(k_0\omega_0 + mN\omega_0)\,\mathrm{sinc}\,(k_0 + mN - p)\,. \quad (9)$$

The double summation term on the right hand side of (9) will be zero if $p - k_0 \neq mN$ for all $m \in \mathbb{Z} - \{0\}$ and $k_0 \in \mathcal{K}$. Since the maximum value of $|p - k_0|$ is $2L$, if we choose N such that $N > 2L$, then the double summation term in (9) will be zero for all $m \in \mathbb{Z} - \{0\}$. Hence, with SoS sampling kernel in (4) and sampling rate $\omega_s \geq (2L+1)\omega_0$, the aliasing effect at frequencies $k\omega_0$ for $k \in \mathcal{K}$ is nullified.

The next step is to estimate $\{X(k\omega_0)\}_{k=-L}^{L}$ from samples $y(nT)$. Since the FRI signal $x(t)$ and sampling kernel $h(t)$ are of finite duration, the sampled signal $y(t)$ has finite support and hence there are finite number of non-zero samples $y(nT)$. Measuring DTFT of the sequence $y(nT)$ at frequencies $k\omega_0$ will give the CTFT of $x(t)$ at $k\omega_0$.

2.2. Estimating delays and amplitudes of FRI signal

The frequency-domain samples of the FRI signal in (1) are given as

$$X(k\omega_0) = G(k\omega_0) \sum_{\ell=1}^{L} a_\ell\, e^{-\mathrm{j}k\omega_0 t_\ell}. \quad (10)$$

Since $g(t)$ is a time-limited signal, it is not bandlimited, and we can choose ω_0 such that $G(k\omega_0) \neq 0$ for $k \in \mathcal{K}$ and $\omega_0 t_L < 2\pi$. From $X(k\omega_0)$ and $G(k\omega_0)$ we can find samples $F(k\omega_0)$ defined in (3). These $2L+1$ samples are of the form of a weighted sum of complex exponentials. Estimating the frequencies of the complex exponentials is a high-resolution spectral estimation problem [22] and can be solved using Prony's annihilating filter [21] method. Once the frequencies (and hence, the pulse delays) are estimated, the amplitudes are estimated by performing least-squares regression.

In the presence of measurement noise and modeling error, we employ Cadzow's [23] denoising method to boost the performance of annihilating filter. In this scenario, we require more number of samples (given by oversampling factor) than the rate of innovation to estimate the delays and amplitudes.

3. EXPERIMENTAL RESULTS

We next validate the proposed method on simulated as well as experimental data.

3.1. Simulated NP-FRI signal

The FRI signal in this experiment is a stream of five Dirac impulses with time delays and amplitudes selected randomly between $(0, 1)$ and $(0, 3)$, respectively. One such realization of FRI signal is shown in Figure 2(a). The SoS sampling kernel for $L = 5$ with $\omega_0 = \frac{2\pi}{0.99}$ is plotted in Figure 2(b). The filtered output of the input FRI signal and its samples taken at a rate $\omega_s = (2L+1)\omega_0$ are displayed in Figure 2(c). From these samples, we compute DTFT at frequencies $\{k\omega_0\}_{k=-L}^{L}$ and estimate $F(k\omega_0)$. We employed annihilating filter method on $F(k\omega_0)$ and estimated the delays $\{t_\ell\}_{\ell=1}^{L}$ and subsequently applied least-squares (LS) method to estimate the amplitudes $\{a_\ell\}_{\ell=1}^{L}$. The reconstructed signal is shown in Figure 2(d).

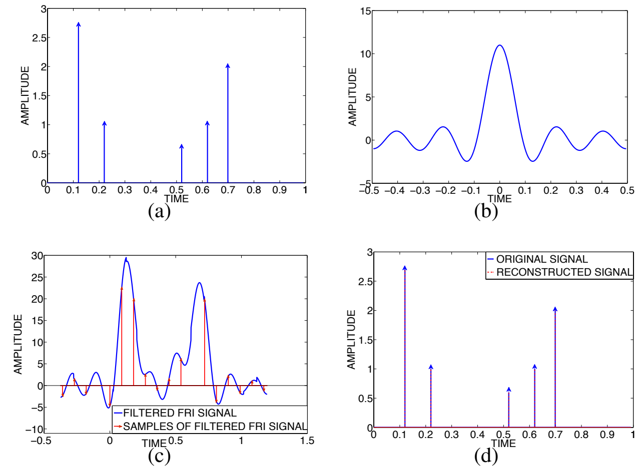

Fig. 2. (a) Stream of five Dirac impulses, (b) SoS sampling kernel with $L = 5$ and $\omega_0 = \frac{2\pi}{0.99}$, (c) Filtered output (in blue) and its samples (in red), and (d) Original (in blue) and reconstructed (in red) FRI signal.

We observe perfect reconstruction of the FRI signal up to numerical precision.

3.2. Simulated ultrasound signals

This experiment is designed to verify the FRI model for ultrasound signals and applicability of SoS sampling kernel for ultrasound image reconstruction, where the number of samples is proportional to the innovation rate of the signal. The ultrasound signal is generated using the open source simulation software Field II [24, 25]. An imaging system comprising a linear array of 64 elements is used to scan an artificial specimen. The specimen (cf. Figure 4(a)) consists of randomly located pins (to mimic bones or high reflection regions in the human body) and cysts (to mimic blood, water and such low reflection regions). The excitation pulse is a truncated sinusoid of frequency 4 MHz. The transducer sampling frequency is set at 40 MHz and the ultrasound beam is focussed to a depth of 10 cm. At 40 MHz sampling frequency, this results in 5194 samples per scan line. A reasonable approximation to the reflected pulses was found to be a truncated Gaussian with a standard deviation of 0.001. A portion of the received signal of a typical scanline is shown in Figure 3(a). On applying the proposed sampling technique on the scanline in Figure 3(a), the reconstruction with $L = 20$ and $L = 80$ are shown in Figure 3(b) and Figure 3(c), respectively. Due to the presence of measurement noise and model mismatch, we employ Cadzow denoising method prior to annihilating filter with an oversampling factor of 4. The number of samples used in the reconstruction of Figure 3(b) and Figure 3(c) are 160 (sampling rate 1.24 MHz) and 640 (sampling rate 4.93 MHz), respectively (measured by downsampling the original signal). On applying the proposed FRI sampling and reconstruction technique to all the scan lines and delay-and-sum beamforming, the reconstructed ultrasound images for simulated specimen are shown in Figure 4(b) and 4(c) for different choices of L. We observe that, by sampling the ultrasound signal with an SoS sampling kernel and applying the FRI method, the signal can be reconstructed accurately with a fewer number of samples as compared with the conventional approach (cf. Figure 4(a)). The reduction (compression ratio) in the number of samples used in re-

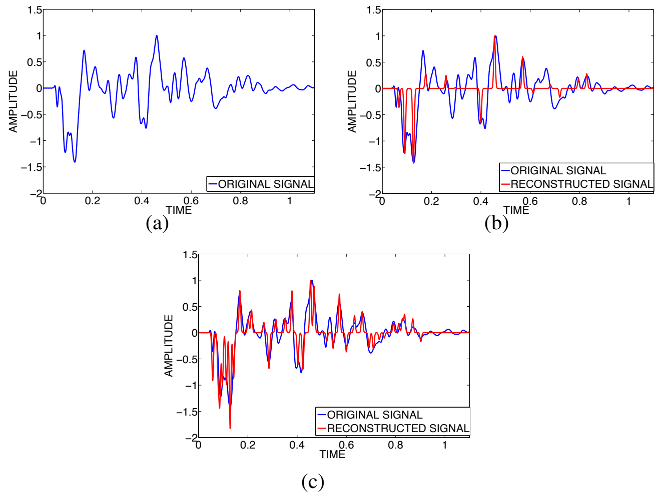

Fig. 3. (a) Ultrasound signal generated using Field II; Ultrasound signals reconstructed with model order (b) $L = 20$, and (c) $L = 80$.

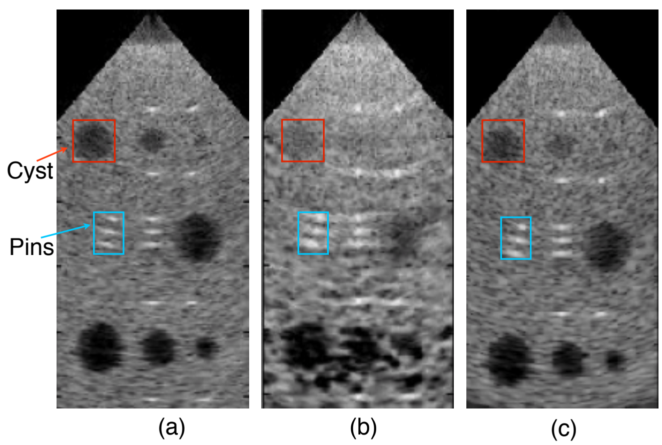

Fig. 4. Experiment results on ultrasound imaging on simulated specimen: (a) Original specimen with cysts (highlighted by red boxes) and pins (highlighted by blue boxes) constructed from 5194 samples at a sampling rate of 40 MHz; reconstructed images with FRI sampling and SoS kernel with (b)160 samples at a sampling rate 1.24 MHz and model order $L = 20$, and (c) 640 samples at a sampling rate of 4.93 MHz and model order $L = 80$.

construction are of the order of 32 for $L = 20$ and 8 for $L = 80$, respectively. As the model order increases, the reconstruction accuracy improves as highlighted by the red and blue boxes shown in Figure 4.

3.3. Real cardiac ultrasound signals

We next evaluate the performance of the proposed sampling and reconstruction method on beamformed data obtained from a General Electric (GE) ultrasound scanner. The image obtained using the standard reconstruction method is shown in Figure 5(a). The filtering with SoS kernel and subsequent sampling is simulated digitally starting with very finely sampled measurements obtained from the GE scanner. The data set consists of 81 scanlines with 1850 samples per scanline. The proposed FRI sampling technique is applied on each scanline of the beamformed data and the shape of the basic FRI pulse is chosen to be a truncated Gaussian with a standard deviation 0.001. The reconstructed ultrasound images are shown in Figure 5(b) and Figure 5(c) for $L = 40$ and $L = 80$, respectively. An oversampling factor of 4 is chosen and Cadzow denoising is applied to overcome noise and model mismatch before employing the annihilation filter method. The compression ratio in reconstructing the ultrasound images by FRI method is 5.7 for $L = 40$ and 2.8 for $L = 80$, respectively.

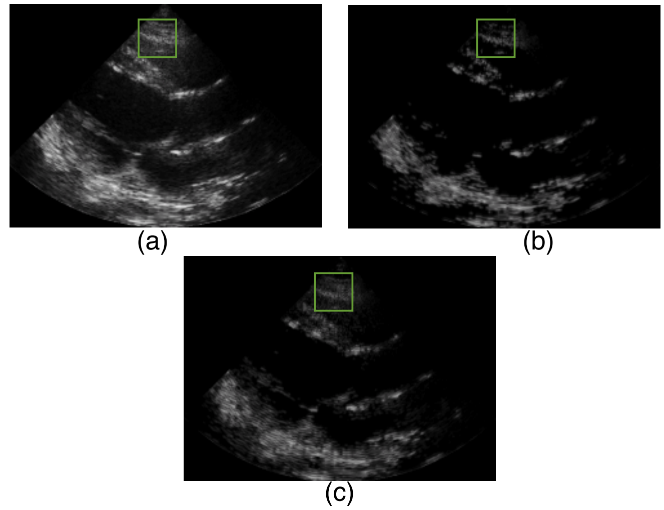

Fig. 5. Validation on real cardiac data: (a) Standard reconstruction obtained with 1850 samples; ultrasound image obtained with FRI sampling and SoS kernel corresponding to (b) 320 samples and model order $L = 40$, and (c) 640 samples and model order $L = 80$.

We observe that, the bright regions in Figure 5(a) (which signifies the chambers of the heart) are recovered in Figure 5(b), but the grey regions corresponding to the smaller amplitude pulses are missed in reconstruction. The areas of the grey regions captured in Figure 5(c) increases when the model order is increased. Thus, the model order is a convenient tool to control the amount of grey scatterers a medical practitioner would want to see. There are practicle occasions when the medical practitioner is interested in observing only the chambers of the heart and not the occluding grey regions. Hence, the FRI sampling and reconstruction method offers an attractive way to also selectively highlight regions of interest in the reconstructed ultrasound image.

4. CONCLUSIONS

We proposed an aperiodic version of the SoS kernel for sampling and reconstructing non-periodic FRI signals. The reconstruction is achieved by canceling the aliasing at frequencies of interest by carefully choosing the sampling rate. The problem then boils down to reconstructing the parameters of a sum of exponentials, which was accomplished by using the annihilating filter method. In the presence of noise, Cadzow's denoising technique was employed to boost the signal-to-noise ratio. We validated the proposed methodology on MATLAB simulated data, signals generated using Field II software, as well as real cardiac ultrasound measurements. We showed that the quality of reconstruction is comparable to the standard reconstruction method, even though the number of samples actually used in the reconstruction is about 10 times less. The model order further helps control the quality of reconstruction.

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