2D SEMI-SUPERVISED CCA-BASED INPAINTING INCLUDING NEW PRIORITY ESTIMATION

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ABSTRACT

This paper presents an inpainting method based on 2D semi-supervised canonical correlation analysis (2D semi-CCA) including new priority estimation. The proposed method estimates relationship, i.e., the optimal correlation, between missing area and its neighboring area from known parts within the target image by using 2D CCA. In this approach, we newly introduce a semi-supervised scheme into the 2D CCA for deriving the 2D semi-CCA which corresponds to a hybrid version of 2D CCA and 2D principle component analysis (2D PCA). This enables successful relationship estimation even if sufficient number of training pairs cannot be provided. Then, by using the obtained relationship, accurate estimation of the missing intensities can be realized. Furthermore, in the proposed method, errors caused in the new variate space obtained by the 2D semi-CCA are effectively used for deriving patch priority determining inpainting order of missing areas. Experimental results show our inpainting method can outperform previously reported methods.

Index Terms— Inpainting, canonical correlation analysis, semi-supervised scheme, texture reconstruction.

1. INTRODUCTION

Inpainting has intensively been studied in the field of image processing since it can afford a number of fundamental applications [1]–[19]. Most of the methods are broadly classified into several categories: structure reconstruction methods [1]–[4] including partial differential equation (PDE)-based approaches, exemplar-based methods [6]–[10], multivariate analysis-based reconstruction methods [11]–[19], etc. Generally, structure reconstruction methods enable successful restoration in edge regions. On the other hand, exemplar-based methods and multivariate analysis-based methods tend to output better results in texture regions. It is well known that the multivariate analysis-based methods can restore missing textures more successfully compared to the exemplar-based methods when sufficient number of training examples cannot be provided. The remainder of this paper focuses on missing texture restoration using the multivariate analysis-based methods with discussion of its details.

There have traditionally been proposed many missing texture restoration methods using multivariate analysis, and they are based on texture approximation using various methods such as principal component analysis (PCA), kernel PCA (KPCA) and sparse representation. For example, Amano et al. proposed an effective PCA-based method that estimates missing textures by back projection for lost pixels [11]. Furthermore, by applying the kernel methods to PCA [20, 21], its improvement can be also realized [12, 13, 14]. Recently, image restoration based on the sparse representation has been studied [15]–[19], [22, 23]. Mairal et al. proposed a representative work based on the sparse representation [15], and several improved methods have been proposed as state-of-the-art methods [16]–[19]. As described above, although the multivariate analysis-based methods can output better results compared to the exemplar-based methods from few training examples, their performance still depends on the number of training examples. Therefore, as the number of training examples decreases, it becomes difficult to successfully grasp the relationship between missing areas and their neighboring areas. Then the restoration performance tends to be degraded.

In this paper, we present a new inpainting method based on 2D semi-supervised canonical correlation analysis (2D semi-CCA). In recent years, it has been reported that 2D multivariate analysis methods such as 2D PCA [24] and 2D CCA [25] can represent visual features successfully. Therefore, in the proposed method, we introduce the 2D CCA into the inpainting for estimating the relationship, i.e., the optimal correlation, between missing areas and their neighboring areas. Furthermore, in order to solve the conventional problem of not being able to successfully grasp the relationship when sufficient number of training examples cannot be provided, the proposed method newly derives a restoration algorithm based on semi-CCA [26]. Semi-CCA is a hybrid version of CCA and PCA and enables accurate relationship estimation even if sufficient number of training pairs cannot be provided. Therefore, the proposed method derives the 2D Semi-CCA for successfully obtaining the relationship to restore missing areas. Furthermore, the proposed method monitors errors caused in the new variate space obtained by the 2D semi-CCA to derive patch priority determining inpainting order of missing areas. Since these errors correspond to approximation performance, i.e., restoration performance, the proposed method adopts them as the confidence values for determining the patch priority. Consequently, the proposed method realizes successful inpainting by introducing the nonconventional approaches using the 2D semi-CCA.

2. INPAINTING BASED ON 2D SEMI-CCA

We present the inpainting method based on 2D semi-CCA in this section. First, in our method, a patch f ($w \times h$ pixels) including missing areas Ω is clipped from the target image. Next, from the known areas $\bar{\Omega}$ within the target patch f, we try to estimate the intensities in its missing areas Ω by using the relationship of these two areas Ω and $\bar{\Omega}$ obtained from the other known parts based on the 2D semi-CCA. Specifically, the proposed method estimates coefficient matrices and canonical correlation matrices for obtaining the

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relationship (See **2.1**). Then, by using the obtained relationship, the intensities within the missing areas Ω are estimated (See **2.2**). The overview of the proposed method is shown in Fig. 1.

For the following explanation, we define two matrices, which respectively surround areas $\bar{\Omega}$ and Ω, as $\mathbf{X}$ ($\mathcal{R}^{m_x \times n_x}$) and $\mathbf{Y}$ ($\mathcal{R}^{m_y \times n_y}$). Note that the elements not belonging to $\bar{\Omega}$ and Ω in the matrices $\mathbf{X}$ and $\mathbf{Y}$, respectively, are set to zero.

2.1. Relationship Estimation Based on 2D semi-CCA

In this subsection, we show the relationship estimation based on the 2D semi-CCA. First, from the target image, we clip patches f_i in the same intervals and define all indices of the clipped patches as $\mathcal{I} \in \{1, 2, \cdots, N\}$. Then, for each patch f_i, matrices respectively corresponding to $\mathbf{X}$ and $\mathbf{Y}$ are defined as $\tilde{\mathbf{X}}_i$ and $\tilde{\mathbf{Y}}_i$, respectively. Furthermore, their centered matrices are respectively defined as $\mathbf{X}_i = \tilde{\mathbf{X}}_i - \mathbf{M}_x$ and $\mathbf{Y}_i = \tilde{\mathbf{Y}}_i - \mathbf{M}_y$, where $\mathbf{M}_x = \frac{1}{|\mathcal{J}_x|}\sum_{i\in\mathcal{J}_x}\tilde{\mathbf{X}}_i$ and $\mathbf{M}_y = \frac{1}{|\mathcal{J}_y|}\sum_{i\in\mathcal{J}_y}\tilde{\mathbf{Y}}_i$, where we define the following three index sets:

- $\mathcal{J}_{xy}$: Indices i that $\tilde{\mathbf{X}}_i$ and $\tilde{\mathbf{Y}}_i$ are both known,
- $\mathcal{J}_x$: Indices i that $\tilde{\mathbf{X}}_i$ are known,
- $\mathcal{J}_y$: Indices i that $\tilde{\mathbf{Y}}_i$ are known.

Note that they satisfy the following conditions: $\mathcal{J}_{xy} \subset \mathcal{J}_x \subset \mathcal{I}$ and $\mathcal{J}_{xy} \subset \mathcal{J}_y \subset \mathcal{I}$.

When considering the 2D CCA [25], the following optimization problem is provided:

$$\begin{aligned}\left\{\hat{\mathbf{l}}_x, \hat{\mathbf{r}}_x, \hat{\mathbf{l}}_y, \hat{\mathbf{r}}_y\right\} &= \arg\max_{\mathbf{l}_x,\mathbf{r}_x,\mathbf{l}_y,\mathbf{r}_y} \mathrm{cov}_{i\in\mathcal{J}_{xy}}\left(\mathbf{l}_x'\mathbf{X}_i\mathbf{r}_x, \mathbf{l}_y'\mathbf{Y}_i\mathbf{r}_y\right)\\ &\quad \text{s.t. } \mathrm{var}\left(\mathbf{l}_x'\mathbf{X}_i\mathbf{r}_x\right) = 1 \text{ and } \mathrm{var}\left(\mathbf{l}_y'\mathbf{Y}_i\mathbf{r}_y\right) = 1.\end{aligned}$$

On the other hand, when considering the 2D PCA [24], the following optimization problems are respectively provided:

$$\left\{\hat{\mathbf{l}}_x, \hat{\mathbf{r}}_x\right\} = \arg\max_{\mathbf{l}_x,\mathbf{r}_x} \mathrm{cov}_{i\in\mathcal{J}_x}\left(\mathbf{l}_x'\mathbf{X}_i\mathbf{r}_x, \mathbf{l}_x'\mathbf{X}_i\mathbf{r}_x\right) \text{ s.t. } \left\|\mathbf{l}_x\right\|^2 = 1,\ \left\|\mathbf{r}_x\right\|^2 = 1,$$

$$\left\{\hat{\mathbf{l}}_y, \hat{\mathbf{r}}_y\right\} = \arg\max_{\mathbf{l}_y,\mathbf{r}_y} \mathrm{cov}_{i\in\mathcal{J}_y}\left(\mathbf{l}_y'\mathbf{Y}_i\mathbf{r}_y, \mathbf{l}_y'\mathbf{Y}_i\mathbf{r}_y\right) \text{ s.t. } \left\|\mathbf{l}_y\right\|^2 = 1,\ \left\|\mathbf{r}_y\right\|^2 = 1.$$

The semi-CCA can be regarded as a hybrid method of CCA and PCA [26]. Therefore, the combined optimization problem is utilized. Note that in the two-dimensional method, it is difficult to simultaneously obtain the optimal results $\hat{\mathbf{l}}_x, \hat{\mathbf{r}}_x, \hat{\mathbf{l}}_y, \hat{\mathbf{r}}_y$. Thus, in the proposed method, we fix each side and estimate the remaining side's transforms. The details are shown below.

[Left Side Derivation]
By fixing the right side vectors $\hat{\mathbf{r}}_x$ and $\hat{\mathbf{r}}_y$, we derive the following Lagrange multiplier approach in the left side of the 2D semi-CCA:

$$\begin{aligned}\mathcal{L}^l &= \alpha\left\{\mathbf{l}_x'\mathbf{C}_{xy}^r\mathbf{l}_y - \frac{\lambda^l}{2}\left(\mathbf{l}_x'\mathbf{C}_{xx}^r\mathbf{l}_x - 1\right) - \frac{\lambda^l}{2}\left(\mathbf{l}_y'\mathbf{C}_{yy}^r\mathbf{l}_y - 1\right)\right\}\\ &\quad +(1-\alpha)\left[\frac{1}{2}\left\{\mathbf{l}_x'\mathbf{D}_{xx}^r\mathbf{l}_x - \lambda^l\left(\left\|\mathbf{l}_x\right\|^2 - 1\right)\right\}\right.\\ &\quad \left. +\frac{1}{2}\left\{\mathbf{l}_y'\mathbf{D}_{yy}^r\mathbf{l}_y - \lambda^l\left(\left\|\mathbf{l}_y\right\|^2 - 1\right)\right\}\right], \qquad (1)\end{aligned}$$

where α is a parameter, $\mathbf{C}_{xy}^r = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}}\mathbf{X}_i\hat{\mathbf{r}}_x\hat{\mathbf{r}}_y'\mathbf{Y}_i'$ ($= \mathbf{C}_{yx}^{r}{}'$), $\mathbf{C}_{xx}^r = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}}\mathbf{X}_i\hat{\mathbf{r}}_x\hat{\mathbf{r}}_x'\mathbf{X}_i'$, $\mathbf{C}_{yy}^r = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}}\mathbf{Y}_i\hat{\mathbf{r}}_y\hat{\mathbf{r}}_y'\mathbf{Y}_i'$, $\mathbf{D}_{xx}^r = \frac{1}{|\mathcal{J}_x|}\sum_{i\in\mathcal{J}_x}\mathbf{X}_i\hat{\mathbf{r}}_x\hat{\mathbf{r}}_x'\mathbf{X}_i'$ and $\mathbf{D}_{yy}^r = \frac{1}{|\mathcal{J}_y|}\sum_{i\in\mathcal{J}_y}\mathbf{Y}_i\hat{\mathbf{r}}_y\hat{\mathbf{r}}_y'\mathbf{Y}_i'$. In order to obtain the optimal vectors of $\mathbf{l}_x$ and $\mathbf{l}_y$ from Eq. (1), we calculate

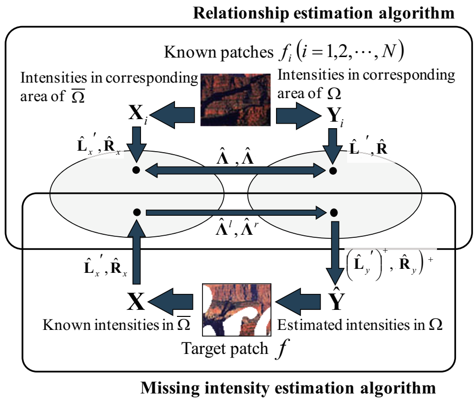

Fig. 1. Overview of the proposed inpainting method.

$\frac{\partial\mathcal{L}^l}{\partial\mathbf{l}_x} = 0$ and $\frac{\partial\mathcal{L}^l}{\partial\mathbf{l}_y} = 0$, and then, the following generalized eigenvalue problem can be derived:

$$\begin{aligned}&\left\{\alpha\begin{bmatrix}\mathbf{0} & \mathbf{C}_{xy}^r\\ \mathbf{C}_{yx}^r & \mathbf{0}\end{bmatrix} + (1-\alpha)\begin{bmatrix}\mathbf{D}_{xx}^r & \mathbf{0}\\ \mathbf{0} & \mathbf{D}_{yy}^r\end{bmatrix}\right\}\begin{bmatrix}\mathbf{l}_x\\ \mathbf{l}_y\end{bmatrix}\\ &\quad = \lambda^l\left\{\alpha\begin{bmatrix}\mathbf{C}_{xx}^r & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_{yy}^r\end{bmatrix} + (1-\alpha)\begin{bmatrix}\mathbf{I}_{\mathbf{l}_x} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{\mathbf{l}_y}\end{bmatrix}\right\}\begin{bmatrix}\mathbf{l}_x\\ \mathbf{l}_y\end{bmatrix}, \qquad (2)\end{aligned}$$

where $\mathbf{I}_{\mathbf{l}_x}$ ($\in \mathcal{R}^{m_x\times m_x}$) and $\mathbf{I}_{\mathbf{l}_y}$ ($\in \mathcal{R}^{m_y\times m_y}$) are the identity matrices. By solving Eq. (2), the optimal vectors $\hat{\mathbf{l}}_x$ and $\hat{\mathbf{l}}_y$ are obtained.

[Right Side Derivation]
In the same way as Eq. (2) in the left side derivation, by fixing the left side vectors $\hat{\mathbf{l}}_x$ and $\hat{\mathbf{l}}_y$, we derive the following generalized eigenvalue problem:

$$\begin{aligned}&\left\{\beta\begin{bmatrix}\mathbf{0} & \mathbf{C}_{xy}^l\\ \mathbf{C}_{yx}^l & \mathbf{0}\end{bmatrix} + (1-\beta)\begin{bmatrix}\mathbf{D}_{xx}^l & \mathbf{0}\\ \mathbf{0} & \mathbf{D}_{yy}^l\end{bmatrix}\right\}\begin{bmatrix}\mathbf{r}_x\\ \mathbf{r}_y\end{bmatrix}\\ &\quad = \lambda^r\left\{\beta\begin{bmatrix}\mathbf{C}_{xx}^l & \mathbf{0}\\ \mathbf{0} & \mathbf{C}_{yy}^l\end{bmatrix} + (1-\beta)\begin{bmatrix}\mathbf{I}_{\mathbf{r}_x} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{\mathbf{r}_y}\end{bmatrix}\right\}\begin{bmatrix}\mathbf{r}_x\\ \mathbf{r}_y\end{bmatrix}, \qquad (3)\end{aligned}$$

where β is a parameter, $\mathbf{C}_{xy}^l = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}}\mathbf{X}_i'\hat{\mathbf{l}}_x\hat{\mathbf{l}}_y'\mathbf{Y}_i$ ($= \mathbf{C}_{yx}^{l}{}'$), $\mathbf{C}_{xx}^l = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}}\mathbf{X}_i'\hat{\mathbf{l}}_x\hat{\mathbf{l}}_x'\mathbf{X}_i$, $\mathbf{C}_{yy}^l = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}}\mathbf{Y}_i'\hat{\mathbf{l}}_y\hat{\mathbf{l}}_y'\mathbf{Y}_i$, $\mathbf{D}_{xx}^l = \frac{1}{|\mathcal{J}_x|}\sum_{i\in\mathcal{J}_x}\mathbf{X}_i'\hat{\mathbf{l}}_x\hat{\mathbf{l}}_x'\mathbf{X}_i$ and $\mathbf{D}_{yy}^l = \frac{1}{|\mathcal{J}_y|}\sum_{i\in\mathcal{J}_y}\mathbf{Y}_i'\hat{\mathbf{l}}_y\hat{\mathbf{l}}_y'\mathbf{Y}_i$. Furthermore, $\mathbf{I}_{\mathbf{r}_x}$ ($\in \mathcal{R}^{n_x\times n_x}$) and $\mathbf{I}_{\mathbf{r}_y}$ ($\in \mathcal{R}^{n_y\times n_y}$) are the identity matrices. By solving Eq. (3), the optimal vectors $\hat{\mathbf{r}}_x$ and $\hat{\mathbf{r}}_y$ are obtained.

[Iterative Algorithm for Solving Both Sides]
The proposed method iteratively estimates the optimal transforms of one side by fixing the other side's transforms. Furthermore, when calculating the transforms of PCA or CCA, we generally do not calculate only the vectors $\hat{\mathbf{l}}_x$, $\hat{\mathbf{l}}_y$, $\hat{\mathbf{r}}_x$ and $\hat{\mathbf{r}}_y$, but the multi-dimensional matrices $\hat{\mathbf{L}}_x$, $\hat{\mathbf{L}}_y$, $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_y$. Therefore, Eqs. (2) and (3), i.e., the problems of the left and right sides, are respectively rewritten as

$$\begin{aligned}&\left\{\alpha\begin{bmatrix}\mathbf{0} & \hat{\mathbf{C}}_{xy}^r\\ \hat{\mathbf{C}}_{yx}^r & \mathbf{0}\end{bmatrix} + (1-\alpha)\begin{bmatrix}\hat{\mathbf{D}}_{xx}^r & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{D}}_{yy}^r\end{bmatrix}\right\}\begin{bmatrix}\mathbf{L}_x\\ \mathbf{L}_y\end{bmatrix}\\ &\quad = \mathbf{\Lambda}^l\left\{\alpha\begin{bmatrix}\hat{\mathbf{C}}_{xx}^r & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{C}}_{yy}^r\end{bmatrix} + (1-\alpha)\begin{bmatrix}\mathbf{I}_{\mathbf{l}_x} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{\mathbf{l}_y}\end{bmatrix}\right\}\begin{bmatrix}\mathbf{L}_x\\ \mathbf{L}_y\end{bmatrix}, \qquad (4)\end{aligned}$$

$$\begin{aligned}&\left\{\beta\begin{bmatrix}\mathbf{0} & \hat{\mathbf{C}}_{xy}^l\\ \hat{\mathbf{C}}_{yx}^l & \mathbf{0}\end{bmatrix} + (1-\beta)\begin{bmatrix}\hat{\mathbf{D}}_{xx}^l & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{D}}_{yy}^l\end{bmatrix}\right\}\begin{bmatrix}\mathbf{R}_x\\ \mathbf{R}_y\end{bmatrix}\\ &\quad = \mathbf{\Lambda}^r\left\{\beta\begin{bmatrix}\hat{\mathbf{C}}_{xx}^l & \mathbf{0}\\ \mathbf{0} & \hat{\mathbf{C}}_{yy}^l\end{bmatrix} + (1-\beta)\begin{bmatrix}\mathbf{I}_{\mathbf{r}_x} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_{\mathbf{r}_y}\end{bmatrix}\right\}\begin{bmatrix}\mathbf{R}_x\\ \mathbf{R}_y\end{bmatrix}, \qquad (5)\end{aligned}$$

Input
- Two dimensional data $\left\{\tilde{\mathbf{X}}_i \in \mathcal{R}^{m_x \times n_x}\right\}$ and $\left\{\tilde{\mathbf{Y}}_i \in \mathcal{R}^{m_y \times n_y}\right\}$
- The size of the canonical variable matrix: $d_l \times d_r$

Output
- Left transforms $\hat{\mathbf{L}}_x \in \mathcal{R}^{m_x \times d_l}$, $\hat{\mathbf{L}}_y \in \mathcal{R}^{m_y \times d_l}$
- Right transforms $\hat{\mathbf{R}}_x \in \mathcal{R}^{n_x \times d_r}$, $\hat{\mathbf{R}}_y \in \mathcal{R}^{n_y \times d_r}$
- Correlation matrices $\hat{\mathbf{\Lambda}}^l \in \mathcal{R}^{d_l \times d_l}$, $\hat{\mathbf{\Lambda}}^r \in \mathcal{R}^{d_r \times d_r}$

Do centering the two-dimensional data to get $\{\mathbf{X}_i\}$ and $\{\mathbf{Y}_i\}$.
Initialize $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_y$.
Repeat
- Compute the following matrices: $\hat{\mathbf{C}}^r_{xy}$ $(= \hat{\mathbf{C}}^r_{yx}{}')$, $\hat{\mathbf{C}}^r_{xx}$, $\hat{\mathbf{C}}^r_{yy}$, $\hat{\mathbf{D}}^r_{xx}$ and $\hat{\mathbf{D}}^r_{yy}$.
- Compute d_l largest eigenvalues $\hat{\mathbf{\Lambda}}^l$ and their corresponding generalized eigenvectors $\hat{\mathbf{L}}_x$ and $\hat{\mathbf{L}}_y$ by Eq. (4).
- Compute the following matrices: $\hat{\mathbf{C}}^l_{xy}$ $(= \hat{\mathbf{C}}^l_{yx}{}')$, $\hat{\mathbf{C}}^l_{xx}$, $\hat{\mathbf{C}}^l_{yy}$, $\hat{\mathbf{D}}^l_{xx}$ and $\hat{\mathbf{D}}^l_{yy}$.
- Compute d_r largest eigenvalues $\hat{\mathbf{\Lambda}}^r$ and their corresponding generalized eigenvectors $\hat{\mathbf{R}}_x$ and $\hat{\mathbf{R}}_y$ by Eq. (5).

until (converged)

Fig. 2. Outline algorithm of the 2D semi-CCA based relationship estimation.

where $\mathbf{\Lambda}^l$ and $\mathbf{\Lambda}^r$ are respectively eigenvalue matrices whose diagonal elements contain the correlation coefficients. Several matrices shown in the above two equations are defined below. In Eq. (4), $\hat{\mathbf{C}}^r_{xy} = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}} \mathbf{X}_i\hat{\mathbf{R}}_x\hat{\mathbf{R}}_y'\mathbf{Y}_i'$ $(= \hat{\mathbf{C}}^r_{yx}{}')$, $\hat{\mathbf{C}}^r_{xx} = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}} \mathbf{X}_i\hat{\mathbf{R}}_x\hat{\mathbf{R}}_x'\mathbf{X}_i'$, $\hat{\mathbf{C}}^r_{yy} = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}} \mathbf{Y}_i\hat{\mathbf{R}}_y\hat{\mathbf{R}}_y'\mathbf{Y}_i'$, $\hat{\mathbf{D}}^r_{xx} = \frac{1}{|\mathcal{J}_{x}|}\sum_{i\in\mathcal{J}_{x}} \mathbf{X}_i\hat{\mathbf{R}}_x\hat{\mathbf{R}}_x'\mathbf{X}_i'$ and $\hat{\mathbf{D}}^r_{yy} = \frac{1}{|\mathcal{J}_{y}|}\sum_{i\in\mathcal{J}_{y}} \mathbf{Y}_i\hat{\mathbf{R}}_y\hat{\mathbf{R}}_y'\mathbf{Y}_i'$. Furthermore, in Eq. (5), $\hat{\mathbf{C}}^l_{xy} = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}} \mathbf{X}_i'\hat{\mathbf{L}}_x\hat{\mathbf{L}}_y'\mathbf{Y}_i$ $(= \hat{\mathbf{C}}^l_{yx}{}')$, $\hat{\mathbf{C}}^l_{xx} = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}} \mathbf{X}_i'\hat{\mathbf{L}}_x\hat{\mathbf{L}}_x'\mathbf{X}_i$, $\hat{\mathbf{C}}^l_{yy} = \frac{1}{|\mathcal{J}_{xy}|}\sum_{i\in\mathcal{J}_{xy}} \mathbf{Y}_i'\hat{\mathbf{L}}_y\hat{\mathbf{L}}_y'\mathbf{Y}_i$, $\hat{\mathbf{D}}^l_{xx} = \frac{1}{|\mathcal{J}_{x}|}\sum_{i\in\mathcal{J}_{x}} \mathbf{X}_i'\hat{\mathbf{L}}_x\hat{\mathbf{L}}_x'\mathbf{X}_i$ and $\hat{\mathbf{D}}^l_{yy} = \frac{1}{|\mathcal{J}_{y}|}\sum_{i\in\mathcal{J}_{y}} \mathbf{Y}_i'\hat{\mathbf{L}}_y\hat{\mathbf{L}}_y'\mathbf{Y}_i$. In this way, by iterating Eqs. (4) and (5), the optimal transforms $\hat{\mathbf{L}}_x$, $\hat{\mathbf{L}}_y$, $\hat{\mathbf{R}}_x$, $\hat{\mathbf{R}}_y$ and the correlation matrices $\hat{\mathbf{\Lambda}}^l$, $\hat{\mathbf{\Lambda}}^r$ in Fig. 1 can be obtained. Finally, we show the outline of our 2D semi-CCA in Fig. 2.

The proposed method can use not only training pairs $(\mathbf{X}_i, \mathbf{Y}_i)$ $(i \in \mathcal{J}_{xy})$ but also the principle components respectively obtained from all examples $\mathbf{X}_i$ $(i \in \mathcal{J}_x)$ and $\mathbf{Y}_i$ $(i \in \mathcal{J}_y)$. Therefore, more successful relationship estimation can be expected. The details of this advantage using the semi-supervised scheme are reported in [26].

2.2. Missing Intensity Estimation Algorithm

In the proposed method, we try to estimate $\mathbf{Y}$ from $\mathbf{X}$ by using the derived results in the previous subsection. Specifically, as shown in Fig. 1, we can estimate the optimal result by the following equation:

$$\hat{\mathbf{Y}} = \left(\hat{\mathbf{L}}_y'\right)^+ \hat{\mathbf{\Lambda}}^l\hat{\mathbf{L}}_x'\left(\mathbf{X} - \mathbf{M}_x\right)\hat{\mathbf{R}}_x\hat{\mathbf{\Lambda}}^r\left(\hat{\mathbf{R}}_y\right)^+ + \mathbf{M}_y,$$

where $(\cdot)^+$ represents a pseudo inverse matrix. In this way, the proposed method enables the estimation of the missing intensities within Ω of the target patch f.

Finally, we clip patches including missing areas and perform their restoration to estimate all missing intensities. Note that in this procedure, we must determine the patch priority. In our method, we adopt an improved version of the method in [9]. Specifically, given a patch $f_\mathbf{p}$ centered at pixel $\mathbf{p}$ that is in the fill-front of the missing areas within the target image, its priority $P(\mathbf{p})$ is defined as follows:

$$P(\mathbf{p}) = C(\mathbf{p}) \cdot D(\mathbf{p}), \tag{6}$$

where $C(\mathbf{p})$ and $D(\mathbf{p})$, which respectively correspond to confidence term and data term, are defined as follows:

$$\begin{aligned} C(\mathbf{p}) &= \frac{\sum_{\mathbf{q}\in f_\mathbf{p}\cap(I-\Theta)} C(\mathbf{q})}{|f_\mathbf{p}|}, \\ D(\mathbf{p}) &= \frac{|\nabla I_\mathbf{p}^\perp \cdot \mathbf{n}_\mathbf{p}|}{\gamma}. \end{aligned}$$

In the above equations, I and Θ are the whole areas of the target image and whole missing areas, respectively. Furthermore, $|f_\mathbf{p}|$ is the number of pixels included within the target patch $f_\mathbf{p}$ ($w \times h$ pixels) . Then γ is a normalization factor (e.g. $\gamma = 255$ for a typical grayscale image), $\nabla I_\mathbf{p}^\perp$ is an isophote at pixel $\mathbf{p}$, and $\mathbf{n}_\mathbf{p}$ is a unit vector orthogonal to the fill-front at pixel $\mathbf{p}$. Note that $C(\mathbf{p})$ is initially set as $C(\mathbf{p}) = 0$ $\forall\mathbf{p} \in \Theta$ and $C(\mathbf{p}) = 1$ $\forall\mathbf{p} \in (I - \Theta)$.

After restoring the target patch $f_\mathbf{p}$, the proposed method newly assigns a new value of the confidence term for its restored area. Specifically, we focus on the following evaluation criterion E and denote it as $\xi(\mathbf{p})$:

$$E = \left\|\hat{\mathbf{L}}_x'\left(\mathbf{X} - \mathbf{M}_x\right)\hat{\mathbf{R}}_x - \hat{\mathbf{\Lambda}}^l\hat{\mathbf{L}}_y'\left(\hat{\mathbf{Y}}^* - \mathbf{M}_y\right)\hat{\mathbf{R}}_y\hat{\mathbf{\Lambda}}^r\right\|_F^2, \tag{7}$$

where $\hat{\mathbf{Y}}^*$ is a matrix whose area not included in Ω is set to zero for $\hat{\mathbf{Y}}$. The confidence term of pixel $\mathbf{q}$ in the restored area Ω of the target patch $f_\mathbf{p}$ centered at $\mathbf{p}$ is simply defined by $C(\mathbf{q}) = \exp\left(-\frac{\xi(\mathbf{p})}{\zeta}\right)$, where ζ is a parameter. The proposed method monitors the criterion obtained in Eq. (7), which corresponds to the error caused in the new variate space obtained by the 2D semi-CCA. Then $C(\mathbf{q})$ derived from this criterion can be regarded as the restoration performance, i.e., the confidence, of the target restored pixel $\mathbf{q}$. Therefore, it is reasonable to adopt $C(\mathbf{q})$ derived from the criterion in Eq. (7) into the proposed method using the 2D semi-CCA. In this way, we can restore all of the missing areas within the target image according to the priorities in Eq. (6).

3. EXPERIMENTAL RESULTS

In this section, we verify the performance of the proposed method by comparing with other existing works. In this experiment, we prepared four test images shown in Fig. 3 and added missing areas to these test images. Note that the positions of the missing areas are previously known. The first and second rows shown in Fig. 3 represent the original images and their corrupted images including missing areas, respectively. For these corrupted images, we performed the restoration of their missing areas by using the proposed method, and the obtained results are shown in the third rows. In this paper, we set $\alpha = \beta = 0.9$.

Next, in order to compare the performance of the proposed method with those of other existing works, we show some quantitative evaluation results. For the four test images shown in the top row of Fig. 3, we randomly added missing blocks of 8×8 pixels with changing the ratio of the missing pixels. Then the obtained corrupted images were restored by the proposed method and some existing methods, the PCA-based method [11], the KPCA-based method [12], the KPCA-based method including the clustering scheme [14], the sparse representation-based method [18] and the 2D CCA [25]-based method. The methods in [11] and [12] are

Fig. 3. Inpainting results obtained by the proposed method. Four test images are used, and the top, center and bottom rows are original images, corrupted images and inpainting results by our method, respectively. The sizes of Test images 1–4 are 480 × 360 pixels, 640 × 480 pixels, 640 × 480 pixels and 480 × 360 pixels, respectively. The percentages of missing areas are 10.7 %, 7.1%, 5.4% and 8.9 %, respectively.

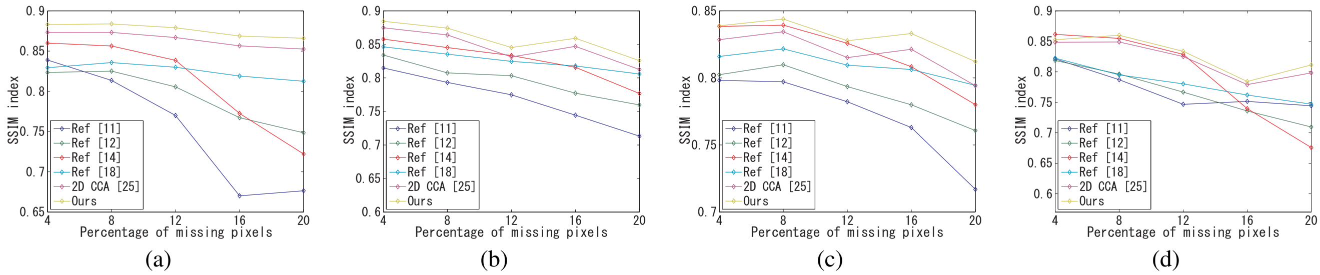

Fig. 4. Relationship between the ratio of missing pixels and the SSIM index of the inpainting results obtained by each method: (a)–(d) respectively show the results obtained from Test images 1–4 shown in Fig. 3.

benchmarking methods in the recent studies using multivariate analysis. Furthermore, we regard the methods in [14] and [18] as the state-of-the-art methods. Since the proposed method corresponds to the improved method of the 2D CCA method [25], we show the results obtained by using this method, that is, the results when using $\alpha = \beta = 1.0$. On the other hand, if we set $\alpha = \beta = 0.0$, it only becomes 2D PCA of $\mathbf{X}_i$ and $\mathbf{Y}_i$, and their relationship cannot be obtained for reconstructing missing areas. Therefore, we used other PCA-based methods [11, 12, 14]. Figure 4 shows the relationship between the ratio of the missing pixels and the SSIM index [27] calculated from the restored images. From these results, we can see the proposed method outputs the best results[1].

[1]In Fig. 3, we only show the inpainting results of the proposed method due to the limitation of pages. All of the inpainting results of the images shown in the second row of Fig. 3 by our method and the previously reported methods [11, 12, 14, 18, 25] in the same conditions can be confirmed in the following Web site.
http://www-lmd.ist.hokudai.ac.jp/wp/wp-content/uploads/ICIP2014-Ogawa.pdf

In the proposed method, we introduce the 2D semi-CCA into the inpainting. This enables accurate relationship estimation between missing areas and their neighboring areas, and it can be confirmed from the results shown in Figs. 3 and 4. Furthermore, the proposed method adopts the patch priority estimation realized by monitoring errors caused in the new variate space obtained based on the 2D semi-CCA. This also improves the performance of the proposed inpainting method.

4. CONCLUSIONS

In this paper, we have presented a new inpainting method using 2D semi-CCA. The proposed method estimates the relationship between missing areas and their neighboring areas by using the 2D semi-CCA to realize successful inpainting even if sufficient number of training pairs cannot be provided. Furthermore, by monitoring errors caused in the new variate space, successful patch priority estimation can be realized. Consequently, from the experimental results, it can be confirmed that our method enables more successful inpainting compared to the previously reported methods.

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