

EFFICIENT POISSON DENOISING FOR PHOTOGRAPHY

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ABSTRACT

In general, image sensor noise is dominated by Poisson statistics, even at high illumination level, yet most standard denoising procedures often assume a simpler additive Gaussian noise, which is in fact a poor approximation. Fortunately, Poisson noise can under some circumstances be simplified via variance stabilizing methods, such as the Anscombe transform, which is well known to statisticians, medical imaging specialists and astronomers. However, in order to use such a procedure effectively, the actual photon count needs to be known and not simply an illumination intensity, which is the main reason why such procedures are not frequently used in the image processing community.

In this article, we propose to use Poisson distribution characteristics to estimate the photon count from relative illumination data, under simple hypotheses. This allows us to use variance-stabilizing methods on standard digital photographs. Thanks to this, the noise becomes close to additive Gaussian and standard filtering methods become significantly more effective. As an example we exhibit the level of improvement that can be achieved using the bilateral filter.

Index Terms— Image sensors ; Gaussian filtering ; Bilateral filter, Poisson noise model.

1. INTRODUCTION

Small consumer-level cameras exhibit many flaws leading to poor image output. Among them are cheap and simple optics, a small sensor due to size consideration, and lack of autofocus, aperture control or motion compensation that can lead to fuzzy images. In addition, these camera do not generally give access to the raw image data, contrarily to more sophisticated equipment. Furthermore, these camera need to be operated simply and so must process the captured image data quickly and effectively in order to present the user with a usable image with a minimum of intervention. In this paper we shall focus on the denoising aspect of this processing procedure. This is a critical as it will condition most of the rest, including colour demosaicing, artifact removal, and coding.

Most standard denoising procedures assume an additive Gaussian noise pattern and optimize their parameters under this hypothesis. However photography noise is dominated by Poisson photon noise under most conditions, even high illumination. This results in noise variance that varies proportionally to light intensity. Poisson noise is neither additive nor multiplicative and so must be dealt with using dedicated procedures.

In the remainder, we present a simplified noise model suitable for digital cameras. We introduce a variance-stabilizing pixel transform that simplify Poisson noise into approximate Gaussian additive noise with known variance. We show that these transforms are not directly applicable to simple intensity images, but require photon counts at each pixel location to operate. We propose a procedure allowing to estimate such counts from relative intensity and show that it does perform well. Using this procedure, we propose an improvement on the bilateral filter [1] that is shown to perform well on photography images, and that is efficient enough to run on small, embedded CPUs.

2. NOISE MODEL

Many sources are cause of noise generation in CCD and CMOS sensors. These can be categorized in four main factors [2]: (1) The photon shot noise – associated with a random Poisson process governing the number of incident photons reaching a photosite; (2) the photon response non uniformity – caused by small sensitivity differences between photosites; (3) the dark current noise – produced by minority carriers thermally generated in the sensor well, also associated with a random Poisson process ; and (4) the read-out noise – resulting from thermal noise cause by MOSFET amplifiers.

A sensor noise model complete with respect to these factors is proposed in [2]. According to this model, photon shot noise has, in consumer photography illumination conditions, the most important influence on the output image. We consider only this noise in our model. We simulated sensor acquisition via a regionalized cumulative spatial Poisson point process [3] using scene images as probability distribution func-

tions, similar to Monte-Carlo process. As scene images we used the Kodak PhotoCD database, extracted at the 256x384 resolution. In Fig. 1, we exhibit some of the characteristics of

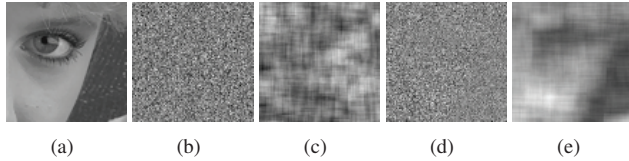


Fig. 1. Differences between Gaussian and Poisson noise. (a) Example clean image. (b) Gaussian noise, PSNR=33dB. (c) 11×11 sliding-window variance of the Gaussian noise. (d) Poisson Noise, PSNR=33dB, (d) 11×11 sliding window variance of the Poisson noise. Observe the correlation between (a) and (e)

Poisson noise, by comparing it to a Gaussian noise of identical intensity. Figure 1(a) is a small subset of the image, Fig. 1(b) is a picture of an additive Gaussian noise of variance $\sigma = 5.7$ that would result in a PSNR of 33dB. We run a sliding window variance filter of size 11×11 on that image, yielding an estimation of the variance of the noise at each location (Fig. 1(c)). There is no evident correlation with the variance of the noise and image data. Conversely, Fig 1(d) shows the Poisson noise resulting from a Monte-Carlo simulation from image (a), resulting in the same PSNR. We observe clear correlation of the variance with image intensity (Fig. 1(e)). Poisson noise is not stationary and hence neither additive nor multiplicative. In the next section, we introduce transforms to simplify Poisson noise.

3. VARIANCE-STABILIZING TRANSFORMS

In spite of Poisson noise being more complex than Gaussian noise, it is still amenable to processing, due to its statistics.

Let P be a random, Poisson-distributed variable. We have the fundamental well-known result

$$E(P) = \text{Var}(P), \quad (1)$$

that is, the variance $\text{Var}(P)$ equal its average or expectancy $E(P)$. We can find many instances of P in the physical world, such as radiation detection events in a Geiger counter, number of failing lighting bulbs in a building, or, to a good approximation, the number of photons falling on a photosite as the result of illumination.

Anscombe [4] proposed a simple transformation to regularize the variance of P :

$$A : P \longrightarrow 2\sqrt{P + \frac{3}{8}}. \quad (2)$$

Under that transformation A is now a random variable with expectancy $2\sqrt{E(P)}$ and constant variance, i.e: $A \approx 2\sqrt{E(P)} + \varepsilon$, with ε a new random variable with Gaussian statistics and a fixed variance of 1.

This is extremely useful for image data, assuming the pixel values can be assimilated to photon counts, each pixel can then be assimilated to a Poisson variable. The pixelwise Anscombe transformation on this data yields a new image with approximately Gaussian additive noise, with known noise variance. The noise become stationary and much easier to filter. However, as we shall see in the next section, we cannot readily equate pixel values with photon counts.

4. PHOTON COUNT ESTIMATION

In digital cameras, recorded photons are transformed into pixel values through several processing, one of which being the automated gain control. Here we assume that the quantum efficiency of the image capture device is constant, and that its output is linear with illumination.

4.1. Automatic gain control

In a digital imaging system, an automatic gain control (AGC) is necessary to capture images in varying light conditions in order for images to present a usable palette. Indeed natural illumination conditions vary far more than most image formats can accommodate. AGC works by adjusting the average intensity of the output signal (see Fig. 2). Assuming access to the gain factor and knowing the sensor resolution, it would be possible to deduce the number of photons that have reached the sensor and then estimate the noise level. However, this information is not usually available to photographers, even using a RAW format.

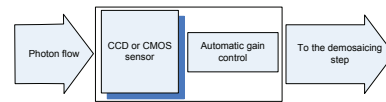


Fig. 2. Digital conversion of a photon flow by a camera sensor. AGC adjusts the average image output intensity

4.2. Photons density from image statistics

In the presence of gain manipulation only, we expect the noise variance to be proportional to average pixels values in near-constant regions, due to its Poisson characteristics. Let I be the image before AGC, and J the adjusted image after AGC (i.e. our data). Assuming γ to be the gain factor, we can write:

$$J = \gamma \times I \quad (3)$$

Considering a Poisson distribution process, we can assume the following relation where $T^-(I)$ represents texture-free areas in the image I .

$$E(T^-[I]) = \text{Var}[T^-[I]] \quad (4)$$

Using elementary substitutions [5] the gain is expressed:

$$\gamma = \frac{E[T^-[J]]}{\text{Var}[T^-[J]]} \quad (5)$$

The image mean $E[J]$ and variance $\text{Var}[J]$ can be simultaneously calculated at each point in I by using a sliding-window algorithm. We seek to establish a correlation between all $E[J]$ and $\text{Var}[J]$ at each point where T^- is non-zero. Since mean and variance are expected to be proportional, a robust linear, intercept-free regression can be performed, the slope coefficient can then be interpreted as the gain factor γ transforming photon numbers to recorded pixel values.

4.3. Texture-free areas of the image

It is difficult to estimate noise variance in textured area of the image. We can detect texture-free areas of the image using the following formula

$$T^-[J] = \varepsilon_w [\nabla[G_\sigma \star J] > \vartheta], \quad (6)$$

where J is the image, G_σ a Gaussian convolution of variance σ , ε_w a morphological erosion using a square window of size $w \times w$ as structuring element. ϑ is an intensity parameter. In our experiments on 8-bit images, we chose $\sigma = 2$, $\vartheta = 2$ and $w = 11$.

For efficiency and to reduce the influence of correlation between neighboring pixels, we sample the $T^-(J)$ regions with 10^3 – 10^5 pixels. At each of these points x we estimate the image average $E[J](x)$ and the noise variance by $\text{Var}[J - G_\sigma \star J]$, in order to eliminate small intensity variations, both via a sliding window algorithm. We finally estimate the total number of photons $\bar{\Phi}$ with the following formula:

$$\bar{\Phi}[J] = \frac{\text{nbpix}[J] \overline{E[J]}}{\gamma} = \frac{\sum_{(x,y) \in I} J(x,y)}{\gamma}, \quad (7)$$

where $\text{nbpix}[J]$ is the total number of pixels in J .

4.4. Experiments and results

We simulated sensor acquisitions of 10^4 to 10^9 photons and AGC followed by quantization via a Monte-Carlo process, corresponding to a photon density per pixel varying from 1 to 10^5 , or a PSNR ranging from 28 to 46 dB, i.e from an extremely noisy to a very clean image, and then estimated the photon count via our method. The excellent correlation between simulated and estimated photons is shown on the log-log plot of Fig. 3 ($R^2 = 0.996$). We now have a reliable way to estimate photon numbers in photographs. We can now propose a denoising algorithm taking advantage of the variance-stabilizing transform.

5. ALGORITHM

Any denoising algorithm assuming Gaussian noise should benefit from variance-stabilisation. As illustration, we use the bilateral filter.

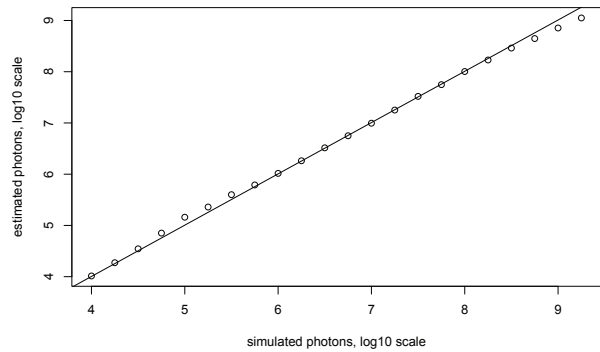


Fig. 3. Correlation between simulated and estimated photon numbers. Note that the slope of the correlation is 1.

5.1. Bilateral filtering

The bilateral filter replaces a pixel value in an image by a weighted mean of its neighbors considering both their geometric closeness and photometric similarities [6, 1]. The Gaussian bilateral filter is defined as follows:

$$v(x) = \frac{1}{C(x)} \sum_{y \in \beta} \exp \left(-\frac{|x-y|^2}{\rho^2} \right) \exp \left(-\frac{|u(y) - u(x)|^2}{h^2} \right) u(y) \quad (8)$$

Where β represents a $w \times w$ sliding window, y an element of β , and x is the centered pixel in the sliding window. $u(x)$ is the intensity of the pixel at the x position in the original image, $v(x)$ is the filtered pixel value at position x , ρ and h are respectively the standard deviation of the Gaussian distribution of the geometrical and the intensity weight. The filter behavior depends highly on these parameters setting. Parameter ρ should be chosen considering the size of window, e.g $\rho = w/4$. Parameter h must be chosen considering the level of filtering needed for the application. Indeed, the higher the standard deviation, the more the filter behaves like a low-pass Gaussian convolution. An illustration of the influence of h is shown on Fig. 4. The parameter h is related to noise vari-

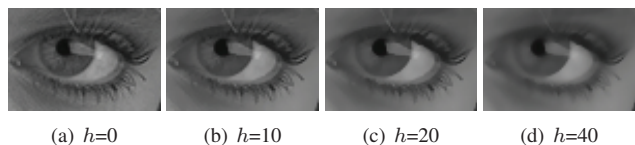


Fig. 4. Bilateral filtering using a 5x5 window and variable h

ance, and is therefore particularly difficult to estimate in the presence of Poisson noise [5, 7]. Conversely, with a noise variance stabilized to a value of 1, the h parameter does not need to be optimized at all.

5.2. Variance stabilization and algorithm

The algorithm is as follows: (1) Estimate photon density and gain control parameter γ as per section 4 ; (2) Pixelwise multiply by inverse gain control $1/\gamma$; (3) Apply pixelwise Anscombe transformation $A = 2\sqrt{I + 3/8}$; (4) Apply bilateral filter with optimized fixed h parameter; (5) Apply pixelwise inverse Anscombe transformation ; (6) Pixelwise multiply by gain control γ . The result is a filtered image with the same dynamic range as the input image.

6. RESULTS AND COMPARISONS

Figure 5 shows a filtering result on a small subsample. We

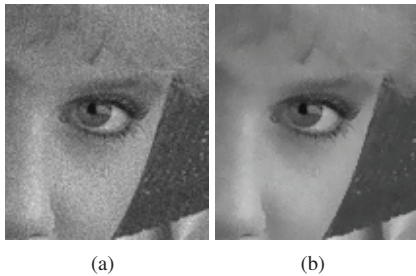


Fig. 5. Result of bilateral filtering with the Anscombe transformation. (a) noisy image, PSNR=31.5dB, (b) filtered image, PSNR=34.8dB

ran this stabilized-variance filter with a variety of images corresponding to photon density ranging from 10^2 to 10^5 , with fixed parameters, namely $w = 7, h = 2.5$. For comparison, we ran the unmodified bilateral on the same data, but in order to achieve similar results, we optimized h for each image by parameter grid search. The result is on Fig. 6. We observe that the fixed-parameter Anscombe+bilateral performs as well or better as a hand-optimized normal bilateral. In that graph we also showed the PSNR of the unfiltered image for comparison, and the result of a fixed- h -value bilateral filter. We further observe that a fixed- h bilateral performs poorly far from its optimal parameter.

7. CONCLUSION

In this paper, we have proposed a methodology for estimating the photon density in digital photographs. This allowed us to use a variance-stabilisation operator to transform the Poisson noise associated with photographs into a Gaussian additive noise with fixed variance = 1. This finally allowed us to effectively process images featuring various noise intensities with a fixed-parameters bilateral filter. The end result is a fast and flexible filter that performs as well or better as the normal bilateral but without the need to finely tune its h parameter, which is tedious, error prone, and hard to automate. This makes this new filter suitable for entirely auto-

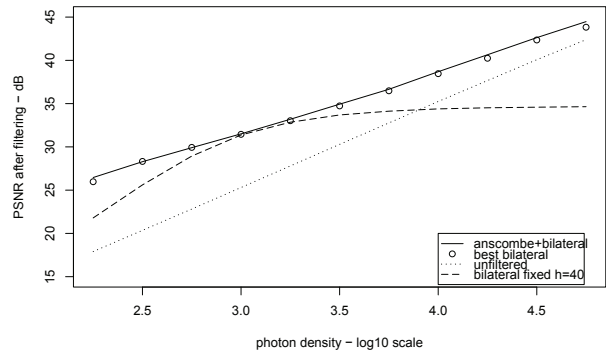


Fig. 6. Comparison of fixed-parameters Anscombe+bilateral (continuous line) filter vs. hand-optimised standard bilateral (dots). The former is always as good or better.

mated filtering tasks, e.g. in consumer-level digital photography equipments.

8. REFERENCES

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