AN EFFICIENT LIFTING STRUCTURE OF BIORTHOGONAL FILTER BANKS FOR LOSSLESS IMAGE CODING

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ABSTRACT

This paper introduces an image transform method by using M-channel biorthogonal filter banks (BOFBs) with an efficient lifting factorization. The proposed lifting factorization of a building block in their lattice structure has unity diagonal scaling coefficients and guarantees perfect reconstruction even if the obtained coefficients are quantized. Since the number of rounding operators of proposed lifting-based BOFBs (LBBOFBs) can be reduced by merging the lifting steps, the proposed structure is efficient for lossless image coding. Image coding results indicate better performance than conventional methods.

Index Terms— Biorthogonal filter banks, lifting structure, reducing rounding operator, lossless-to-lossy image coding.

1. INTRODUCTION

Recently, many researchers have been studying in the field of multirate signal processing. Filter bank (FB) is one of the most efficient concepts to compress multimedia signals [1]. FBs are adopted to image, audio and video coding standards such as JPEG and MPEG [2]. Fig. 1 (a) shows a uniform maximally decimated M-channel filter bank which consists of parallel analysis filters $H_i(z)$, synthesis filters $F_i(z)$, decimaters and interpolators. Fig. 1 (b) illustrates its equivalent polyphase representation where $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are the type-I and -II polyphase matrices, respectively. The polyphase representation is formulated as follows [1]:

$$\begin{aligned} \begin{bmatrix} H_0(z) & H_1(z) & \dots & H_{M-1}(z) \end{bmatrix}^T &= \mathbf{E}(z^M)\mathbf{e}(z)^T \\ \begin{bmatrix} F_0(z) & F_1(z) & \dots & F_{M-1}(z) \end{bmatrix} &= \mathbf{e}(z)\mathbf{R}(z^M) \end{aligned} \tag{1}$$

where $\mathbf{e}(z) = \begin{bmatrix} 1 & z^{-1} & \dots & z^{-(M-1)} \end{bmatrix}$ and $\cdot^T$ denotes transposition of a matrix. If perfect reconstruction (PR) is achieved, the synthesis polyphase matrix $\mathbf{R}(z)$ can be chosen as the inverse of $\mathbf{E}(z)$. The obtained FB is called a BOFB. If $\mathbf{E}^\dagger(z^{-1})\mathbf{E}(z) = \mathbf{I}$ and $\mathbf{R}(z) = \mathbf{E}^\dagger(z^{-1})$, the FB belongs to a special class of PRFBs called a paraunitary (PU) FB, where $\cdot^\dagger$ stands for conjugate transpose. It is commonly known that PUFBs can be designed easily due to the property. Although the number of design parameters is smaller than that of BOFBs, the frequency responses of PUFBs are usually worse than those of BOFBs. Hence, we focus on BOFBs.

PRFBs are designed efficiently by the lattice structure [3]. To apply FBs to lossless image coding, the lattice structure should be represented by lifting structures which has unity diagonal scaling coefficients to avoid quantization errors. In [4], the authors introduced the lifting structure of order-1 PUFBs based on a Householder matrix for both lossy and lossless image coding. The lifting structures for

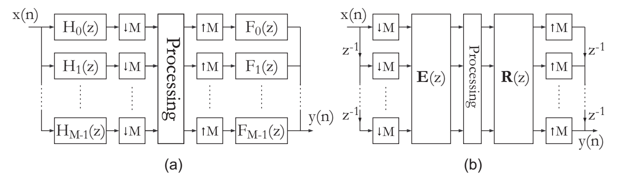

Fig. 1. (a) An M-channel filter bank. (b) A polyphase structure of a filter bank.

BOFBs have researched in [5] and [6]. However, the conventional factorization of BOFBs involves non-unity diagonal scaling coefficients. Hence, they have not been applied to lossless coding directly. Furthermore degree-1 BOFBs which have unity diagonal scaling coefficients throughout the lifting structure have been proposed in [7].

In this paper, we propose a novel lifting structure of order-1 building blocks in BOFBs. The proposed lifting structure has more design parameters than those of the conventional order-1 PUFBs and degree-1 BOFBs. Additionally by merging the lifting steps, the structure can be reduced the number of rounding operators efficiently which is useful for lossless image coding. Proposed FBs can also be applied to lossy image coding by interrupting the obtained bit stream.

This paper is organized as follows; first, we review the lattice structure of BOFBs in section 2. In section 3, we introduce the lifting factorization for BOFBs with unity diagonal scaling coefficients and the structure of merged lifting steps. Design examples and lossless-to-lossy image coding application are compared with 5/3-tap and 9/7-tap DWT [1] and the same order of the conventional PUFB [4] in section 4. Section 5 concludes the paper.

2. LATTICE STRUCTURE OF BOFBS

2.1. Order-1 Building Block of BOFBs

A class of causal M-channel L-th order FIR BOFBs are factorized into [6]

$$\mathbf{E}(z) = \mathbf{W}_L(z)\cdots\mathbf{W}_1(z)\mathbf{E}_0 \tag{2}$$

where $\mathbf{E}_0$ is an $M \times M$ nonsingular matrix which is called the first block. $\mathbf{W}_m(z)(m = 1 \dots L)$ is an $M \times M$ first-order BO building block given by

$$\mathbf{W}_m(z) = \mathbf{I} - \mathbf{U}_m\mathbf{V}_m^\dagger + z^{-1}\mathbf{U}_m\mathbf{V}_m^\dagger \tag{3}$$

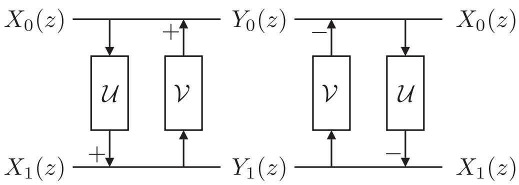

Fig. 3. Two lifting steps with lifting multipliers $\mathcal{V}$ and $\mathcal{U}$ and its inverse

where the $M \times \gamma_m$ parameter matrices $\mathbf{U}_m$ and $\mathbf{V}_m$ satisfy

$$\mathbf{V}_m^\dagger \mathbf{U}_m = \begin{bmatrix} 1 & \times & \cdots & \times \\ 0 & 1 & \cdots & \times \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{\gamma_m \times \gamma_m}$$

for some integer $1 \leq \gamma_m \leq M$, where $\times$ indicates possibly nonzero elements. This is a generalization of the paraunitary factorization given in [8] where $\mathbf{U}_m = \mathbf{V}_m$, and has been used for a factorization of the biorthogonal lapped transform (BOLT) [9]. The properties of the matrix are described as follows:

- Since the rank of $\mathbf{V}_m^\dagger \mathbf{U}_m$ is γ_m, the McMillan degree of $\mathbf{W}_m(z)$ as in (3) is γ_m. When $\gamma_m = 1$ and $M/2$, the structure is called *degree-1* and *order-1* building block, respectively. In this paper, we focus on the order-1 structure, since more design parameters can be used than the degree-1 structure.
- The structure in (2) completely spans all causal FIR PRFBs having anticausal FIR inverses. The spanned analysis filters have filter lengths no greater than $M(L+1)$, and the McMillan degree of $\mathbf{E}(z)$ ranges from L to ML, where L is the order of the FB.
- The Type-II synthesis polyphase matrix $\mathbf{R}(z)$ is given by

$$\mathbf{R}(z) = \mathbf{E}_0^{-1}\mathbf{W}_1^{-1}(z)\ldots\mathbf{W}_L^{-1}(z). \tag{4}$$

  Due to the possibly nonzero off-diagonal elements of $\mathbf{V}_m^\dagger \mathbf{U}_m$, the order of $\mathbf{W}_m^{-1}(z^{-1})$ can be greater than one. Thus the synthesis bank could have different filter lengths from $M(L+1)$.

  In this paper, we set $\mathbf{V}_m^\dagger \mathbf{U}_m = \mathbf{I}_\gamma$. Hence, $\mathbf{W}_m^{-1}(z^{-1}) = \mathbf{I} - \mathbf{U}_m\mathbf{V}_m + z\mathbf{U}_m\mathbf{V}_m$, which is anticausal and satisfies $\mathbf{R}(z)\mathbf{E}(z) = \mathbf{I}$ for PR.

2.2. Generalized Householder Matrix H

Let $\mathbf{H}$ be an $M \times M$ generalized Householder matrix [3]. $\mathbf{H}$ can be written as follows:

$$\mathbf{H} = \mathbf{I}_M - 2\mathbf{U}\mathbf{V}^\dagger, \tag{5}$$

where $\mathbf{V}^\dagger\mathbf{U} = \mathbf{I}_\gamma$, the size of matrices $\mathbf{U}$ and $\mathbf{V}$ is $M \times \gamma$. When $\gamma = M/2$, it can be regarded as the order-1 building block at $z = -1$. In this paper, we adopt this matrix as the first block of the lattice structure.

3. AN EFFICIENT LIFTING STRUCTURE OF BOFBS

In this section, we introduce a new lifting factorization of the order-1 building block and the generalized Householder matrix. To apply the FBs to lossless image coding, the building block $\mathbf{W}_m(z)$ and first block $\mathbf{E}_0$ should be factorized into the lifting steps, and should not have non-unity diagonal scaling coefficients. Fig. 3 shows a series of two lifting steps and their easy-to-compute inverses.

3.1. Lifting Structure of Order-1 Building Blocks of BOFBs

We assume $\mathbf{U}_m$ and $\mathbf{V}_m$ in (3) as

$$\mathbf{U}_m = \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} \text{ and } \mathbf{V}_m = \begin{bmatrix} \mathbf{C} \\ \mathbf{D} \end{bmatrix},$$

where $\mathbf{C}^\dagger\mathbf{A} + \mathbf{D}^\dagger\mathbf{B} = \mathbf{I}$. Substituting them into (3), we obtain

$$\mathbf{W}(z) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{AC}^\dagger & \mathbf{AD}^\dagger \\ \mathbf{BC}^\dagger & \mathbf{BD}^\dagger \end{bmatrix} + z^{-1}\begin{bmatrix} \mathbf{AC}^\dagger & \mathbf{AD}^\dagger \\ \mathbf{BC}^\dagger & \mathbf{BD}^\dagger \end{bmatrix}.$$

Then, two block lifting matrices are multiplied from the both side of $\mathbf{W}(z)$. If the product can be represented as a block lifting matrix, $\mathbf{W}(z)$ can be factorized into the product of some block lifting matrices. First, let $\mathbf{P}$ be an arbitrary $M/2 \times M/2$ matrix. The block lifting matrix is multiplied from the right side of $\mathbf{W}(z)$ as follows:

$$\mathbf{S}(z) \equiv \mathbf{W}(z)\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P} & \mathbf{I} \end{bmatrix}.$$

To keep the property of unity diagonal scaling coefficients, upper left components of $\mathbf{S}(z)$ has to be an identity matrix as

$$\mathbf{I} - \mathbf{AC}^\dagger + z^{-1}\mathbf{AC}^\dagger - \mathbf{AD}^\dagger\mathbf{P} + z^{-1}\mathbf{AD}^\dagger\mathbf{P} = \mathbf{I}.$$

The condition of $\mathbf{P}$ is defined by

$$\mathbf{P} = -\mathbf{D}^{-\dagger}\mathbf{C}^\dagger. \tag{6}$$

Consequently, $\mathbf{S}(z)$ can be rewritten as

$$\mathbf{S}(z) = \begin{bmatrix} \mathbf{I} & -\mathbf{AD}^\dagger + z^{-1}\mathbf{AD}^\dagger \\ \mathbf{P} & \mathbf{I} - \mathbf{BD}^\dagger + z^{-1}\mathbf{BD}^\dagger \end{bmatrix}.$$

Next, other block lifting matrix $\mathbf{T}(z)$ is multiplied from the left side of $\mathbf{S}(z)$ such as

$$\begin{aligned}\mathbf{T}(z) &\equiv \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{Q} & \mathbf{I} \end{bmatrix}\mathbf{S}(z) \\ &= \begin{bmatrix} \mathbf{I} & -\mathbf{AD}^\dagger + z^{-1}\mathbf{AD}^\dagger \\ \mathbf{Q}+\mathbf{P} & \mathbf{I} + (z^{-1}-1)(\mathbf{QAD}^\dagger + \mathbf{BD}^\dagger) \end{bmatrix}\end{aligned}$$

The lower left component of $\mathbf{T}(z)$ should be a null matrix. Hence, the condition

$$\mathbf{Q} = -\mathbf{P} \tag{7}$$

is obtained. Under the condition, (6), (7) and $\mathbf{C}^\dagger\mathbf{A} + \mathbf{D}^\dagger\mathbf{B} = \mathbf{I}$, the lower right component can be rewritten as follows:

$$\begin{aligned}&\mathbf{I} + (z^{-1}-1)(\mathbf{QAD}^\dagger + \mathbf{BD}^\dagger) \\ &= \mathbf{I} + (z^{-1}-1)(\mathbf{D}^{-\dagger}\mathbf{C}^\dagger\mathbf{AD}^\dagger + \mathbf{D}^{-\dagger}(\mathbf{I} - \mathbf{C}^\dagger\mathbf{A})\mathbf{D}^\dagger) \\ &= z^{-1}\mathbf{I}.\end{aligned}$$

Consequently, $\mathbf{T}(z)$ is formulated as

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{P} & \mathbf{I} \end{bmatrix}\mathbf{W}(z)\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & -\mathbf{AD}^\dagger + z^{-1}\mathbf{AD}^\dagger \\ \mathbf{0} & z^{-1}\mathbf{I} \end{bmatrix}.$$

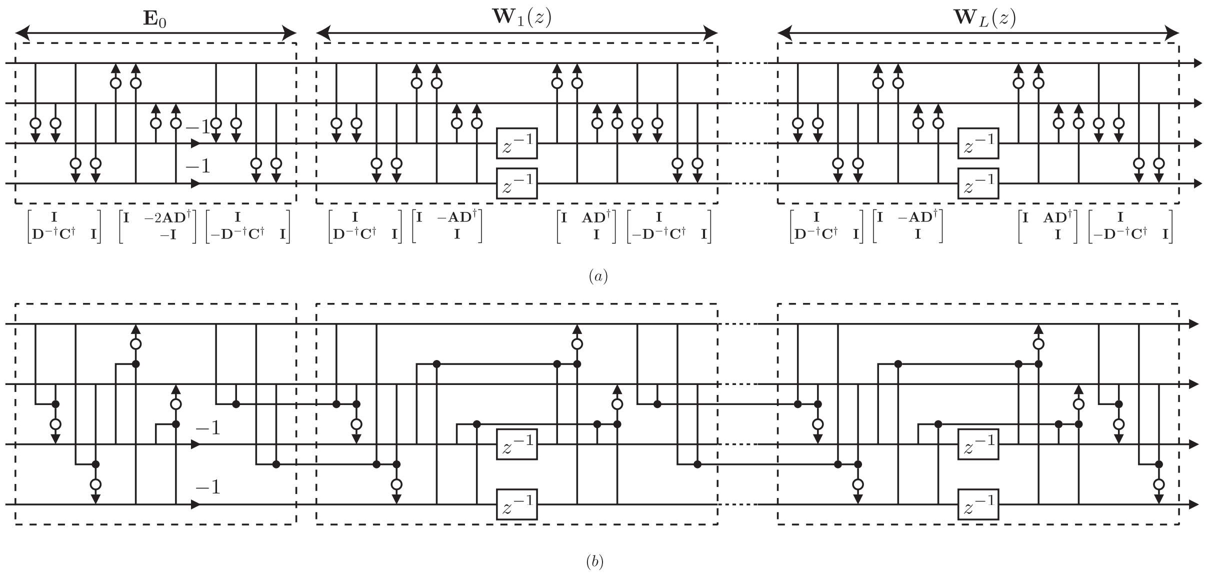

Fig. 2. (a) The proposed lifting structure, (b) Reducing of rounding operators

Therefore, an order-1 building block $\mathbf{W}(z) = \mathbf{I} - \mathbf{U}\mathbf{V}^\dagger + z^{-1}\mathbf{U}\mathbf{V}^\dagger$ can be factorized into

$$\mathbf{W}(z) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{P} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -\mathbf{A}\mathbf{D}^\dagger + z^{-1}\mathbf{A}\mathbf{D}^\dagger \\ & z^{-1}\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{P} & \mathbf{I} \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{D}^{-\dagger}\mathbf{C}^\dagger & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{A}\mathbf{D}^\dagger \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I} \end{bmatrix}$$
$$\begin{bmatrix} \mathbf{I} & -\mathbf{A}\mathbf{D}^\dagger \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{D}^{-\dagger}\mathbf{C}^\dagger & \mathbf{I} \end{bmatrix}. \quad (8)$$

3.2. Lifting Structure of The First Block Based on Generalized Householder Matrix

Let $\mathbf{E}_0$ be an $M \times M$ generalized householder matrix. $\mathbf{E}_0$ can be written as follows:

$$\mathbf{E}_0 = \mathbf{I}_M - 2\mathbf{U}\mathbf{V}^\dagger, \quad (9)$$

where $\mathbf{V}^\dagger\mathbf{U} = \mathbf{I}_\gamma$, the size of $\mathbf{U}$ and $\mathbf{V}$ is $M \times M/2$ which is the same as the above subsection. Thus, a generalized Householder matrix is decomposed into the product of block lifting matrices as

$$\mathbf{E}_0 = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{D}^{-\dagger}\mathbf{C}^\dagger & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -2\mathbf{A}\mathbf{D}^\dagger \\ \mathbf{0} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{D}^{-\dagger}\mathbf{C}^\dagger & \mathbf{I} \end{bmatrix},$$

where **A**, **C** and **D** are arbitrary $M/2 \times M/2$ nonsingular matrices. The derivation of $\mathbf{E}_0$ is almost similar to the previous section except substituting $z = -1$ into (8), thus we omit it. Fig. 2 (a) shows the example of the proposed lifting structure of four-channel LBBOFB.

3.3. Merging Rounding Operators

The number of rounding operators should be as small as possible for lossless image coding because these operators cause a reduction of coding efficiency. We can reduce it by merging lifting steps. Since our FB is applied to lossless image coding, we can merge the lifting steps at both side of z^{-1}. Fig. 2 (b) shows the proposed lifting structure with reduced rounding operators. Black and white circles denote adder and rounding operators, respectively.

4. RESULTS

In this section, we present the design examples of proposed LBBOFBs. The cost function to design the FBs is a weighted linear combination of the coding gain C_{CG}, the stopband attenuation of analysis and synthesis filter bank E_{STOP} and R_{STOP}, and the DC leakage C_{DC} [3].

$$\Phi = \sum_{M-1}^{i=1} (\omega_1 E_{STOP} + \omega_2 R_{STOP} + \omega_3 C_{DC}) - \omega_4 C_{CG} \quad (10)$$

where ω_1, ω_2, ω_3, and ω_4 are weighted coefficients.

4.1. Application to Lossless Image Coding

Our LBBOFBs are applied to lossless image coding by using a rounding operator in each merged lifting step. We adopted the periodic extension at the image boundaries and EZW-IP as a wavelet based coder [10]. The coding results are compared by entropy [bpp]=(Total number of bits [bit])/(Total number of pixels [pixel]) which indicate how the FBs efficiently reduce the spatial redundancy of the input signals. Table 1 shows the comparison between LBBOFBs and LBPUFBs and 5/3-tap WT [11]. The comparison of the number of rounding operators and lossless coding results is shown in Table 3. The proposed FBs present better performance than those of the conventional FBs.

4.2. Lossless-to-Lossy Coding

Proposed LBBOFBs can be also applied to lossy image coding by interrupting the obtained bit stream. As well as lossless image coding, we adopted the periodic extension and EZW-IP. The coding results are compared by $\mathrm{PSNR} = 10\log_{10}(255^2/\mathrm{MSE})$[dB] where MSE is the mean squared error. Table 2 and Fig. 5 show the comparison of PSNRs between 8×24 LBBOFBs and 9/7-tap WT and the part of the enlarged images of *Barbara*. It is obvious that our LBBOFBs indicate better results on both PSNR and perceptual visual quality of reconstructed images against 9/7-tap WT.

Table 1. Comparison of lossless image coding (Entropy [bpp]).

Image	5/3-tap	Conv. LBPUFB [4]		Prop. LBBOFB	
(512×512)	WT [11]	8×16	8×24	8×16	8×24
Barbara	4.87	4.88	4.82	4.85	**4.76**
Elaine	5.11	5.12	5.06	5.13	**5.05**
Finger	5.84	5.70	5.68	5.72	**5.66**
Finger2	5.60	5.48	5.43	5.49	**5.39**

Table 2. Comparison of lossy image coding (PSNR [dB])

Barbara(512×512)		
Bit rate	9/7-tap WT	Conv. (8×24)
0.1	24.04	**24.97**
0.25	27.23	**28.38**
0.5	30.47	**32.03**
1	34.91	**36.21**

Table 3. Comparison of the total number of rounding operators in LBBOFBs.

	8×16LBBOFB		8×24LBBOFB	
	# of Rounding	Entropy [bpp]	# of Rounding	Entropy [bpp]
before	28	4.877	44	4.785
reduced	10	**4.854**	14	**4.764**

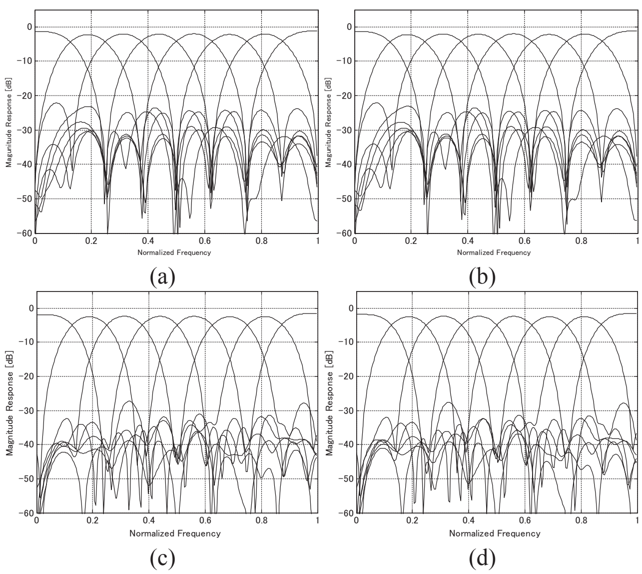

Fig. 4. Frequency responses of LBBOFBs: (a), (b) 8×16 analysis and synthesis filter banks, (c), (d) 8×24 analysis and synthesis filter banks.

5. CONCLUSION

In this paper, we proposed a novel lifting structure of BOFBs based on the generalized Householder matrix. This class of FBs, called LBBOFBs, has unity diagonal scaling coefficients and guarantees PR even if the lifting coefficients are quantized at each lifting step. Due to this property, our LBBOFBs are suitable for lossless-to-lossy image coding. Furthermore, LBBOFBs presents superior coding results on entropy to 5/3-tap WT, 9/7-tap WT and the conventional LBPUFB.

Fig. 5. Barbara (Bit rate: 0.25[bpp]): (left) 9/7-tap WT, (right) 8×24 LBBOFB.

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