A HIERARCHICAL APPROACH FOR FAST AND ROBUST ELLIPSE EXTRACTION

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ABSTRACT

This paper presents a hierarchical approach for fast and robust ellipse extraction from images. At the lowest level, the image is described as a set of edge pixels, from which line segments are extracted. Then, line segments that are potential candidates of elliptic arcs are linked to form arc segments according to connectivity and curvature relations. After that, arc segments that belong to the same ellipse are grouped together. Finally, a robust statistical method, namely RANSAC, is applied to fit ellipses. This method does not need a high dimensional parameter space like Hough Transform based algorithms, and so it reduces the computation and memory requirements. Experiments on both synthetic and real images demonstrate that the proposed method has excellent performance in handling occlusion and overlapping ellipses.

***Index Terms*—** ellipse extraction, line segment extraction, arc segment grouping, RANSAC

1. INTRODUCTION

The extraction of ellipses from images is a key problem in computer vision and pattern recognition for different applications, such as camera calibration, traffic sign recognition and object detection in industrial manufacturing [1] [2].

Most methods proposed for detecting ellipses in images generally fall into three groups: Hough Transform (HT) [3-5], Genetic Algorithm (GA) [6], and edge-following algorithms [7] [8]. Hough transform (HT) is one of the most widely used techniques for feature extraction owing to its simple theory for operation, but its high computation and memory requirements make it practically unfavorable. Even though many modifications (e.g. see [4] [5]) have been proposed to improve the standard HT, they still suffer from various problems such as dependence on the choice of parameters, interference from complex background and occlusion. The genetic algorithms (GA) are relatively more effective for detecting ellipses with partial occlusion. Unfortunately, they are computationally expensive. Besides, GA has an inherent risk of finding a suboptimal solution. The edge-following methods work on the original edge images and trace the boundary of the objects to get a list of chained edge points. They are computationally more efficient than both HT and GA based methods. However, the existing edge-following methods are unable to handle one or more of the following: overlapping features, occlusion, multiple ellipses or images with complex background. A reason for the shortcomings of the edge-following methods is that they pay too much attention to local features but ignore the importance of viewing the whole in a global way.

The work presented here is based on the idea of edge following, but the difficulty of lack of global view in existing methods is addressed by means of a new hierarchical framework. In this framework, the image is described as a set of edge pixels at the lowest level, from which line segments are extracted. Then, line segments that are potential candidates of elliptic arcs are linked to form arc segments according to connectivity and curvature relations. After that, arc segments that belong to the same ellipse are grouped together. Finally, RANSAC is applied to fit ellipses robustly. Experimental results demonstrate that our method is capable of accurately extracting multiple ellipses from an image with complex background, even when the ellipses are partially overlapped or occluded.

The paper is organized as follows. In section 2, we describe the proposed hierarchical approach consisting of four different stages. In Section 3, experimental results using both synthetic and real images are provided to illustrate the performance of our method in comparison with HT-based methods and the Upwrite [7] method. Some concluding remarks are given in Section 4.

2. THE HIERARCHICAL APPROACH--FROM EDGE POINTS TO ELLIPSES

In this paper, we adopt a hierarchical approach to ellipse detection, generating progressively higher level descriptions

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of the data. The hierarchical algorithm can be regarded as consisting of 4 stages:

(i) Line segment extraction - at the lowest level, the image is described as a set of edge pixels, from which line segments are extracted;
(ii) Line segment linking - line segments that are potential candidates of elliptic arcs are linked to form arc segments;
(iii) Arc segment grouping - arc segments that belong to the same ellipse are grouped together;
(iv) Ellipse fitting - RANSAC is used to robustly fit ellipse in each arc group.

The detailed operations in each stage will be described in the following subsections.

2.1 Line Segment Extraction

The first stage, line segment extraction, is to represent the boundaries of the objects in the images by piecewise line segments. we use P. D. Kovesi's code for edge linking and line segment fitting [9], with a slightly modification to output not only the extracted line segments, but also their corresponding supporters. Here a supporter of a certain feature (line segment, arc segment, or ellipse) is generally defined as an edge point sufficiently close to the feature where the closeness is measured by the Euclidean distance of the edge point to the feature. Thus in this paper, the term 'line segment' or 'arc segment' not only refers to the geometric line feature or arc feature, but also their corresponding supporters.

2.2 Line Segment Linking

After line segment extraction, the boundaries of the objects in the images are represented by line segments. An elliptic arc will be approximated by a sequence of connected line segments. The purpose of the line segment linking stage is to link connected line segments into arc segments. Since elliptic arcs are convex, we are only interested in arcs with a consistent sign in its curvature. A sequence of line segments will be linked into an arc segment if they satisfy the following conditions for connectivity and curvature.

Connectivity Condition:

Given two line segments l_1 and l_2 each with two end points, the proximity d of the two line segments is measured by the smallest of the four distances between their end points. The connectivity condition is:

$$d \le d_0 \tag{1}$$

where d_0 is a predefined threshold for closeness. We will refer to two line segments satisfying the above condition as *connected line segments*. A sequence of n connected line segments refers to an n-tuple in which each line segment is connected to the next element in the n-tuple except that the last may have no one to connect or it may be connected to the first element in the n-tuple.

Curvature Condition:

If an elliptic arc is approximated by a sequence of connected line segments, the angle between two connected line segments represents the curvature of the arc. This angle should not be too small in order for the sequence of line segments to represent a genuine arc, but yet not too large so that the arc does not have a sharp turning point. Furthermore, the curvature should be of the same sign throughout the sequence for it to represent a convex arc.

Let $l_i (i = 1,\cdots,n)$ be a sequence of connected line segments, and let $\beta_i (i = 1,\cdots,n-1)$ be the angle from l_i to l_{i+1}. Then, the curvature condition can be stated as:

$$\beta_{\min} \le |\beta_i| \le \beta_{\max} \quad (i = 1,\cdots,n-1) \tag{2a}$$

and

$$\text{all } \beta_i (i = 1,\cdots,n-1) \text{ are of the same sign.} \tag{2b}$$

This means the orientation deviation should follow the same direction along the linking direction for the same chain of line segments to guarantee convexity. In most cases, a sequence of connected line segments representing an elliptical arc consists of at least 3 line segments, so we define a sequence of n ($\ge$ 3) connected line segments satisfying the curvature condition as an elliptical arc segment. In the algorithm to link line segments into elliptic arcs, we define:

$\{l\}$ = the set of line segments which have not been linked, initialized to include all extracted line segments;

$\{c\}$ = a set of elliptical arcs, each of which is a sequence of line segments satisfying the connectivity and curvature conditions, initialized to be the empty set Φ.

We search for sequence of n connected line segments $s = (l_1, l_2, \cdots, l_n)$ with $l_i \in \{l\}$ and $n \ge 3$. If s satisfies (2a, b), we enter it into $\{c\}$, and in any case, the line segments of s are removed from $\{l\}$. This process is repeated until $\{l\}$ does not contain any further sequence of line segments with length $\ge$ 3.

2.3 Arc Segment Grouping

After the local linking process, connected line segments which support the same ellipse have been linked together locally as an elliptical arc segment. In this section, we will group the arc segments that come from the same ellipse together. The criterion for arcs to be grouped into the same ellipse is as follows:

Given a pair of arcs, let (x_i^j, y_i^j) $i = 1,2,\ldots,M_j$ $j = 1,2$ be i^{th} supporter of the j^{th} arc, where M_j is the number of supporters for the j^{th} arc. Fit an ellipse Γ to all the supporters using least-squares technique. Let e_i^j be the residual error given by the geometric distance from (x_i^j, y_i^j) to Γ. A supporter for the fitted ellipse Γ is defined as a point (x_i^j, y_i^j) whose residue error e_i^j is less than a given threshold. Suppose there are m_j supporters of the ellipse from the j^{th} arc ($j = 1,2$), then we can compute the supporter ratio of the j^{th} arc for the ellipse as:

$$r_j = \frac{m_j}{M_j} \quad (j = 1,2) \tag{3}$$

If both r_1 and r_2 are larger than a given threshold, we consider the two arc segments as belonging to the same ellipse and merge them into a group. Regarding a group of arc segments as if they are a single arc segment, we can repeat the above process to merge further arcs into the group. This technique avoids the problem of mistakenly merging a short arc segment with a long one dominant in length [11].

2.3 Ellipse fitting by applying RANSAC

Up to now we have grouped arc segments together with their supporters into a group that presumably belong to the same ellipse. However, due to noise and extraneous features in the image, some supporters of the arc segments which do not belong to the ellipse may have been included in the group. Hence, the supporters of the arc segments can be classified as inliers that fit the ellipse and outliers that do not. In order to extract the ellipse accurately using only the inlier points, we apply RANSAC (random sample consensus) [10] to each group to remove the outliers while recovering the ellipse model.

3. EXPERIMENTAL RESULTS

We performed experiments on both synthetic data and real images to evaluate the performance of the proposed method. The parameters used in the algorithm are set as follows. In the linking stage, the default threshold for the connectivity constraint is $d_0 = 5$ pixels, and the default thresholds for the curvature constraint are $\beta_{\min} = 0$, $\beta_{\max} = 0.5$ rad. In most cases, the default parameters can work well and they need not extra tuning. The experiments are performed on a Pentium4 PC. To compare the accuracy and computation time of our method with existing techniques, three HT-based algorithms (Standard HT, Randomized HT and Probabilistic HT) and UpWrite [7] are selected as reference algorithms. When testing these reference algorithms, we run the source code directly after edge detection. We have tried to tune the parameters in the source code but found that the default values [7] perform the best, therefore the default values are used here.

We generate many synthetic edge maps, and show one testing result as representative in Fig. 1. It contains four separate ellipses, two of which are incomplete simulating occlusion. Having the ground truth, we use the ratio $\alpha = S_n / S_g$ to evaluate the extraction accuracy, where S_n is the non-overlapping area of the ground truth and the extracted ellipse, and S_g is the area of the ground truth ellipse. According to the experiments, we regard the extracted ellipse correct if $\alpha < 0.1$. The performances are compared in Table 1. We also test our method on real images, one of which is shown in Fig. 2. The scene is complex consisting of concentric ellipses, ellipses with occlusion and overlapping. The reference algorithms failed to recognize any of the elliptical objects even after we tune the parameters, whereas our algorithm can detect all three ellipses in the images successfully within a reasonable time period of 18 seconds. Furthermore, test results not shown here also show that our method outperforms the reference algorithms in all cases that we have tried. In Table 2, we present some statistics on performance comparison.

5. CONCLUSIONS AND DISCUSSION

This paper presents a fast and robust method to detect elliptical objects in the images. We detect the ellipses in a hierarchical approach, whereby the features of an ellipse are detected progressively from a local to the global level. The success of our method lies in (i) use of feature-based criteria for removing edge points associated with non-elliptical objects, (ii) retention of local information, namely supporter points, at the final stage ellipse fitting, and (iii) use of RANSAC to remove outliers.

This method does not need a high dimensional parameter space like Hough Transform based algorithms, and so it reduces the computation and memory requirements. The algorithm is easy to implement and the performance is good. Generally, it is capable of extracting multiple ellipses with complex background in the images quickly and correctly, even when some of them are occluded by other objects. Experiments on synthetic and real images show that the proposed method is superior to the existing methods, especially when the testing images are complicated.

However, our method may have difficulties when the edges of the elliptical objects are not clear-cut. It may fail to extract ellipses which are too small in the images. We may improve our method to handle these in the future.

6. REFERENCES

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Table1 Testing results of synthetic image1

		SHT	**RHT**	**PHT**	**Up-Write**	**Ours**
Time(s)		36.61	0.71	2.07	0.39	1.02
Accuracy	#detected ellipses	1	3	2	2	4
	# correctly detected ellipses	1	1	1	2	4

Table2 Extraction accuracy statistics on synthetic images*

Method / Accuracy %	**SHT**	**RHT**	**PHT**	**Up-Write**	**Ours**
# correctly detected ellipses/ total # ellipses	39%	62%	35%	73%	100%

* The statistics is base on the testing results of 95 randomly generated synthetic full ellipses, coming from:

- 20 images, each of which contains one ellipse;
- 15 images, each of which contains three separate ellipses;
- 15 images, each of which contains two overlapping ellipses

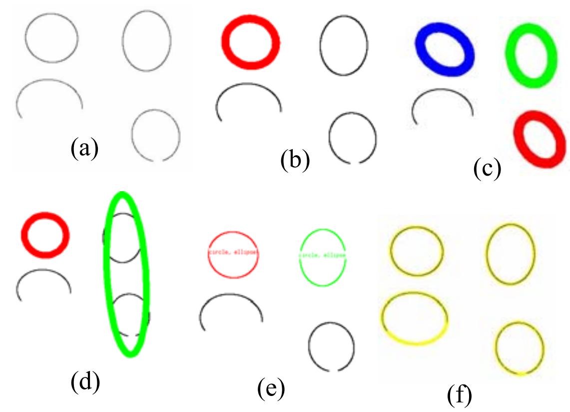
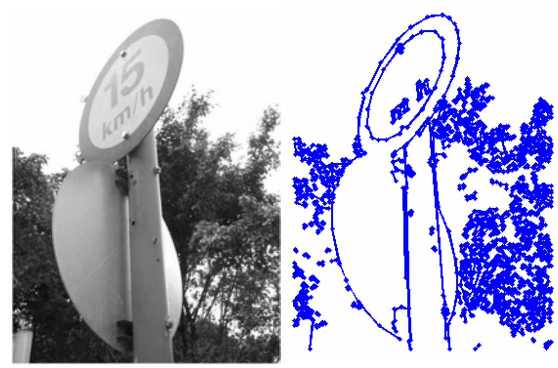


Fig1 testing results for synthetic image 1 (size 500×400)
(a) original image for testing
(b) testing result by SHT (the thick one);
(c) testing result by RHT (three thick ellipse);
(d) testing result by PHT (two thick ellipses);
(e) testing result by UpWrite (the upper two thin ellipses);
(f) testing result by the proposed method (all the four thin ellipse)



(a) (b)

(c)

Fig2. Testing results for real image 1(traffic sign) with size 640×480
(a) original image
(b) the extracted line segments
(c) extracted ellipses overlaid on the original image

computation time: 18.22(s)