# ANALYZING SYMBOL AND BIT PLANE-BASED LDPC IN DISTRIBUTED VIDEO CODING

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## ABSTRACT

Many distributed video coders are implemented using sophisticated error correction codes that use soft information (conditional probabilities) as *a priori* knowledge. This *a priori* information models the dependency behavior between the input data and side-information. In this paper we analyze both a symbol and bit plane-based approach using LDPC codes. We show both theoretically and experimentally that a bit plane-based encoder has the same performance as a symbol-based coder, if an appropriate dependency model is chosen. We argue that due to a significant complexity reduction a bit plane-based coder is preferable over a symbol-based approach.

***Index Terms—*** Distributed video coding, bit plane-based, symbol-based, LDPC

## 1. INTRODUCTION

The attractive idea of distributed source coding is that, in the case of jointly decoding, two correlated sources $X$ and $Y$ can be compressed separately without the awareness of the other source and still attain the same compression as if the other source was known. For the specific case where $Y$ is available at the decoder and $X$ and $Y$ are jointly Gaussian, Wyner [1] proved that by using channel coding at the encoder it does not matter whether $Y$ is not known at the encoder. Moreover, since the generation of a correlated source is only necessary at the decoder, this has led to new insights in the video coding community especially for the purpose of low complexity encoding of video. By using channel codes the differences between the correlated distorted version $Y$ and the original input $X$ can be corrected. In distributed video coding the correlated version $Y$ is obtained from the motion compensated prediction of $X$ generated at the decoder. A lossy distributed video compression scheme is usually equipped with a lossy quantization step before actual lossless compression (Slepian-Wolf (SW) coding) of the input data. In literature two different approaches can be found, which are shown in Figure 1. In the first symbol-based approach (Figure 1(a)) the symbols of the original frame $X$ are first quantized by a $2^L$-level quantizer and SW coding is done on the symbols of $Q$ and $Y$ [4, 5]. In the second bit plane-based approach (Figure 1(a)) the bit planes of $Q$ are first extracted before encoding [6, 7]. Initially

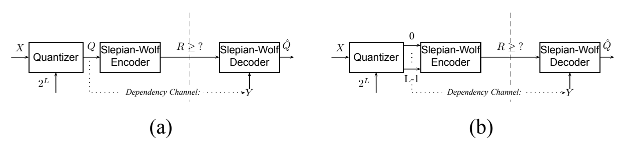


**Fig. 1**. Distributed source coding using (a) symbol-based SW decoder (S-SWD) (b) bit plane-based SW decoder (B-SWD)

distributed video coders (DVC) were carried out with simple syndrome decoding techniques [2]. Not much later, DVCs with more sophisticated codes, such as Turbo codes, were introduced [3]. The benefits of these type of codes is that they incorporate the underlying statistics of the channel noise into the decoding process. This means for DVC that the symbol-based dependency model (S-DM) $P(X|Y)$ between the symbols of $X$ and $Y$ can be incorporated if it is known or modeled in advance. Moreover, in the first approach the symbol-based Slepian-Wolf decoder (S-SWD) needs a symbol-based dependency model (S-DM) that describes the relation between the quantized symbols $Q$ and side information symbols $Y$. This can directly be extracted from the dependency model between $X$ and $Y$. For the second approach the bit plane-based SW decoder (B-SWD) needs a bit plane-based dependency model (B-DM) that describes the relation between the bit planes of $Q$ and side information $Y$.

In this paper we analyze both approaches theoretically and experimentally. First question we answer is how to choose the dependency model in the case of S-SWD and B-SWD in order to compare both approaches. In many studies simplified dependency models are used to describe the bit plane-based dependency, like a BSC dependency [6] or directly calculated from the earlier S-DM [7], which oversimplifies the actual dependency. Therefore secondly more complex B-DMs are analyzed, where the symbols of $Y$ are taken into account together with already correctly decoded bit planes. To do a full analysis also the complexity of decoding is taken into account. In [8] it was shown that the complexity of the LDPC decoding grows linearly with the number of bits per symbol. This only holds true for a specific implementation with FFT-transforms, while other standard implementations using convolutions are far more complex.

The structure of this paper is as follows. In Section 2 the symbol-based dependency model and three different bit plane-based dependency models are discussed and compared by computing the minimal achievable rates. Section 3 describes how these dependency behaviors can be modeled at the decoder side. For each of these models an analytical expression is given. These models are experimentally compared in Section 4 using synthetic and real video data. Finally, conclusions can be found in Section 5.

## 2. ACHIEVABLE RATES

In this section we discuss and compare the minimal achievable rate to have lossless Slepian-Wolf coding for a B-SWD and a S-SWD as shown in Figure 1. As shown SW coding in S-SWD is done on the $2^L$-level quantized input symbols $Q$of $X$. While in B-SWD SW coding is done on the $L$ bit planes of $Q$ separately. The minimal achievable rate is expressed by Slepian and Wolf in terms of the conditional entropy between $Q$ and $Y$. This is straightforward in the case of S-SWD and is given by $H(Q|Y)$. In the case of B-SWD the rate is dependent on the type of information that is taken into account in this conditional entropy measure. The dependency model can be expressed by

1. the corresponding bit plane $Q$ and $Y$ (B-DM$_\mathrm{b}$), i.e. $P(Q^{(b)}|Y^{(b)})$ with $b = 0$ as LSB and $b = L-1$ as MSB.
2. the bit plane of $Q$ and all bit planes – i.e. the symbol – $Y$ (B-DM$_\mathrm{s}$), i.e. $P(Q|Y^{(b)})$.
3. the bit plane of $Q$ and symbol $Y$ together with already (correctly) decoded bit planes of $Q$ (B-DM$_\mathrm{s+b}$), i.e. $P(Q^{(b)}|Y, Q^{(b+1)}, \dots, Q^{(L-1)})$

The order of the list indicates the complexity of the dependency incorporated in the model. In theory by using all available information at the decoder side the minimal achievable rates for S-SWD and B-SWD are *identical*, since

$$\begin{aligned} H(Q|Y) &= H(Q^{(0)}, \dots, Q^{(L-1)}|Y) \\ &= \sum_{b=0}^{L-2} H(Q^{(b)}|Y, Q^{(b+1)}, \dots, Q^{(L-1)}) \\ &\quad + H(Q^{(L-1)}|Y). \end{aligned} \tag{1}$$

The models B-DM$_\mathrm{b}$ and B-DM$_\mathrm{s}$ result in an increase in entropy since

$$\begin{aligned} H(Q|Y) &= \sum_{b=0}^{L-2} H(Q^{(b)}|Y, Q^{(b+1)}, \dots, Q^{(L-1)} \\ &\quad + H(Q^{(L-1)}|Y) \\ &\leq \sum_{b=0}^{L-1} H(Q^{(b)}|Y) \leq \sum_{b=0}^{L-1} H(Q^{(b)}|Y^{(b)}). \end{aligned} \tag{2}$$

Consequently, the B-DM$_\mathrm{b}$ and B-DM$_\mathrm{s}$ models will result in a performance loss when compared to the B-DM$_\mathrm{s+b}$ and S-DM in Eq. (1). If we rank the expected performance of all models, we expect S-DM and B-DM$_\mathrm{s+b}$ to perform the best, and B-DM$_\mathrm{b}$ to have the worst performance.

## 3. PRACTICAL DEPENDENCY MODELS

To implement the dependency models in a practical LDPC-based distributed video decoder, we need estimates of the probabilities $P(Q|Y)$, $P(Q^{(b)}|Y^{(b)})$, $P(Q^{(b)}|Y)$ and $P(Q^{(b)}|Y, Q^{(b+1)}, \dots, Q^{(L-1)})$ of the four dependency models on which the entropies were based in the previous section. Following the work in [9] and [5], we determine the S-DM $P(Q|Y)$ based on a Laplacian PDF $P_N(n)$ that models the behavior of the noise $N$ in the dependency channel. Then, the S-DM discrete conditional probabilities $P(Q|Y)$ are found by adding the probabilities of $P(X|Y) = P_N(n)$ for $X$-values lying inside the same quantization bin.

First estimates are found for the B-DM$_\mathrm{s}$ conditional probabilities $P(Q^{(b)}|Y)$ from which the B-DM$_\mathrm{b}$ probabilities $P(Q^{(b)}|Y^{(b)})$ can be derived. The conditional probabilities for the B-DM are derived from the S-DM $P(Q|Y)$. For B-DM$_\mathrm{s}$ holds that both bit values of bit plane $b = 0$ of $Q$ and $Y$ are equal if the distance between $Q$ and $Y$ is 0 or a multiple of 2, since

$$P(Q^{(0)} = b_q|Y = y, Y^{(0)} = b_q) = \sum_{q=\cdots,y-2,y,y+2,\dots} P(Q = q|Y = y). \tag{3}$$

In Eq. (3) $b_q$ is a constant that is either 0 or 1. Dependent on the bit value of $Y$ in the corresponding bit plane $b$ the B-DM$_\mathrm{s}$ can be derived a similar yet somewhat more complicated expression , namely:

$$\begin{aligned} &P[Q^{(b)} = b_q|Y = y, Y^{(b)} = b_q] \\ &\quad = \sum_{m=-q^-}^{q^+} \sum_{n=0}^{r_{max}} P(y + m \cdot d + (n - r(y, 2^b))|y) \\ &\quad = \sum_{m=-q^-}^{q^+} \sum_{n=0}^{r_{max}} P(m \cdot d + (n - r(y, 2^b))), \end{aligned} \tag{4}$$

with $q^- = q(y, 2^{b+1})$, $q^+ = q((2^L - 1) - y, 2^{b+1})$, $r_{max} = 2^b - 1$, $d = 2^{b+1}$ and where the functions $r(a, b)$ and $q(a, b)$ are the remainder and quotient of the division between two integer values $a$ and $b$. The conditional probability $P(Q^{(b)}|Y^{(b)})$ of the B-DM$_\mathrm{b}$ can now be found using Eq. (4) as follows:

$$\begin{aligned} &P[Q^{(b)} = b_q|Y^{(b)} = b_q] \\ &\quad = \sum_{\forall Y} P[Q^{(b)} = b_q|Y = y, Y^{(b)} = b_q] \cdot P[Y = y|Y^{(b)} = b_q], \end{aligned} \tag{5}$$

where $P[Y = y|Y^{(b)} = b_q]$ is 0 or 1 depending on whether the bit plane of $Y$ equals $b_q$. Note that Eq. (5) does not yield a BSC model. From Eq. (5) we can obtain a BSC model with crossover probability $p_c$ as follows:

$$p_c = \sum_{b_q=0,1} P[Q^{(b)} \neq b_q|Y^{(b)} = b_q] P[Y^{(b)} = b_q], \tag{6}$$

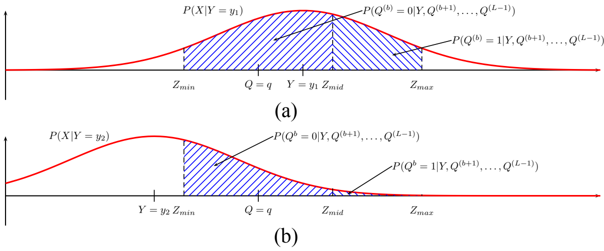


**Fig. 2**. Finding conditional probabilities of $Q^b = 0, 1$ given the conditional probability density model $P(Q|Y)$ and the values of all previous decoded bit planes ($Q^{b+1}$,. . .,$Q^L$)

where $P[Y^{(b)}]$ are the *a priori* probabilities of bit plane $b$ of $Y$. Finally, the information from the previous correctly decoded bit planes is incorporated to derive the conditional probabilities $P(Q^{(b)}|Y, Q^{(b+1)}, \ldots, Q^{(L-1)})$ for the B-DM$_{\mathrm{s+b}}$ dependency model. Figure 2 shows the conditional probabilities for bit plane $b$ for two different cases together with the continuous dependency model $P(X|Y)$ for two different values of $Y$ ($y_1$ and $y_2$), the quantized original symbol $Q = q$ and three threshold values $Z_{min}$, $Z_{mid}$ and $Z_{max}$. The threshold values have the same bit values for $b = b + 1, \ldots, L-1$ as the already decoded bit planes of $Q = q$. Furthermore, $Z_{min}$, $Z_{mid}$ and $Z_{max}$ is the minimum, the middle value and maximum value with the same previous decoded bit plane values as $Q = q$. So that the area under the density $P(X|Y)$ from $Z_{min}$ to $Z_{mid}$ and $Z_{mid}$ to $Z_{max}$ indicates the value of $P(Q^{(b)}|Y, Q^{(b+1)}, \ldots, Q^{(L-1)})$ with $Q^{(b)} = 0$ and $Q^{(b)} = 1$ of the B-DM$_{\mathrm{s+b}}$ dependency model respectively:

$$\begin{aligned}
&P(Q^{(b)} = 0|Y = y, Q^{(b+1)}, \ldots, Q^{(L-1)}) \\
&\quad = \sum_{i=0}^{2^b-1} P(q(x_p, 2^{b+1}) \cdot 2^{b+1} + i|Y = y) \\
&\quad = \sum_{i=0}^{2^b-1} P_N(q(x_p, 2^{b+1}) \cdot 2^{b+1} + i - y), \\
&P(Q^{(b)} = 1|Y = y, Q^{(b+1)}, \ldots, Q^{(L-1)}) \\
&\quad = \sum_{i=2^b}^{2^{b+1}-1} P_N(q(x_p, 2^{b+1}) \cdot 2^{b+1} + i - y). \qquad (7)
\end{aligned}$$

with $x_p = \sum_{i=b+1}^{L-1} Q^{(i)} \cdot 2^i$.

## 4. EXPERIMENTS

In this section we experimentally compare the compression ratios of a bit plane and symbol LDPC-based distributed video coder on synthetically generated data and real video data using the theoretical dependency models of Section 3.

In the first experiment we use synthetically generated data. The original data $X$ is a discrete-valued vector of length $K =$ 25344 (QCIF size); each value is randomly drawn from an uniform distribution on $[0, 255]$. Side information $Y$ is generated by adding Gaussian distributed random noise $N$ with with zero mean and variance $\sigma_N^2 = 4$ to $X$. Both the original data as well as side information $Y$ are quantized using a 16-level uniform quantizer, hence the input to the SW coder are 16-level symbols or 4 bit planes. The S-DM conditional probabilities $P(Q|Y)$ are calculated from the noise $N$. The conditional probabilities required for the B-DMs are calculated using the Eqs. (4), (5) and (7) in Section 3. The resulting conditional entropies, calculated using Eqs. (1) and (2), are given in Table 1, third row (labeled 'Entropy $H$'), for the four different dependency channel models. We observe that these estimates satisfy the (in)equalities in Eqs. (1) and (2). We also observe that the simple model B-DM$_{\mathrm{b}}$ performs very poorly compared to the more sophisticated channel models. The fourth row in Table 1 gives the result of actual bit plane and symbol-based DVC coding using LDPC codes [10, 11]. The listed bit rates are those minimally required required for error-free decoding of the received LDPC bit stream. We observe that the performance rating in this case is conform the measured entropy values of each of the dependency models. We can also observe that there is still a big gap between the minimal coded bit rate needed for perfect decoding and the measured entropy value. In the second experiment real video

| | Dependency | | | |
|---|---|---|---|---|
| | S-DM | B-DM$_{\mathrm{b}}$ | B-DM$_{\mathrm{s}}$ | B-DM$_{\mathrm{s+b}}$ |
| Entropy $H$ | 0.28 | 0.88 | 0.48 | 0.28 |
| Coded Rate $R$ | 0.41 | 1.21 | 0.67 | 0.41 |

**Table 1**. Total entropies in and real coded rate in bit/symbol for the four different dependency model assumptions: (1) S-DM, (2) B-DM$_{\mathrm{b}}$, (3) B-DM$_{\mathrm{s}}$ and (4) B-DM$_{\mathrm{s+b}}$

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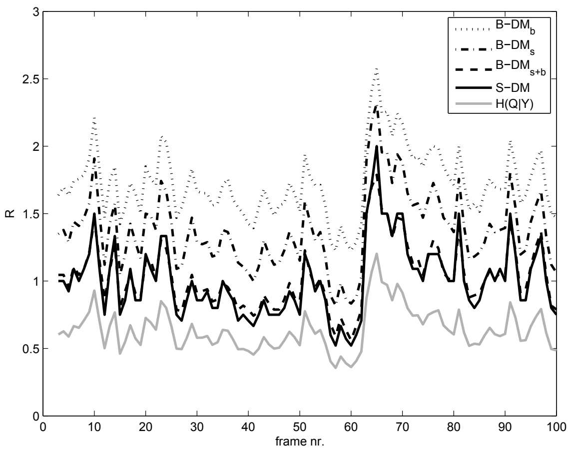
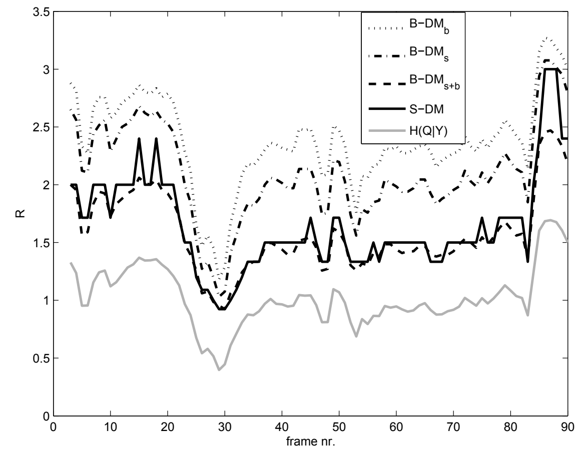
data is used. The side information $Y$ is an extrapolated prediction of $X$ based on three previous frames [5]. The dependency between $X$ and $Y$ is estimated by a Laplacian PDF with zero mean. The variance of the PDF is estimated from the correctly decoded frame $Q$ and predicted side information $Y$ of the previous frame. In this case the dependency model is an estimate of the real behavior of the data. Before SW coding with LDPC the input is quantized with a 16-level uniform quantizer. The dependency behavior between the symbols of $X$ and $Y$ is modeled by a Laplacian PDF with zero mean. In Figure 3 the minimal coded bit rates needed for perfect decoding for the four assumed S-DM and B-DM models are shown as well as the achievable rate $H(Q|Y)$ for the first 100 frames of the Foreman sequence and the first 90 frames of the Stefan sequence, both in qcif format. This figure also shows that there is a big gap between the theoretical boundary $H(Q|Y)$ and the minimal coded bit rate. We further observe from Figure 3 the following. First, S-DM and B-DM$_{\mathrm{s+b}}$ perform equally and both outperform the B-DM$_{\mathrm{b}}$ and B-DM$_{\mathrm{s}}$ which satisfies Eq. 1. The differences between S-DM and




(a) Foreman




(b) Stefan

**Fig. 3**. Coded bit rate results using different dependency models for the symbol and bit plane-based distributed video coders for both the (a) Foreman and (b) Stefan sequences

B-DM$_{s+b}$ are caused by the limited amount of coding rates of the symbol and bit plane-based DVC coder.

## 5. CONCLUSIONS

In this paper we have shown that if the dependency channel model is known both the a symbol-based and a bit plane-based approach can performs equally, conditioned on the dependency measure that is taken into account. Also in a distributed video coder, where the dependency channel is not known, both approaches performs approximately equally if the correct bit plane dependency model is assumed. Since the complexity of a symbol-based LDPC decoder is roughly $L$ times ($L$ denotes the number of bit planes) more complex than a bit plane-based decoder. A bit plane-based LDPC coder is preferred for the purpose of distributed video coding.

## 6. REFERENCES

[1] A. Wyner, "Recent results in the shannon theory," *IEEE Transactions on Information Theory*, vol. 20, no. 1, January 1974.

[2] S.S. Pradhan and K. Ramchandran, "Distributed source coding using syndromes (discus): design and construction," *IEEE Transactions on Information Theory*, vol. 49, no. 3, pp. 626–643, March 2003.

[3] A. Aaron and B. Girod, "Compression with side information using turbo codes," in *Proceedings IEEE Data Compression Conference*, April 2002, pp. 252–261.

[4] A. Aaron, D. Varodayan, and B. Girod, "Wyner-ziv residual coding of video," in *Proceedings Picture Coding Symposium*, Beijing, China, April 2006.

[5] R. P. Westerlaken, S. Borchert, R. Klein Gunnewiek, and R.L. Lagendijk, "Dependency channel modeling for a ldpc-based wyner-ziv video compression scheme," in *Proceedings 2006 IEEE International Conference on Image Processing, ICIP '06*, Atlanta, USA, October 2006, pp. 277–280.

[6] A. Aaron, S. Rane, E. Setton, and B. Girod, "Transform-domain wyner-ziv codec for video," in *Proceedings Visual Communications and Image Processing*, San Jose,CA, January 2004.

[7] M. Dalai, R. Leonardi, and F. Pereira, "Improving turbo codec integration in pixel-domain distributed video coding," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, Toulouse, France, 14-19 May 2006, vol. 2, pp. 537–540.

[8] T. Richardson and R. Urbanke, "The capacity of low-density parity check codes under message-passing decoding," *IEEE Transactions on Information Theory*, vol. 47, pp. 599–618, Februari 2001.

[9] A. Aaron and B. Girod, "Wyner-ziv video coding with low encoder complexity," in *Proceedings International Picture Coding Symposium, PCS04, invited paper*, San Francisco,CA, December 2004.

[10] R.P. Westerlaken, R. Klein Gunnewiek, and R.L. Lagendijk, "The role of the virtual channel in distributed source coding of video," in *Proc. IEEE International Conference on Image Processing*, September 2005, vol. 1, pp. 581–584.

[11] D. Varodayan, A. Aaron, and B. Girod, "Rate-adaptive distributed source coding using low-density parity-check codes," in *Proceedings Asilomar Conference on Signals, Systems, and Computers*, October 2005, pp. 1203–1207.