Lagrangian Strain Tensor Computation with Higher Order Variational Models

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1 Motivation

In specific application areas, obtaining higher order motion information is of great interest. An example of such information is the Lagrangian strain tensor [3] that plays a vital role in mechanical engineering. Since this tensor is computed by means of first-order motion derivatives, it is tempting to estimate the optical flow field with a highly accurate variational model and compute its derivatives afterwards. However, this procedure yields in general not very good results because of the following reasons: First, these derivatives have to be obtained via numerical differentiation. Differentiation is a classical ill-posed problem where small fluctuations in the data can cause large deviations in the result [1]. Secondly, most high-accurate models make use of a first-order smoothness assumption that causes the results to be biased towards translational motion; see e.g. [5].

2 Contribution

In this paper, we propose a novel approach for deriving higher order variational models that offer the following advantages: They directly estimate regularised versions of the motion derivatives, which circumvents the problems of numerical differentiation. Moreover, they deploy higher order smoothness assumptions that do not suffer from translational bias.

3 Our Recursive Approach

First we consider a *baseline model* that we want to extend. For instance, a variational approach for optical flow computation can be based on a Horn and Schunck model [2] with a data term that can handle large displacements. It computes the motion flow field $\mathbf{u}$ between two images f_1 and f_2 defined on $\Omega \subset \mathbb{R}^2$ as minimiser of the energy functional

$$E_1(\mathbf{u}) = \int_\Omega \left(\left(f_2(\mathbf{x}+\mathbf{u}) - f_1(\mathbf{x}) \right)^2 + \alpha \underbrace{\sum_{i=1}^{2} \|\nabla u_i\|^2}_{=:S_1(\mathbf{u})} \right) \mathrm{d}\mathbf{x}\,.$$

This method suffers from the aforementioned drawbacks: It only estimates the motion between the images, and the first-order smoothness term S_1 prefers solutions biased towards translational motion.

As a remedy, we derive a *new model* that replaces S_1 by two new terms. It minimises the energy

$$E_2(\mathbf{u},\mathbf{A}) = \int_\Omega \Bigg(\left(f_2(\mathbf{x}+\mathbf{u}) - f_1(\mathbf{x}) \right)^2 + \alpha_1 \underbrace{\Big(\sum_{i,j=1}^{2} \left(a_{i,j} - \partial_{x_j} u_i \right)^2}_{=:M(\mathbf{u},\mathbf{A})} + \alpha_2 \underbrace{\sum_{i,j=1}^{2} \|\nabla a_{i,j}\|^2 \Big)}_{=:S_2(\mathbf{A})} \Bigg) \mathrm{d}\mathbf{x}\,.$$

The new terms have the following effects: The similarity term M causes the matrix-valued function $\mathbf{A} = [a_{i,j}]_{2\times 2}$ to contain estimates of the first-order motion derivatives that can be used for the strain tensor computation. Furthermore, the first-order smoothness term S_2 ensures that these estimates are regularised. Finally, combining M and S_2 yields a second-order smoothness assumption for the motion $\mathbf{u}$, which no longer causes the translational bias.

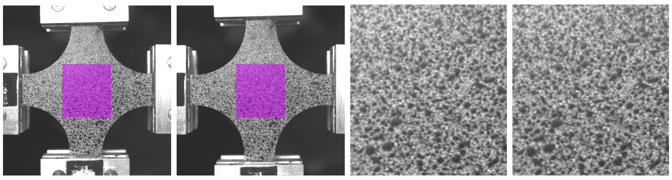

Figure 1: **(a) Left:** Two frames of the biaxial tensile experiment with the areas of interest colored in purple. **(b) Right:** Enlarged versions of the respective areas of interest.

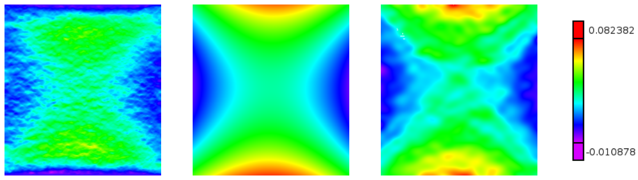

Figure 2: Results for the strain in y-direction obtained from different methods. **(a) Left:** Numerical differentiation of optical flow field. **(b) Centre:** Higher order model obtained from presented approach. **(c) Right:** Commercial software Vic-2D.

This procedure may be repeated by using E_2 as the baseline model and replacing S_2, which indicates that this approach is of generic nature and therefore can be used to derive models of arbitrary order n.

4 Experiment

In order to assess the performance of our new approach, we consider image data of a biaxial tensile experiment that is often conducted in mechanical engineering [4]. We consider the area of interest depicted in Figure 1. In Figure 2, we observe that a higher order model obtained from the presented approach gives much better results than a direct numerical differentiation of the optical flow field. Last but not least, our model is also able to outperform the commercial software Vic-2D that belongs to the leading tools for evaluating data of such experiments.

References

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