

Closed form solution for radial distortion estimation from a single vanishing point

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Radial lens distortion found in real, most notably off-the-shelf medium to wide angle optics can be quite severe. A-priori calibration remedies the problem, but requires access to the camera. Alternative approaches make use of correspondences in multiple images of a moving camera [3], relying on sufficiently overlapping views.

Our paper fits in the category of techniques, where straight scene lines are used to determine distortion parameters from a single image. These plumbline methods [2] critically depend on the existence of long edges and get thrown off easily if a significant portion originates from non-linear structures. Here, the use of vanishing points provides an advantage: When a large number of image edges groups *w.r.t.* to a common vanishing point, it is very likely that they stem from parallel scene lines. Recent methods for vanishing point detection either ignore radial distortion completely, or assume only weak distortion, which is accounted for after vanishing point estimation [1, 4]. If strong distortion is present the effects are detrimental to the extraction of vanishing points and such algorithms are bound to fail. Furthermore, when dealing with radial distortion effects in refinement stages, the error is assessed in the undistorted image. This creates significant bias towards estimates shrinking the undistorted image [5]

Contributions: To overcome this limitation, we suggest a simple RANSAC procedure for vanishing point detection by clustering of distorted lines. The proposed technique builds upon: a) Our newly devised closed-form solution for simultaneously estimating a vanishing point and radial distortion from three distorted image lines. b) A consistency measure which avoids bias by quantifying the error of lines *w.r.t.* a vanishing point in the distorted image.

Specifically, our work bases on the division model [3], mapping a distorted image point $\mathbf{x} = (x, y)^T$ to the undistorted point $\tilde{\mathbf{x}}$ by

$$C: \quad \tilde{\mathbf{x}} = \mathbf{x} / (1 + \lambda r^2). \quad (1)$$

Here, $r = \sqrt{x^2 + y^2}$ is the distance of the distorted point to the distortion center, which we fix at the center of the image. Under this model, straight line segments are distorted into circular arcs [5]. Such arcs serve as basic features for our approach. We extract them using a Canny edge detector followed by circle fitting. For each detected arc smoothed estimates of the arc's midpoint $\mathbf{x} = (x, y)^T$ and the normal $\mathbf{n} = (u, v)^T$ of the tangent at that point are computed from a fitted circle.

Closed-form solution: If we undistort \mathbf{x} and map \mathbf{n} accordingly to its undistorted image, the resulting transformed tangent line

$$\tilde{\mathbf{t}} = \begin{pmatrix} u \\ v \\ -ux - vy \end{pmatrix} + \lambda \begin{pmatrix} ux^2 + 2vxy - uy^2 \\ vy^2 + 2uxy - vx^2 \\ 0 \end{pmatrix}. \quad (2)$$

coincides with the line generating the circle, passing through the undistorted vanishing point $\tilde{\mathbf{v}} = (\tilde{v}_1, \tilde{v}_2, \tilde{v}_3)^T$. Stacking the equations $\tilde{\mathbf{t}}_i^T \tilde{\mathbf{v}}_i = 0$, $i = 1, 2, 3$ of three tangent lines, we obtain the generalized eigenvalue problem

$$(D + \lambda E) \tilde{\mathbf{v}} = \begin{pmatrix} d_1 & d_2 & d_3 \\ d_4 & d_5 & d_6 \\ d_7 & d_8 & d_9 \end{pmatrix} + \lambda \begin{pmatrix} e_1 & e_2 & 0 \\ e_3 & e_4 & 0 \\ e_5 & e_6 & 0 \end{pmatrix} \tilde{\mathbf{v}} = \mathbf{0}. \quad (3)$$

The characteristic polynomial of the problem is a quadratic $\lambda^2 c_2 + \lambda c_1 + c_0 = 0$, which can be easily solved. Once λ is obtained, the undistorted vanishing point $\tilde{\mathbf{v}}$ can be found by plugging into (3).

Consistency measure: The consistency of a circular arc with midpoint \mathbf{x} and normal \mathbf{n} *w.r.t.* a vanishing point $\tilde{\mathbf{v}}$ and λ is computed as

$$\text{dist}(s, \tilde{\mathbf{v}}, \lambda) = \frac{l_s}{2} \sin \angle(\mathbf{n}, \mathbf{n}'). \quad (4)$$

Here, \mathbf{n}' is the normal of the arc "corrected" to be compatible with $\tilde{\mathbf{v}}$:

$$\mathbf{n}' = \begin{pmatrix} 1 + \lambda r^2 - 2\lambda x^2 & -2\lambda xy \\ -2\lambda xy & 1 + \lambda r^2 - 2\lambda y^2 \end{pmatrix} \begin{pmatrix} \tilde{v}_3 \frac{y}{1 + \lambda r^2} - \tilde{v}_2 \\ \tilde{v}_1 - \tilde{v}_3 \frac{x}{1 + \lambda r^2} \end{pmatrix} \quad (5)$$

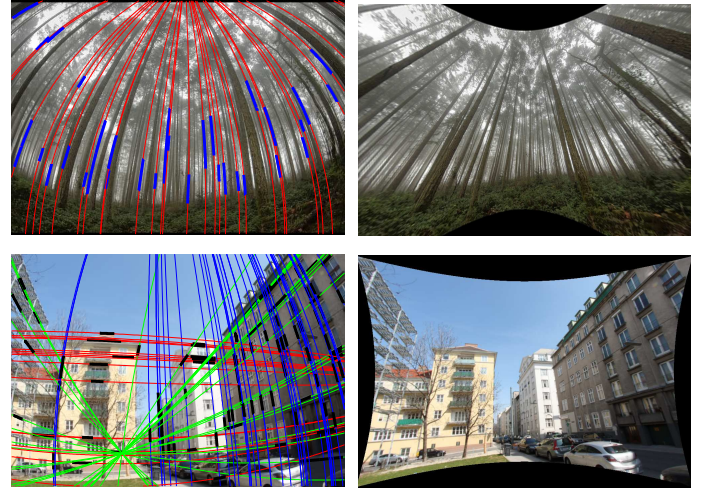


Figure 1: Exemplary results for one (top row) and three vanishing points (bottom row). Left: Circular arcs grouped *w.r.t.* to vanishing points, indicated by converging circles. Right: Automatically undistorted images.

The weighting factor l_s is the arc length in pixels and represents a means of allowing less angular deviation for longer arcs. Furthermore, (4) gracefully handles the case when no distortion ($\lambda = 0$) is present, degenerating to the line segment-based consistency employed in [6].

In Fig. 1 (top row) a typical result obtained with our RANSAC-based method to estimate distortion from a single vanishing point is depicted. One can see that even images of natural scenes may be used to compute visually pleasing results. Unfortunately, many images have more than one vanishing point and if the dominant one is close to the image center, radial distortion cannot be reliably estimated. To remedy this problem, we suggest to augment our method with the vanishing-point based calibration scheme introduced in [6]. Using two additional circular arcs (for a total of five), we arrive at a unified camera calibration approach, which in addition to stably estimating radial distortion, computes the camera's focal length and up to three orthogonal vanishing points. The algorithm was tested on two data sets containing over 200 images of urban environments taken with calibrated cameras. For an example, see Fig. 1 (bottom row). As detailed in the paper, our approach exhibits high accuracy in terms of focal length and radial distortion estimates, handling strong radial distortion just as well as negligible one.

Our conclusion is that it is possible to directly estimate vanishing points in images with strong radial distortion with only little more effort than in the standard, undistorted case. Furthermore, our algorithm may be used to detect vanishing points in images unaffected by radial distortion without the danger of overfitting.

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