Isowarp and Conwarp: Warps that Exactly Comply with Weak-Perspective Projection of Deforming Objects

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Deformable surface reconstruction from monocular views has been intensively studied during the last few years [1, 6]. In the template-based case, the goal is to obtain the 3D shape of the deformed template from a single image. This problem is ill-posed without constraints on how the surface deforms. Shape inference methods have been proposed for a variety of deformation constraints: low-rank shape priors [4, 6], temporal deformation smoothness [7], isometric deformations [1, 2, 5] and conformal deformations [1]. Isometric and conformal constraints accurately model deformations in many real cases and they are described with simple differential constraints. Isometric reconstruction from perspective camera views has attracted much of the attention [1, 5, 6].

We study surface reconstruction with isometric and conformal deformations and weak-perspective projection. In real scenarios, cameras with large focal lengths and shallow scene depth produce images close to affine projection conditions. In those conditions the solution is ambiguous [3] and the state-of-art with perspective cameras provide unstable results. This paper gives analytical solutions and shows that the registration between the template and the input image is not independent of the reconstruction. We define registration warps that comply with the weak-perspective projection of isometric and conformal surfaces, ensuring analytical solutions to compute surface's shape. We show that this method is more accurate than state-of-art methods based on perspective models, when the camera conditions are close to affine.

Figure (1) shows the different functional relationships between the template and the deformed surface. The template is represented as a surface $\mathcal{T} \subset \mathbb{R}^3$ embedded in 3D. We assume that $\mathcal{T}$ has disk topology, admitting a flat template denoted as $\mathcal{P} \subset \mathbb{R}^2$. We denote the flattening as the invertible mapping function $\Delta \subset \mathbb{C}_1$.

The input image $\mathcal{I}$ shows the projection of the deformed template $\mathcal{S} \subset \mathbb{R}^3$. The camera model Π is a weak-perspective projection. We define $\eta \in \mathbb{C}_1$ as the warp mapping a point $\mathbf{p} \in \mathcal{P}$ in the flattened template to a corresponding point $\mathbf{q} = \eta(\mathbf{p}) \in \mathcal{I}$ in the image. Function $\psi \in \mathbb{C}_1$ models deformations between the template $\mathcal{T}$ and the deformed surface $\mathcal{S}$. Conformal and isometric deformations are described with conditions on the first derivatives of ψ, namely J_ψ:

$$J_\psi^\top J_\psi = \mu \mathbf{I}_{3\times 3}, \quad \text{where} \quad \mu = \begin{cases} 1 & \text{Isometric} \\ \mu \in \mathbb{C}_1 \text{ unknown} & \text{Conformal.} \end{cases} \tag{1}$$

We define $\varphi = (\psi \circ \Delta) \in \mathbb{C}_1$ which maps points $\mathbf{p} \in \mathcal{P}$ in the 2D template to points $Q = \varphi(\mathbf{p}) \in \mathcal{S}$ in the deformed surface. Using the properties of the conformal flattening and the conditions (1), the following differential properties are derived:

$$J_\varphi^\top J_\varphi = \lambda \mathbf{I}_{2\times 2}, \quad \text{where} \quad \lambda = \begin{cases} \sqrt{\det(J_\Delta^\top J_\Delta)} & \text{Isometric} \\ \lambda \in \mathbb{C}_1 \text{ unknown} & \text{Conformal.} \end{cases} \tag{2}$$

Reconstruction with Isometric and Conformal Deformations

The reconstruction problem with Isometric or Conformal deformations is equivalent to find a solution to the following system of PDEs:

$$\begin{cases} \chi_x + s^2 \left(\dfrac{\partial \varphi_z}{\partial x}\right)^2 = \chi_y + s^2 \left(\dfrac{\partial \varphi_z}{\partial y}\right)^2 = \lambda s^2 \\[2ex] \xi + s^2 \left(\dfrac{\partial \varphi_z}{\partial x}\right)\left(\dfrac{\partial \varphi_z}{\partial y}\right) = 0, \end{cases} \tag{3}$$
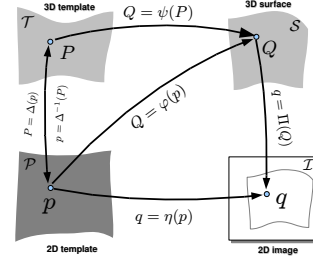

Figure 1: General model of template-based 3D reconstruction

where λ is given by equation (2) and:

$$\chi_x = \frac{\partial \eta}{\partial x}^\top \frac{\partial \eta}{\partial x} \qquad \chi_y = \frac{\partial \eta}{\partial y}^\top \frac{\partial \eta}{\partial y} \qquad \xi = \frac{\partial \eta}{\partial x}^\top \frac{\partial \eta}{\partial y}. \tag{4}$$

The isowarp

We define the isowarp as a function η that represents the weak-perspective projection of an isometric deforming surface. If η is an isowarp, system (3) has an exact solution and directly solves the isometric reconstruction problem. An isowarp η satisfies the following system of PDEs:

$$(s^2\lambda - \chi_x(s^2\lambda - \chi_y) - \xi^2 = 0$$

$$(s^2\lambda - \chi_y)\left(s^2\frac{\partial \lambda}{\partial y} - \frac{\partial \chi_x}{\partial y}\right)^2 - (s^2\lambda - \chi_x)\left(s^2\frac{\partial \lambda}{\partial x} - \frac{\partial \chi_y}{\partial x}\right)^2 = 0, \tag{5}$$

where $s^2 \in \mathbb{R}^+$

The conwarp

We define the conwarp as a function η that represents the weak-perspective projection of a conformal deforming surface. A conwarp satisfies the following PDE:

$$(s^2\lambda - \chi_y)\left(s^2\frac{\partial \lambda}{\partial y} - \frac{\partial \chi_x}{\partial y}\right)^2 - (s^2\lambda - \chi_x)\left(s^2\frac{\partial \lambda}{\partial x} - \frac{\partial \chi_y}{\partial x}\right)^2 = 0, \tag{6}$$

where $\lambda = \frac{1}{2s^2}\left(\chi_x + \chi_y + \sqrt{(\chi_x - \chi_y)^2 + \xi^2}\right)$

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