Seam Carving with Forward Gradient Difference Maps

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ABSTRACT

We propose a new energy function for seam carving based on forward gradient differences to preserve regular structures in images. The energy function measures the curvature inconsistency between the pixels that become adjacent after seam removal, and involves the difference of gradient orientation and magnitude of the pixels. Our objective is to minimize the differences induced by the removed seam, and the optimization is performed by dynamic programming based on multiple cumulative energy maps, each of which corresponds to the seam pattern associated with a pixel. The proposed technique preserves straight lines and regular shapes better than the original and improved seam carving, and can be easily combined with other types of energy functions within the seam carving framework. We evaluated the performance of our algorithm by comparing with the original and improved seam carving algorithms using public data.

Categories and Subject Descriptors
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1. INTRODUCTION

Image retargeting is a technique to transform source image to target, which typically has a different size and/or aspect ratio, with constraint of content preservation. A lot of image retargeting algorithms have been proposed in recent years [2, 6, 9, 12, 14, 15], where pixels in uniform or uninformative regions within original image are typically removed or original image is reduced to a smaller one using a warping function \(^1\); the common objective of both approaches is to preserve visual information of important regions and minimize the distortions of contents in the input image.

A primitive image retargeting algorithm is cropping [13], where saliency map is utilized to find the region of interest and the cropping area is determined by a simple criterion based on spatial arrangement of the salient region. This method is effective to identify an important area in a given image, but is not straightforward to handle the images that salient regions are spatially apart from each other. Another class of image retargeting technique is warping [7, 5, 14, 15], which performs nonlinear transformation to obtain the resized images. In this technique, important areas are transformed conservatively while unimportant regions undergo relatively more significant warping. Therefore, unimportant areas may be overly distorted and even disappear depending on the amount of transformation, which makes the image unnatural.

Seam carving [2] removes paths of pixels with low energy repetitively, and minimizes the loss of important information in an image without severe distortion. The main advantage of the algorithm is simplicity and efficiency, but the performance of seam carving depends heavily on energy function. While the performance of the original seam carving is typically good for the images with large textureless regions (or regions with non-regular textures such as tree, bush, and water), it frequently fails to preserve structural information such as straight lines and regular shapes since it only employs the magnitude of gradient in its energy function. To overcome the limitation, several variations using different energy functions have been studied; they are based on visual saliency [1], diffusion [3], visibility map [8], and so on. On the other hand, some algorithms introduced forward energy and attempted to reduce the visual artifacts caused by seam removal [3, 11].

Hybrid approaches combines multiple operators to retarget images. Dong et al. [4] proposed a combination of seam carving and scaling, and Rubinstein et al. [12] incorporated cropping in addition to the two operators. These methods adjust the balance of multiple operators for image resizing, and inhibit excessive seam removals destroying regular structures by adding structure preserving operators such as scaling and cropping.

We propose a new energy function based on the forward gradient differences for seam carving, and the main purpose of the new energy function is to preserve regular structures, especially, straight lines and smooth curves. This idea is similar to [11] in the sense that we utilize a forward energy function to reduce artifacts after seam removal. Contrary to [11], we focus on gradient differences in both orientation and magnitude before and after the removal of a seam, and find the optimal seam by dynamic program-

\(^1\)In general, image retargeting means both extension and reduction of an image, but our primary concern is the reduction in this paper; the extension of an image can be performed in most image retargeting algorithms with the almost identical framework.
ming based on three cumulative cost maps corresponding to three potential seam directions. We employ 9 operators to compute the forward gradient differences in both \(x\) and \(y\) direction, and each operator represents a seam pattern associated with a pixel.

The rest of the paper is organized as follows. Section 2 briefly reviews the original seam carving algorithm. Section 3 presents our new energy function and optimization technique based on dynamic programming. Experimental results are presented in Section 4.

2. SEAM CARVING

The seam carving [2] is a simple contents-aware image resizing technique, which is composed of the following three steps:

- The energy of each pixel is calculated based on the magnitude of gradient.
- An optimal seam with the lowest energy is identified and removed by dynamic programming.
- The energy map in the next iteration is recalculated based on the new image after seam removal.

These steps are repeated until the desired image size is achieved.

Energy function is a formula to evaluate the amount of information in each pixel, and the pixels with low energy values are typically removed by the seam carving algorithm. The original seam carving algorithm measures the magnitude of gradient in each pixel \((x, y)\) of image \(I\) for the energy as

\[
e(x, y) = \left| \frac{\partial I(x, y)}{\partial x} \right| + \left| \frac{\partial I(x, y)}{\partial y} \right|.
\]

The seam is a vertical or horizontal path of pixels. A vertical seam is an 8-connected path of pixels in an \(m \times n\) image from top to bottom, which is formally defined by

\[
s^u = \{s^n_u\}_{i=1}^n = \{(u(i), i)\}_{i=1}^n, \text{ s.t. } \forall i, |u(i) - u(i - 1)| \leq 1,
\]

where \(u\) is a mapping as \(u : [1, \ldots, n] \rightarrow [1, \ldots, m]\). Note that there is only one pixel in each row in the vertical seam. Therefore, the pixels in a vertical seam are given by

\[
I_{s^u} = \{(I(s^n_u), i)\}_{i=1}^n = \{(I(u(i), i))\}_{i=1}^n.
\]

The vertical seam with the lowest energy, \(s^*_u\), is defined by

\[
s^*_u = \arg \min_{s} E(s^u) = \arg \min_{s} \sum_{i=1}^{n} e(I(s^n_u)).
\]

The optimal seam is found by dynamic programming. The process of dynamic programming starts from the construction cumulative cost map for vertical seam, \(C^u\), as the following equations:

\[
C^u(i, j) = e(i, j) + \min\left\{\begin{array}{l} C^u(i - 1, j - 1) \\ C^u(i, j - 1) \\ C^u(i + 1, j - 1) \end{array}\right\},
\]

where the vertical cumulative matrix is computed from bottom to top and \(C^u(1, 1)\) corresponds to the bottom-left corner of image. After computing the cumulative cost map, the minimum value of the top row in \(C^u\) is selected as the starting point to find the vertical seam by backtracking. The horizontal seam carving can be implemented by the same way as the vertical one.

In the original seam carving, the energy function is based only on the magnitude of gradient and the optimal seam may include foreground pixels if the regions in the objects are flat and homogeneous. Also, the objective function focuses on the pixel-level local information, and the structural information in the scene such as straight lines and smooth curves can be lost or severely distorted.

3. OUR ALGORITHM

We describe an improved seam carving algorithm based on a new energy function using forward gradient differences. The advantage of our algorithm is the capability to preserve regular structures in an image, which is obtained by the estimation of the energy based on the differences of gradient orientation and magnitude after seam removal. In our algorithm, multiple energy maps, each of which corresponds to each local pattern of a seam, are utilized, and the optimal seam in each step is obtained by dynamic programming with the multiple energy maps.

3.1 Forward gradient operators

Since our energy function considers the smoothness of gradient at the location of seam removal, the energy function needs to involve the patterns of seam. There are 9 different patterns associated with each pixel, which are denoted by the shaded blocks in Fig. 1. For each seam pattern, we defined the forward gradient operator in \(x\) and \(y\) direction, which are equivalent to the ordinary Sobel operators for the image with a relevant seam removed. The operators in both directions are illustrated in Fig. 1. Note that the operators in Fig. 1 are for vertical seam extraction; transposed operators are used to find horizontal seams.

3.2 Energy function

Suppose that \(I_t (t = 0, \ldots, T)\) is the image after the removal of the \(t\)-th seam, where \(I_0\) denotes the original image. The gradient orientation and magnitude maps of \(I_t\), which are denoted by \(A_t\) and \(M_t\), respectively, are given by

\[
A_t(x, y) = \tan^{-1}\left(\frac{\partial I_t(x, y)}{\partial y} \right) \quad \text{and} \quad M_t(x, y) = \sqrt{\left(\frac{\partial I_t(x, y)}{\partial x}\right)^2 + \left(\frac{\partial I_t(x, y)}{\partial y}\right)^2},
\]

where \(|A_t(x, y)| \leq \frac{\pi}{2} \text{ for every } (x, y)\). Note that the gradient in \(x\) and \(y\) direction is computed with Sobel operators. The forward gradient orientation and magnitude maps with respect to the \(k\)-th forward gradient operators are computed by the convolution of the input image and the \(k\)-th forward gradient operators. The maps are denoted by \(A_{t+k}^k\) and \(M_{t+k}^k\), respectively, which are formally defined as follows:

\[
A_{t+k}^k(x, y) = \tan^{-1}\left(\frac{(I_t \circ h_k^y)(x, y)}{(I_t \circ h_k^x)(x, y)}\right) \quad \text{and} \quad M_{t+k}^k(x, y) = \sqrt{\left((I_t \circ h_k^x)(x, y)\right)^2 + \left((I_t \circ h_k^y)(x, y)\right)^2},
\]

where \(h_k^x\) and \(h_k^y\) are the \(k\)-th forward gradient operator in \(x\) and \(y\) direction, respectively, as illustrated in Fig. 1, and \(\circ\) is the convolution operator. Note that there are 9 different local patterns of seams as in Fig. 1 and that 9 forward gradient orientation and magnitude maps, \(A_{t+k}^k\) and \(M_{t+k}^k\), \(k = 1, \ldots, 9\), should be created. Each map contains the gradient information of the image after a particular type of seam is removed from the current image.

We evaluate how the energies for gradient orientation and magnitude in each pixel are updated by removing a seam with the following equations,

\[
\Delta A_t^k(x, y) = |A_{t+k}^k(x, y) - A_0(x', y')| \quad \text{and} \quad \Delta M_t^k(x, y) = |M_{t+k}^k(x, y) - M_0(x', y')|,
\]

where \((x', y')\) denotes the corresponding location to \((x, y)\) in the initial image. We simply compute the differences in orientation and magnitude of gradient in each pixel for all the \(k\) seam patterns.
and right—and the same number of cumulative cost matrices are maps generated by Eq. (12) to find the optimal seam in each step.

The original seam carving, but our algorithm uses 9 different energy results are presented in Fig. 4, and our algorithm is still better than elongated areas.


\begin{equation}
C_k(x, y) = \Delta \mathbf{A}_k(x, y) \cdot \Delta \mathbf{M}_k(x, y) \cdot \mathbf{M}_k(x, y).
\end{equation}

The final energy function at each location with respect to the \( k \)-th forward gradient operators to remove \( t + 1 \)st seams is given by

\[
e^t_l(x, y) = \Delta \mathbf{A}_t(x, y) \cdot \Delta \mathbf{M}_t(x, y) \cdot \mathbf{M}_t(x, y) \cdot \mathbf{M}_t(x, y).
\]

We prefer the seam preserving the orientation and magnitude of the gradient after removal, which is achieved by the first and second term, respectively, in Eq. (12). The third term is required to avoid removing strong edges; the optimal seam is sometimes aligned exactly with an edge, which may disappear unexpectedly without the consideration of gradient magnitude.

3.3 Optimization

The optimization is performed by dynamic programming as in the original seam carving, but our algorithm uses 9 different energy maps generated by Eq. (12) to find the optimal seam in each step.

For each pixel, the available directions of a seam are 3—left, up, and right—and the same number of cumulative cost matrices are required in our dynamic programming. Let \( C_l, C_u \) and \( C_r \) be cumulative matrices to deliver cost in the left, up and right direction, respectively. \(^2\) The initial conditions of the cumulative matrices in each direction are given by

\[
\begin{align*}
C_l(x, 1) &= \min\{c^3_l(x, y), c^7_l(x, y), c^9_l(x, y)\} \\
C_u(x, 1) &= \min\{c^4_u(x, y), c^5_u(x, y), c^6_u(x, y)\} \\
C_r(x, 1) &= \min\{c^3_r(x, y), c^7_r(x, y), c^9_r(x, y)\}.
\end{align*}
\]

The cumulative cost matrices are filled based on the directions of the incoming and outgoing seams associated with each pixel from bottom to top. The recurrence relations of the three matrices are as follows:

\[
\begin{align*}
C_l(x, y) &= \min\left\{C_l(x - 1, y - 1) + e^1_l(x, y), C_u(x, y - 1) + e^2_l(x, y), C_r(x - 1, y) + e^3_l(x, y)\right\} \\
C_u(x, y) &= \min\left\{C_u(x - 1, y - 1) + e^4_u(x, y), C_l(x, y - 1) + e^5_u(x, y), C_r(x, y - 1) + e^6_u(x, y)\right\} \\
C_r(x, y) &= \min\left\{C_r(x - 1, y - 1) + e^3_r(x, y), C_u(x, y - 1) + e^4_r(x, y), C_l(x, y - 1) + e^5_r(x, y)\right\}
\end{align*}
\]

In the above equations, the matrices in the left hand side denote the direction of the seam heading in the next row and the matrices in the right hand side mean the direction of the seam incoming from the previous row. A single cumulative cost matrix is used in the original seam carving, but our algorithm employ three matrices,

\[\text{which represent the direction of the seam in each pixel. Note that the three cumulative cost matrices have the same values at the last row. In other words,}\]

\[
\forall x (x = 1, \ldots, m) \text{ since} \]

\[
\begin{align*}
e^1_l(x, n) &= e^1_l(x, n) = e^1_l(x, n) \\
e^4_l(x, n) &= e^4_l(x, n) = e^4_l(x, n) \\
e^3_l(x, n) &= e^3_l(x, n) = e^3_l(x, n).
\end{align*}
\]

Because of this property, we can start from any matrix to find the optimal seam by backtracing. We simply select the pixel with minimum cost in any of the cumulative matrices, and traverse the path backward, from top to bottom, by which the seam with the minimum energy is identified in each step. The optimization by dynamic programming based on multiple cumulative cost matrices is efficient although it is slower than the original seam carving.

4. EXPERIMENT

Our seam carving algorithm based on the forward gradient difference maps is evaluated with a public dataset. All the data used in our experiments are downloaded from the RetargetMe [10] website (http://people.csail.mit.edu/mrub/retargetme).

We first tested the characteristics of the seams found by our new energy function, which are illustrated in Fig. 2. Note that the seams are extracted from relatively flat and textureless regions, and tend to cross edges in a perpendicular direction or lie along vertically elongated areas.

In our experiments, we compared our algorithm with two different types of seam-carving techniques—the original seam carving [2] and the improved seam-carving [11]. As illustrated in Fig. 3, [2] fails to preserve the regular structures in images such as straight lines, while our algorithm generates visually more comfortable outputs. The performance of our algorithm with respect to [11] was tested because our algorithm is similar to [11]. The comparative results are presented in Fig. 4, and our algorithm is still better than [11] in preserving regular structures such as lines and curves.
5. CONCLUSION

We proposed a novel energy function for seam carving, which is based on the forward gradient difference in both orientation and magnitude. The new energy function constructs multiple cumulative energy maps, each of which corresponds to the local direction of a seam, and a dynamic programming based on the multiple energy maps is employed for optimization. We tested our algorithm in a public dataset and obtained qualitatively better results than the original and improved seam carving algorithms in the preservation of regular structures. The proposed algorithm can be easily combined with other types of energy functions within the seam carving framework to enhance image retargeting results.

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7. REFERENCES