

Optimization design of orthogonal filter banks for image coding via multi-objective genetic algorithm

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Abstract

In this paper, we propose an optimisation method based on a multi-objective Genetic Algorithm (GA) for the design of orthogonal filter banks in an image coding scheme. A parameterization is used to achieve perfect reconstruction orthonormal FIR filter banks with a first order regularity. We search the optimal parameter set according to the coding gain, frequency selectivity and the group delay characteristics. Particularly, to design near linear phase filter banks, the group delay flatness in the filter pass-band is introduced as an objective to be minimised. We formulate the optimization problem as multi-objective and we use the Non-dominated Sorting Genetic Algorithm approach (NSGAI) to solve this problem by searching solutions that achieve the best compromise between the different objectives criteria, these solutions are known as Pareto Optimal Solutions. From experimental results, our new optimal filter banks are shown to outperform significantly the Daubechies orthogonal filter banks for the majority of test images.

Keywords

orthogonal filter banks, multiobjective optimization, wavelet image coding.

1 Introduction

In a wavelet image coding scheme, the choice of filter banks is a key problem which affects both the system and the performances of compression. Generally, optimal filter banks are selected for image coding systems from a library of filter banks designed for signal processing purposes using some metrics related to such systems [1]. While the most suitable filter banks for image coding belong to the biorthogonal class of filter banks, the orthogonal Daubechies filter banks belong to the class of wavelet filter banks used most often in image coding applications. Orthogonal filter banks have some interesting properties, such as energy preservation, that are often used in the design of quantization procedures and bit allocations algorithms.

These properties make the orthogonal filter banks very attractive, but in the case of wavelet FIR filter banks, orthogonality is non-compatible with phase-linearity, which seems to be relevant too. A solution of this drawback is to design orthogonal filter banks as symmetric as possible. This can be achieved by the optimization of the group delay flatness in the orthogonal filter banks. To introduce flexibility in the design, the group delay flatness can be considered only in passbands instead of the full band since phase distortion is not important in stopbands. This allows the investigation of a larger region of solution space where better performing filter banks can be obtained.

In this work, we are interested in the design of orthogonal filter banks (FBs) for image coding. The optimal FBs design approach presented in this work is a continuation and improvement of earlier work in the field. Perfect reconstruction (PR) filter banks with a first order regularity requirement are constructed using the parameterization proposed in the reference [3]. In addition, the design of optimal PR-orthogonal FBs for image coding should consider several criteria of practical significance related to such application, namely: energy compaction capabilities or coding gain, frequency selectivity, and phase linearity.

In fact, the design problem requires simultaneous optimization of three objective functions with different individual optima. Practically, there it is no possible solution that satisfies maximally all the objective functions. To solve this problem, a multi-objective genetic approach called NSGAI [6] is used to find *Pareto optimal solutions* that make all possible tradeoffs among competing objectives through evolution.

This article is organized as follows. In section 2, we define the design criteria of filter banks in image coding applications. The optimization problem formulation and the multi-objective GA used for designing filter banks are presented in section 3. In section 4 we evaluate the performances of compression of the optimized filter bank for a set of test images. Finally, section 5 concludes the paper with a summary of our work.

2 Design criteria

2.1 PR condition

Figure 1 shows a two channel filter bank where $H_0(z)$ and $H_1(z)$ are the analysis low and high-pass filters, respectively, and $G_0(z)$ and $G_1(z)$ are the synthesis filters.

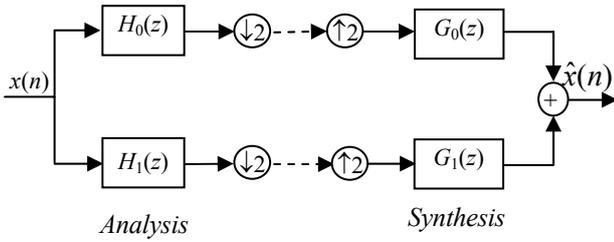


Figure 1 - Two channel filter bank

To design a PR orthogonal filter bank, $\hat{x}(n) = x(n)$, the analysis and synthesis filters have to satisfy the following equations:

$$\begin{aligned} h_1(n) &= (-1)^{(n+1)} h_0(L-n-1) \\ g_0(n) &= h_0(L-n-1) \\ g_1(n) &= h_1(L-n-1) \end{aligned} \quad n = 0, \dots, L-1 \quad (1)$$

Where L is the length of the filters. The above equations therefore establish an important property of an orthogonal system: In an orthogonal two-channel filter bank, all filters are obtained from a single prototype filter.

The lattice parameterization described by Vaidyanathan [2] offers the opportunity to design orthogonal wavelet filters via unconstrained parameters. Imposing the even length filters with $L=2N$ and using polyphase decomposition, $H_0(z)$ and $H_1(z)$ in the analysis filter bank are then given by:

$$\begin{aligned} \begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} &= \begin{bmatrix} \sum_{n=0}^{N-1} h_0(2n)z^{-2n} + z^{-1} \sum_{n=0}^{N-1} h_0(2n+1)z^{-2n} \\ \sum_{n=0}^{N-1} h_1(2n)z^{-2n} + z^{-1} \sum_{n=0}^{N-1} h_1(2n+1)z^{-2n} \end{bmatrix} \\ &= \underbrace{\begin{pmatrix} H_{00}(z^{-2}) & H_{01}(z^{-2}) \\ H_{10}(z^{-2}) & H_{11}(z^{-2}) \end{pmatrix}}_{\mathbf{H}_p(z^{-2})} \begin{pmatrix} 1 \\ z^{-1} \end{pmatrix} \end{aligned} \quad (2)$$

A paraunitary filter bank with FIR filters of length $L=2N$ can be reached by the following parameterization [2, 3]:

$$\begin{aligned} \mathbf{H}_p(z^{-2}) &= \left[\prod_{k=1}^{N-1} \{ \mathbf{I} + (z^{-2} - 1) \mathbf{v}_k \mathbf{v}_k^T \} \right] \mathbf{V}_0 \\ &= \left[\prod_{k=1}^{N-1} \mathbf{V}_k(z^{-2}) \right] \mathbf{V}_0 \end{aligned} \quad (3)$$

Where

$$\mathbf{v}_k = \begin{bmatrix} \cos \theta_k \\ \sin \theta_k \end{bmatrix}, \mathbf{V}_0 = \begin{bmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{bmatrix}$$

The polyphase matrix can be expressed as [3]:

$$\mathbf{H}_p(z^{-2}) = \mathbf{X}_k(z^{-2}) \mathbf{V}_0 \quad (4)$$

Where

$$\mathbf{X}_k(z^{-2}) = \begin{bmatrix} \sum_{m=1}^{K+1} c_m^K z^{-2(m-1)} & \sum_{m=1}^{K+1} d_m^K z^{-2(m-1)} \\ \sum_{m=1}^{K+1} f_m^K z^{-2(m-1)} & \sum_{m=1}^{K+1} g_m^K z^{-2(m-1)} \end{bmatrix}$$

The values of the polynomial coefficients are calculated iteratively using the following equation:

$$\begin{cases} c_m^K = \cos^2 \theta_k c_m^{K-1} + \sin^2 \theta_k c_m^{K-1} + \sin \theta_k \cos \theta_k (f_m^{K-1} - f_m^{K-1}) \\ d_m^K = \cos^2 \theta_k d_m^{K-1} + \sin^2 \theta_k d_m^{K-1} + \sin \theta_k \cos \theta_k (g_m^{K-1} - g_m^{K-1}) \\ f_m^K = \cos^2 \theta_k f_m^{K-1} + \sin^2 \theta_k f_m^{K-1} + \sin \theta_k \cos \theta_k (c_m^{K-1} - c_m^{K-1}) \\ g_m^K = \cos^2 \theta_k g_m^{K-1} + \sin^2 \theta_k g_m^{K-1} + \sin \theta_k \cos \theta_k (d_m^{K-1} - d_m^{K-1}) \end{cases} \quad (5)$$

Where:

$$\begin{cases} c_1^1 = \sin^2 \theta_1, c_2^1 = \cos^2 \theta_1, d_1^1 = -\sin \theta_1 \cos \theta_1, d_2^1 = \sin \theta_1 \cos \theta_1 \\ f_1^1 = -\sin \theta_1 \cos \theta_1, f_2^1 = \sin \theta_1 \cos \theta_1, g_1^1 = \cos^2 \theta_1, g_2^1 = \sin^2 \theta_1 \\ c_m^{K-1} = d_m^{K-1} = f_m^{K-1} = g_m^{K-1} = 0, \quad \text{for } m < 2 \\ c_m^{K-1} = d_m^{K-1} = f_m^{K-1} = g_m^{K-1} = 0, \quad \text{for } m > K \end{cases}$$

and $K = 2, \dots, N-1$ and $m = 1, \dots, K+1$.

Therefore eq. (4) implies that $L=2N$ sequence $h_i(n)$ which satisfies the PR condition is parametrized by N free parameters $\{\theta_k\}$.

The regularity constraint is the crucial distinction between wavelet transforms and perfect reconstruction filter banks. It is related to the number of zeros of $H_0(z)$ at $z = -1$ [1]. In image coding, some regularity is desired and higher regularity does not appear to yield significant improvements for coding quality [1]. The following relation provides a constraint on θ_0 which guarantees the presence of one a zero at $z = -1$ in $H_0(z)$:

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} \Big|_{z=-1} = \begin{bmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ \beta \end{bmatrix}, \quad (6)$$

Where β is a constant. A possible solution is $\theta_0 = -\pi/4$, for a given $\beta = -\sqrt{2}$ [3]. This relation reduces the number of free parameters by one. It is shown in [3] that the first order regularity condition, yields:

$$\begin{aligned} h_0(2n) &= \frac{1}{\sqrt{2}} (c_{n+1}^{N-1} - d_{n+1}^{N-1}), \\ h_0(2n+1) &= \frac{1}{\sqrt{2}} (c_{n+1}^{N-1} + d_{n+1}^{N-1}), \end{aligned} \quad n = 0, \dots, N-1 \quad (7)$$

So, the filter $H_0(z)$ with first order regularity can be designed via the following stages:

1. Generate $N-1$ angles $\{\theta_k, k = 1, \dots, N-1\}$,
2. Compute the polynomial coefficients $c_m^{N-1}, d_m^{N-1}, f_m^{N-1}, g_m^{N-1}$ using eq.(5).
3. Compute the coefficients of $H_0(z)$ using eq.(7).

After computing the coefficients of the low pass filter $H_0(z)$, all the remaining filters of filter banks are deduced from this filter by using eq.(1).

2.2 Coding gain

The Coding Gain (CG) measures the energy concentration capability of filter banks and is a widely accepted general measure of coding performance [4, 5]. By modelling a natural image as a one-dimensional Markovian source with a correlation factor ρ and by assuming uncorrelated quantization errors, Katto and Yasuda [5] derived a filter dependent coding gain:

$$CG(\rho) = 10 \log_{10} \left(\prod_{k=0}^{M-1} (A_k B_k)^{-\frac{1}{\alpha_k}} \right) \quad (8)$$

Where: $A_k = \sum_i \sum_j h'_k(i) h'_k(j) \rho^{|j-i|}$, $B_k = \sum_i g'_k(i)^2$

For orthogonal filters we have $B_k = \sum_i g'_k(i)^2 = 1$. Consequently, we obtain:

$$CG(\rho) = 10 \log_{10} \left(\prod_{k=0}^{M-1} (A_k)^{-\frac{1}{\alpha_k}} \right) \quad (9)$$

Where h'_k and g'_k are the k^{th} analysis and synthesis filter of the M channel nonuniform filter bank equivalent to the N_d ($M = N_d + 1$) level tree structured filter bank respectively (e.g., figure 2), α_k is the corresponding subsampling ratio, and ρ is the correlation factor.

In addition, we have:

$$H'_i(z) = \begin{cases} H_1(z) & \text{if } i = 0 \\ H_1(z^{\alpha_i/2}) \prod_{k=0}^{i-1} H_0(z^{\alpha_k/2}) & \text{if } 1 \leq i \leq M-2 \\ \prod_{k=0}^i H_0(z^{\alpha_k/2}) & \text{if } i = M-1 \end{cases} \quad (10)$$

Where:

$$\alpha_k = \begin{cases} 2^{i+1} & \text{if } 0 \leq i \leq M-2 \\ 2^i & \text{if } i = M-1 \end{cases}$$

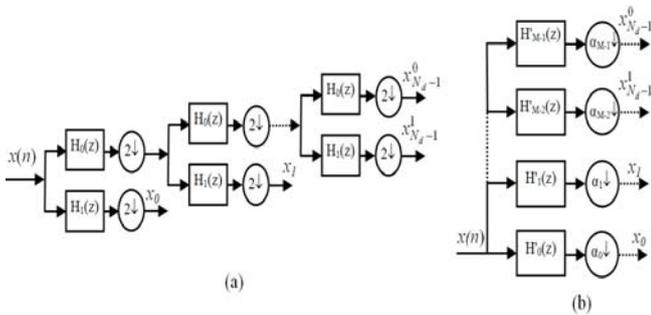


Figure 2 - M -band filter banks ($M = N_d + 1$): (a) N_d level tree structured filter bank (b) the equivalent M channel nonuniform filter bank.

In our work, a correlation factor $\rho = 0.95$ is used and a six-level dyadic tree-structured subband decomposition is adopted here since experimentally this number of levels provides the best performance for a wide range of image types and is often used in the evaluation of wavelet image coding algorithms.

2.3 Measure of Symmetry

As mentioned, linear-phase and PR are mutually exclusive in the orthonormal filter bank design. But severe phase nonlinearities are known to create undesired degradations in image and video applications. Therefore, a measure that indicates the level of nonlinearity in the filter-phase response is included as a parameter in the optimal filter design.

Nonlinear-phase is related to the asymmetry of the impulse response. As a measure of filter symmetry, we use the group delay flatness. In the case of symmetry, the group delay is simply a constant. Otherwise, the mean squared Error of the group delay can be used to evaluate group delay flatness:

$$E_{gd} = \frac{2}{\pi} \int_0^{\pi/2} (\tau(\omega) - \tau_0)^2 d\omega \quad (11)$$

Where $\tau(\omega)$ is the group delay of the lowpass filter, defined as $-d\theta/d\omega$, with $\theta(\omega)$ being the phase of $H_0(\omega)$, the discrete time Fourier transform of h_0 . Also, τ_0 is the average group delay over the interval $[0, \pi/2]$. The integral is evaluated only over the passband interval since the group delay behavior over the stop band is of little importance. Obviously, given a number of coefficients, the more group delay flatness we have, the closer E_{gd} is to zero. Eq. (11) can be approximated as a summation:

$$E_{gd} = \frac{1}{K} \sum_{n=0}^{K-1} \left(\tau\left(\frac{n\pi}{2K}\right) - \tau_0 \right)^2 \quad (12)$$

Where we have K points uniformly distributed over $[0, \pi/2]$ and τ_0 is the mean value defined as $\tau_0 = \frac{1}{K} \sum_{n=0}^{K-1} \tau\left(\frac{n\pi}{2K}\right)$.

2.4 Frequency selectivity

The advantage of frequency selectivity in image coding is that the coarse quantization into unimportant subbands is less expensive, since errors will be confined to the band where they occur. A generally used criterion in subband coding theory is to make the two analysis filters approximate the ideal low-pass and high-pass filters, respectively.

To quantify the frequency selectivity of filter, we define the filter bank Transition Band Energy (TBE) as:

$$TBE = \int_0^{\pi} |H_0(\omega) H_1(\omega)|^2 d\omega \quad (13)$$

Where $H_i(e^{j\omega})$ is the frequency response of filters $H_i(z)$.

Using parseval's relation we obtain:

$$TBE = \pi \sum_{n=0}^{L-1} |h_0(n) * h_1(n)|^2 \quad (14)$$

This function is a measure of the deviation from an ideal lowpass and highpass filter pair [4]. If the overlap between the filters is zero, which is only possible for ideal filters, then the TBE is zero.

3 A multi-objective Genetic Algorithm for the design of filter banks

A general multi-objective optimization problem consists of a number of objectives to be optimized simultaneously and is associated with a number of inequality and equality constraints. Such a problem can be stated as follows:

$$\begin{aligned} & \text{minimise (or maximise)} f_i(x) \quad i = 0, \dots, N \\ & \text{subject to: } \begin{cases} T_j(x), & j = 1, \dots, N \\ S_k(x) \leq 0, & k = 1, \dots, N \end{cases} \end{aligned} \quad (15)$$

The f_i are the objective functions, N is the number of objectives, x is a vector whose p components are the design or decision variables, T_j and S_k are the constraint functions. Generally, the objectives under consideration conflict with each other, and optimizing a particular solution with respect to a single objective can degrade results with respect to the other objectives. Generally, it is difficult to combine the above objectives both to formulate a single objective function. A reasonable solution to a multi-objective problem is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution. Such solutions form a trade-off space and are known as the *Pareto optimal solutions*.

In a minimization problem, for M objective functions, a feasible solution \mathbf{X} is dominated by feasible solution \mathbf{Y} if:

$$\begin{aligned} \forall i = 1, \dots, M \quad & f_i(X) \geq f_i(Y) \text{ and} \\ \exists j = 1, \dots, M \quad & f_j(X) > f_j(Y) \end{aligned} \quad (16)$$

A solution is said to be Pareto optimal if it is not dominated by any other solution in the solution space. Furthermore, that solution is said to be *nondominated solution*. The set of all feasible nondominated solutions are called the *Pareto optimal set*. The corresponding objective function in the objective space constitutes a *Pareto front*.

In our design, we search the angles $\{\theta_k, k = 1, \dots, N - 1\}$ that maximise the coding gain and minimise the two individual objective functions, namely, the transition band energy and the group delay error. Our multi-objective optimization problem is formulated as follows:

$$\min_{\theta_k} (Objf_1, Objf_2, Objf_3), \text{ and } \begin{cases} Objf_1 = TBE \\ Objf_2 = \frac{1}{CG_{dB}^2} \\ Objf_3 = E_{gd} \end{cases} \quad (17)$$

In our case, A set of angles are treated as chromosomes which are optimized by a multi-objective genetic algorithm to obtain a set of filter banks that minimise all the prescribed objective functions with a satisfactory level. This algorithm is based on the NSGA II approach [6]. This algorithm is based on the Non-dominated Sorting Genetic Algorithm known as NSGAI. The particularity of this approach is that, in addition to the Pareto nondomination principle used in the conventional multi-objective genetic algorithms, a crowding operator is used to maintain diversity in the population and an elitism mechanism is introduced to prevent the loss of better solutions once they are found during the genetic evolution.

4 Experimental results

Before evaluation of our results, we give some important parameters used in our simulation work. In our genetic algorithm, real valued angles are used for chromosome construction. A simulated binary crossover operator with a distribution index of 40 and a probability of 0.9 is used [7]. Also, a polynomial mutation of distribution index 20 and of probability of 0.01 is applied [7]. The population size n_{pop} is set to 100 and chromosomes of the initial population are obtained by randomly generating angles between $[0, 2\pi]$. The maximum generation (G_{max}) is set to 500 generations.

In this work, our design method is applied to the design of an orthogonal filter bank of length 8 (e.g., $N=4$). Figure 3 shows a 3D scatter plot of a set of Pareto optimal solutions obtained for the optimized filter bank. The NSGAI algorithm produces a set of Pareto optimal solutions which are considered as candidates of the final decision making solution. To select a final solution θ^{opt} from this set of solutions, we use the following relation [8]:

$$\theta^{opt} = \arg \min_{\theta^j} \max_{i=\{1,2,3\}} \omega_i (Objf_i(\theta^j) - \bar{f}_i) \quad (18)$$

We set:

$$\omega_i = \frac{1}{\frac{1}{n_{pop}} \sum_{j=1}^{n_{pop}} Objf_i(\theta^j)}$$

Where $\theta^j, j = 1 \dots n_{pop}$ is the set of generated Pareto optimal solutions and $\bar{\mathbf{f}} = \{\bar{f}_1, \bar{f}_2, \bar{f}_3\}$ is the aspiration level. Note that the θ^{opt} can be approximately given as the one which gives the closest Pareto optimal solution to the given aspiration level. By choosing the aspiration level (0.05, 0.010, 0.01), we have selected the optimal set $\theta^{opt} = \{4.14392765, 2.75161017, 5.1338803\}$.

In our work, it is very interesting to design filter banks that can perform well for any test image. Therefore, a series of images which have different frequency contents has been selected for evaluation of filter banks coding performance.

In experimentation, the SPIHT codec [10] is employed for evaluation of the performance of the optimized filter banks. Six levels of wavelet decomposition have been employed.

Images	Filter banks	Compression ratios				
		128:1	64 : 1	32 :1	16 :1	8:1
Lena	Db4	27,51	30,26	33,24	36,59	40,18
	Opt4	27,53	30,34	33,34	36,66	40,21
Barbara	Db4	23,44	24,48	27,41	30,81	35,87
	Opt4	23,41	24,49	27,58	31,09	36,21
Goldhill	Db4	26,20	28,03	29,59	32,36	35,97
	Opt4	26,14	28,01	29,71	32,46	36,05
Finger	Db4	20,18	21,95	23,58	26,58	30,01
	Opt4	20,19	22,05	23,79	27,02	30,70
Bike	Db4	18,19	19,35	21,95	25,44	30,46
	Opt4	18,22	19,37	22,02	25,56	30,56
Target	Db4	16,49	17,71	21,93	27,79	35,76
	Opt4	16,69	18,02	22,65	28,10	36,93
Straw	Db4	18,73	20,02	21,45	24,00	27,24
	Opt4	18,71	19,99	21,48	24,16	27,47
Aerial	Db4	21,19	23,06	24,82	27,74	31,89
	Opt4	21,31	23,25	25,37	27,97	32,13

Table 1- PSNR (in dB) versus compression ratio for our optimized filter bank “Opt4” and the Daubechies orthogonal filter bank “Db4”.

To qualify the effectiveness of our design method, we compare the performance of our optimized filter bank labeled “Opt4” with that of the Daubechies popular orthogonal filter bank “Db4”. Table 1 presents the PSNR values generated by these two filter banks compared for different compression ratio where the best result is highlighted in each case.

For the eight test images, our optimal filter bank Opt4 outperforms the Db4 filter bank in the majority of cases and the greatest degree of improvement is occurred for compression ratios less than 64:1. For some images such as “Finger” and “Target” the improvement is very significant, e.g., 0.69 and 1.17dB. In the cases where the optimized filter bank is worse, the degradation is very small. Statistically, we have obtained in average an improvement of 0,20dB.

To justify the improvement of performance obtained with our filter banks, we compare their characteristics to those of the filter bank db4 in Table 2. It is clear that our optimized filter bank provide a significant improvement in term of all considered criteria.

To assess the compression performance of our optimized filter bank, we present an example of a test image which has been compressed at rate where the distortion becomes visible. Figure 4 shows the image “Barbara” compressed at a compression ratio 32:1 for the Db4 filter and Opt4 filters. For this compression rate, all compressed images

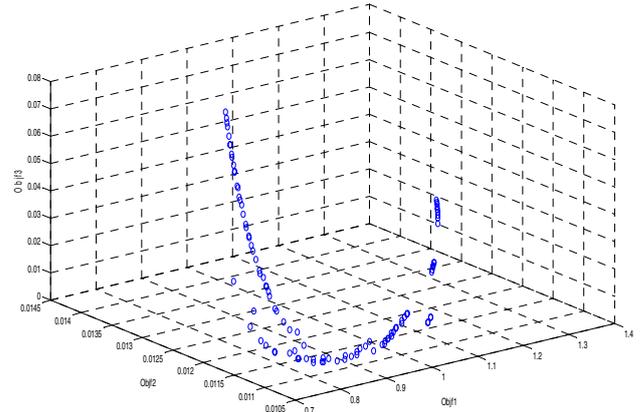


Figure 3 - 3-D scatter plot of the Pareto optimal solutions obtained by using the NSGAI.

	TBE	CG _{dB}	E _{gd}
db4	0.8	9.664	0.143
FB _{opt}	0.681	9.738	0.0289

Table 2- Comparison between Characteristics of filter banks

suffer from ringing artifacts. One can clearly see these artifacts surrounding the contours of the images. The image compressed using the optimized filter Opt4 has less ringing effect in comparison to that of the Db4 filter which can be observed in the zoomed portions of such images.

5 Conclusion

In this work, a method based on genetic algorithms was presented for the optimization of filter banks for a lossy image coding scheme. The problem of optimization is to find a set of filter bank coefficients that satisfy multiple objectives which are used to measure the effectiveness of filter banks in such scheme. The problem was formulated as multi-objective and solved using the NSGAI algorithm. From simulation results, it is shown that our optimized filter banks outperform significantly the Db4 filter bank for the majority of tested cases.

By sacrificing the high degree of regularity of orthogonal wavelet filter banks, superior image compression performance can be achieved with filters that exhibit good energy compaction and near linear phase characteristics.

While our interest in this work is in orthogonal filter banks, because of the availability of orthonormality, regularity and perfect reconstruction, the importance of orthonormality for image compression should be given greater consideration for biorthogonal filter banks.



(a)



(b)



(a) 4×zoom



(b) 4×zoom

Figure 4 - Image Barbara compressed at 32:1 with SPIHT: (a) Db4 (b) Opt4

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