Reconstructed Image by hypergeometric function Of Legendre

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Abstract— **This paper introduces a new set of orthogonal moments function hypergeometric based on the discrete Legendre polynomials. The Legendre moments can be effectively used as pattern features in the analysis of two-dimensional images. The implementation of moments proposed in this paper does not involve any numerical approximation, since the basis set is orthogonal in the discrete domain of the image coordinate space. The paper presents the experimental results of Legendre moments with hypergeometric function and demonstrates their feature representation capability using the method of image reconstruction.**

Index Terms: **Discrete orthogonal systems, Image feature representation. Orthogonal moments Legendre.**

I. INTRODUCTION

THE function moments have been used as shape descriptors in a variety of applications in image analysis, like visual pattern recognition ,[1], [4], object classification [7], template matching [6], edge detection [5], pose estimation [13], robot vision[12], data compression [9] . In all these applications, geometric moments and their extensions in the form of radial and complex moments have played important roles in characterizing the image shape, and in extracting features that are invariant with respect to image plane transformations. Teague [18] introduced moments with orthogonal basis functions, with the additional property of minimal information redundancy in a moment set. In this class, Zernike moments have been extensively researched in the recent past, and several new techniques have emerged involving orthogonal moment based feature detectors [10], [14][19].In the following, we consider some of the major problems that are commonly encountered while implementing moment functions.

A. Two-dimensional Numerical Approximation of Continuous Integrals

The general two-dimensional (2-D) moment definition using a moment weighting kernel (also known as the basis function) $\psi_{pq}(x,y)$, and an image intensity function $f(x,y)$ is given as

$$\Psi_{pq} = \int_x\int_y \psi_{pq}(x,y) f(x,y)\,dxdy.$$

p, q=0,1,2… (1)

The integrals in the above equation are usually approximated by discrete summations, and this process not only leads to numerical errors in the computed moments, but also severely affects the analytical properties which they were intended to satisfy, such as invariance, orthogonal etc.

B. Coordinate Space Transformation

Orthogonal basis functions do not have the aforesaid problem of large dynamic range variation, but they generally have a domain which is completely different from the image coordinate space. For example, the Legendre and Tchebichef polynomials are valid only in the range [-1,1],while the Zernike radial polynomials are defined inside the unit circle. The Laguerre polynomials are defined in the range $[0,\infty[$, [2], [10], [11], [18].

The above problems motivate us to consider using discrete orthogonal polynomials as the basis set, and to define the corresponding moments directly on the image coordinate space. Since the implementation of discrete orthogonal moments does not involve any numerical approximations, the basis functions will exactly satisfy the orthogonal property, and thus yield a superior image reconstruction. Consider a discrete orthogonal system { $f_n(i)$ }, where $a \le i \le b$.

The orthogonal property in the above domain can then be written as

$$\sum_{i=a}^{i=b} \omega(i) f_m(i) f_n(i) = \rho(n,a,b)\delta_{mn}. \quad (2)$$

Where $\omega(i)$ is the weighting function (also called the jump function), and $\rho(.)$ is the squared norm.

II. DISCRETE ORTHOGONAL MOMENTS

The following well-known theorem on orthogonal functions provides the mathematical basis for arriving at a definition for discrete orthogonal moments of an image intensity distribution $f(x,y)$: If $\{P_n(x)\}$ is a set of discrete orthogonal polynomials with unit weight, satisfying the condition

$$\sum_{x=0}^{N-1} P_n(x) P_m(x) = \rho(n,N)\delta_{mn},$$

$$0 \le m,n \le N-1 \qquad (3)$$

Then any bounded function f(x, y), $0 \le \{x,y\} \le N-1$,has the following polynomial representation in terms of the functions $P_n(x)$

$$f(x,y) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \lambda_{mn} P_m(x) P_n(y), \qquad (4)$$

Where the coefficients moments λ_{pq} are given by

$$\lambda_{pq} = \frac{1}{\rho(p,N)\rho(q,N)} \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} P\left(\frac{2i-N+1}{N-1}\right) \times P\left(\frac{2j-N+1}{N-1}\right) f(i,j),$$

p, q=0, 1,2,…N-1 (5)

The above theorem can be generalized for orthogonal polynomials with weight $\omega(x)$,by replacing each orthogonal function $P_n(x)$ by the function $P_n(x)\sqrt{\omega(x)}$, in (3)–(5).

Equation (15) is easily obtained by substituting for $f(x,y)$ using (4) in the expression: $\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} P_p(x) P_q(y) f(x,y)$, and noting that

$$\rho(p,N) = \sum_{p=0}^{N-1} \left\{ P_p\left(\frac{2p-N+1}{N-1}\right)\right\}^2, \qquad (6)$$

Conversely, (4) follows from (5). In the context of image moments, it means that if we define a discrete orthogonal moment function as in (5) with $\{P_n(x)\}$ as the basis set, then the image may be reconstructed from the moments, using (4) as the inverse moment transform. The moment definition as given in (5) completely eliminates the need for any approximation of continuous integrals, and does not require coordinate space transformations.

We propose a modified version of Legendre polynomials as a convenient set of discrete orthogonal basis functions with unit weight, for defining moments of the above type.

The discrete generalized Legendre polynomial [1], [3],[8] can be defined as

$$P_n(x) = {}_2F_1\left(-n, n+1; 1; \left(\frac{1-x}{2}\right)\right)$$

$$x \in [-1,1], \qquad \text{n,} = 0, 1, 2, \ldots \text{N - 1}, \qquad (7)$$

With ${}_2F_1(.)$is the generalized hyper geometric function

$${}_2F_1(a,b;c;z) = \sum_k^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!} \qquad (8)$$

From the relation (8), I give a new definition of the polyno-ials of Lgendre in the discrete base of the hypegeometric functions.

$$P_n(x) = \sum_{k=0}^{N-1} (-1)^k \frac{(k+n)!}{(k!)^2 (n-k)!} \left(\frac{1-x}{2}\right)^k, \qquad (9)$$

The Legendre polynomials satisfy the property of orthogonal (3), with

$$\rho(n,N) = \sum_{l=0}^{2n}\sum_{j=0}^{l} \frac{(-1)^l}{(N-1)^l} \times C^n{}_{n+(l-j)} \times C_n^{(l-j)} \sum_{i=0}^{N-1} (i)^l, \qquad (10)$$

And the following recurrence formula holds:

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x),$$

n=2, 3, 4…., (11)

The equation (5) also leads to the following inverse moments transform:

$$f(x,y) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} \lambda_{mn} P_n(x) P_m(y)$$

x, y=0,1,…N-1. (12)

EXPERIMENTAL RESULTS

This section presents the test data and results used to validate the theoretical framework presented above, and also to establish the feature representation capability of Legendre moments with hypergeometric function through image reconstruction. A multi- level real image of "LENA" (see Fig 2) on a 100x100 pixel.

The sequence of reconstructed images, as the maximum order of moments used in the reconstruction is varied from one to 40, is shown in (Fig.1).We used the following formula to characterize the MSE between an input A multi level real image $f(x, y)$, and the reconstructed image $\hat{f}(x, y)$.

$$MSE = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left|f(x,y) - \hat{f}(x,y)\right|^2 .$$

(13)

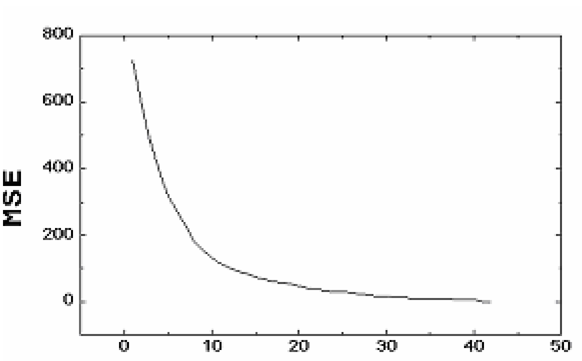

Fig.1. Order of rebuilding

III. CONCLUSION

A new set of discrete orthogonal moment features based on Legendre polynomial with the hypergeometric function has been proposed in this paper. The basis functions are orthogonal in the domain of the image coordinate space, and this feature completely eliminates the need for any discrete approximation in their numerical implementation.

Experimental results conclusively prove the effectiveness of Legendre moments with the hypregemetric function as the feature descriptors.

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Various orders of rebuilding of image “LENA" by Legendre.

Fig.2